The Review of Symbolic Logic
Volume 17, Number 2, June 2024

ON QUANTIFICATION AND EXTENSIONALITY

KAI F. WEHMEIER
Department of Logic and Philosophy of Science, University of California, Irvine

Abstract. We investigate whether ordinary quantification over objects is an extensional phenomenon, or rather creates non-extensional contexts; each claim having been propounded by prominent philosophers. It turns out that the question only makes sense relative to a background theory of syntax and semantics (here called a grammar) that goes well beyond the inductive definition of formulas and the recursive definition of satisfaction. Two schemas for building quantificational grammars are developed, one that invariably constructs extensional grammars (in which quantification, in particular, thus behaves extensionally) and another that only generates non-extensional grammars (and in which quantification is responsible for the failure of extensionality). We then ask whether there are reasons to favor one of these grammar schemas over the other, and examine an argument according to which the proper formalization of deictic utterances requires adoption of non-extensional grammars.

§1. Introduction. Is quantification over objects a non-extensional operation? In other words (speaking somewhat loosely for now), does the presence of ordinary quantification in an interpreted formal language *ipso facto* prevent it from having the property that, whenever in any complex well-formed expression one replaces some proper subexpression by another, coextensive one, the resulting expression is coextensive with the original one?

There are no doubt many readers to whom this question seems silly. Some will want to point out that Quine[1] gave a categorical *affirmative* answer long ago:

> It is well known, and easily seen, that the conspicuously limited means which we have lately allowed ourselves for compounding sentences—viz., 'and', 'not', and quantifiers—are capable of generating only extensional contexts. Quine [14, p. 12]

Others, coming from a different tradition, will likewise dismiss the question, but rather because they take it to have been answered *in the negative*. Nathan Salmon[2] may be cited as an example:

> The nonextensionality of a quantifier phrase is a surprising but trivial consequence of the way the quantifier works with a variable. (Salmon [17, p. 416])

Received: August 20, 2021.
2020 *Mathematics Subject Classification*: Primary 03A05, 03A10, 03B10, 03B65, 03B80.
Key words and phrases: quantification, extensionality, deixis.

1 See also Carnap [2, p. 51].
2 Nuel Belnap [1, p. 6] expresses a similar view, and indeed the point was already made some 30 years earlier, if rather in passing, by David Lewis [9, p. 252].

doi:10.1017/S1755020324000066

The very fact that these eminent authors provide *different* answers to our question suggests that it might be worth taking a closer look. This is what we propose to do in this essay.[3]

That contradictory answers should have been given is the more surprising in light of the fact that the two parties appear to agree on what we might call the generating syntax (i.e., the inductive definition of the well-formed expressions, particularly of quantified formulas), the ingredients for the semantic evaluation of well-formed expressions (to wit, model-theoretic structures and variable assignments), and the recursive (Tarskian [23]) steps to be taken in evaluating expressions relative to structures and variable assignments, as well as, presumably, on what it takes for well-formed expressions to be coextensive:[4]

- Two sentences are coextensive if and only if they are both true or both false.
- Two singular terms are coextensive if and only if they designate the same object.
- Two predicative expressions ("predicators," as we will call them for brevity) are coextensive if and only if they are true of the same arrangements of arguments (such as *n*-tuples or sequences of objects) and false of the same arrangements of arguments.

We will see that what allows for reasonable disagreement as to the extensionality of quantification is the fact that the generating syntax and the recursive procedure underlying Tarskian satisfaction do not by themselves suffice to constrain coextensiveness sufficiently in order to answer the question. To do that, we need a *grammar*.[5] As we shall use the term here, a grammar G comprises all of the following:

(G1) A *lexicon* Λ and a definition of the set WF^{G} of G-*well-formed expressions* (G-expressions for short) over Λ, together with a relation of G-*constituency* on WF^{G}.

(G2) A specification of the *logical space* relevant to G, i.e., the class $\mathsf{Mod}(\mathsf{G})$ of G-*models* in which G-expressions are to be evaluated.

(G3) An *ontology*, i.e., a function $\mathsf{Obj}^{\mathsf{G}}$ that assigns to each G-model $\mathbf{M}$ the set $\mathsf{Obj}^{\mathsf{G}}_{\mathbf{M}}$ of G-*objects*$_{\mathbf{M}}$.

(G4) For each G-model $\mathbf{M}$, a partition of the G-expressions into three G-*categories*$_{\mathbf{M}}$: G-*sentences*$_{\mathbf{M}}$, G-*terms*$_{\mathbf{M}}$, and G-*predicators*$_{\mathbf{M}}$.

3 In my [25] I argue that quantifiers are extensional operators, but the argument is based on finding "Fregean" formalisms for quantification (as presented in [24]) superior to "Tarskian" ones (but see Pickel and Rabern [13] for an opposing view). In that paper, I grant that within standard Tarskian languages, quantification is non-extensional; my point there is just that we shouldn't put much stock in this diagnosis. Here I tackle the extensionality question directly with respect to standard ("Tarskian") languages for quantification. Our discussion might also be usefully compared to that by Glanzberg and King [5], who react to Rabern's [16] diagnosis that the treatment of quantification in Kaplan [8] is not compositional (see Pickel and Rabern [12] for a critique).

4 We will make these criteria more precise below.

5 We intend "grammar" to be understood in the broad sense of a complete description of a language system, including in particular both syntax and semantics (as e.g. in "Montague grammar"), not the narrow one of a description of a syntax only (as in "context-free grammar").

(G5) A G-*truth operator* $\mathsf{true}^{\mathsf{G}}$ that maps each G-model $\mathbf{M}$ to a function $\mathsf{true}^{\mathsf{G}}_{\mathbf{M}}$ from the set of G-sentences$_{\mathbf{M}}$ into $\{0, 1\}$, where a G-sentence$_{\mathbf{M}}$ ϕ is said to be G-*true in* $\mathbf{M}$ just in case $\mathsf{true}^{\mathsf{G}}_{\mathbf{M}}(\phi) = 1$, and G-*false in* $\mathbf{M}$ if $\mathsf{true}^{\mathsf{G}}_{\mathbf{M}}(\phi) = 0$.

(G6) A G-*designation operator* $\mathsf{des}^{\mathsf{G}}$ that maps each G-model $\mathbf{M}$ to a function $\mathsf{des}^{\mathsf{G}}_{\mathbf{M}}$ all of whose admissible arguments are G-terms$_{\mathbf{M}}$ and whose values are G-objects$_{\mathbf{M}}$ (where, if $\mathsf{des}^{\mathsf{G}}_{\mathbf{M}}(t) = m$, we say that t G-*designates* m in $\mathbf{M}$).

(G7) A G-*predication operator* $\mathsf{true\text{-}of}^{\mathsf{G}}$ that assigns to each G-model $\mathbf{M}$ a function $\mathsf{true\text{-}of}^{\mathsf{G}}_{\mathbf{M}}$ which maps each G-predicator$_{\mathbf{M}}$ Π to a function $\mathsf{true\text{-}of}^{\mathsf{G}}_{\mathbf{M}}(\Pi)$ whose arguments are arrangements of members of the type hierarchy over $\mathsf{Obj}^{\mathsf{G}}_{\mathbf{M}}$ and whose values are in $\{0, 1\}$. If $\mathsf{true\text{-}of}^{\mathsf{G}}_{\mathbf{M}}(\Pi)(\rho) = 1$ (respectively, $= 0$), we say that Π *is* G-*true of* ρ *in* $\mathbf{M}$ (respectively, G-*false of* ρ *in* $\mathbf{M}$).

Each of our two camps assumes, more or less tacitly, a background grammar; and what makes it possible for them to arrive at different answers to the extensionality question is that their grammars differ. In our reconstruction, the camps make the same choices, we shall see, with respect to (G1) and, but for a technicality, (G3). Their model spaces differ in a crucial way that we will explain in due course; but despite this difference in model spaces, the recursive semantics they pick to drive their respective items (G4) through (G7) are essentially the same; it's just that they let the semantics drive these items in different ways, resulting in different outcomes with respect to (G4) through (G7) and hence in (closely related but nevertheless) different grammars.

Now suppose G is a grammar. For each G-model $\mathbf{M}$, we define the G-*coextensiveness relation in* $\mathbf{M}$, $\mathsf{coext}^{\mathsf{G}}_{\mathbf{M}}$, as follows:

(DC1) G-sentences$_{\mathbf{M}}$ ϕ and ψ are G-*coextensive in* $\mathbf{M}$ if and only if $\mathsf{true}^{\mathsf{G}}_{\mathbf{M}}(\phi) = \mathsf{true}^{\mathsf{G}}_{\mathbf{M}}(\psi)$; i.e., just in case they are either both G-true in $\mathbf{M}$ or both G-false in $\mathbf{M}$.

(DC2) G-terms$_{\mathbf{M}}$ t_0, t_1 are G-*coextensive in* $\mathbf{M}$ if and only if they G-designate the same G-object$_{\mathbf{M}}$ in $\mathbf{M}$, i.e., if and only if t_0 and t_1 are both in the domain of $\mathsf{des}^{\mathsf{G}}_{\mathbf{M}}$ and $\mathsf{des}^{\mathsf{G}}_{\mathbf{M}}(t_0) = \mathsf{des}^{\mathsf{G}}_{\mathbf{M}}(t_1)$.

(DC3) G-predicators$_{\mathbf{M}}$ Π_0, Π_1 are G-*coextensive in* $\mathbf{M}$ if and only if they are G-true in $\mathbf{M}$ and G-false in $\mathbf{M}$, respectively, of the same arrangements of arguments; in other words, just in case $\mathsf{true\text{-}of}^{\mathsf{G}}_{\mathbf{M}}(\Pi_0)$ and $\mathsf{true\text{-}of}^{\mathsf{G}}_{\mathbf{M}}(\Pi_1)$ are the same function.

(DC4) The G-*coextensiveness relation in* $\mathbf{M}$ is the union of the three category-specific G-coextensiveness relations in $\mathfrak{M}$ defined in (DC1) through (DC3).

Let G be a grammar and suppose that α_0, α_1, β are well-formed G-expressions. Suppose further that o is an occurrence, as a constituent, of α_0 in β. Then we say that the replacement of α_0 by α_1 at the occurrence o in β is G-*legitimate* if the result of this replacement is also a well-formed G-expression.

Now let $\mathbf{M}$ be a G-model. The G-expressions α_0 and α_1 are G-*equicategorial in* $\mathbf{M}$ if they are both G-sentences$_{\mathbf{M}}$, both G-terms$_{\mathbf{M}}$, or both G-predicators$_{\mathbf{M}}$. A complex G-expression β is said to be G-*extensional in* $\mathbf{M}$ if, whenever (i) α_0 and α_1 are well-formed G-expressions that are G-equicategorial and G-coextensive in $\mathbf{M}$, (ii) o is an occurrence, as a constituent, of α_0 in β, and (iii) the replacement of the occurrence o of α_0 with α_1 in β is G-legitimate, then the result of this replacement is G-coextensive in $\mathbf{M}$ with β. The *grammar* G is *extensional in* $\mathbf{M}$ just in case every complex G-expression is G-extensional in $\mathbf{M}$. Finally, G is *extensional* if G is extensional in every G-model $\mathbf{M}$.

Here's what we will do in the rest of this essay: The grammar-component (G1) common to both parties, as well as the recursive semantics upon which they agree, are laid out in Section 2. In Section 3 we define a "Quinean" grammar schema Ext that constructs, from any given quantificational lexicon Λ, a grammar Ext_Λ, shown in Section 4 to be extensional. Against the background of Ext, the answer to the question whether quantification is extensional is affirmative. In Section 5 we define a "Salmonian" grammar schema NExt, whose grammars NExt_Λ can be construed as generalizations of their counterpart grammars Ext_Λ. It is shown that no NExt_Λ is extensional, and that it is indeed quantification that is responsible for the failure of extensionality in NExt-grammars. The question whether quantification is extensional thus has no absolute, grammar-independent answer. A related but different question, to wit, whether we should embrace grammars that make quantification extensional or rather those that make it non-extensional, however, might be answered by showing that one of Ext and NExt is preferable to the other. Considerations of simplicity would seem to favor Ext; on the other hand, the non-extensional grammars NExt_Λ are more general than their extensional counterparts Ext_Λ. But generality is not *per se* a theoretical virtue: Unless the added generality serves a previously unfulfilled purpose, there is no reason to embrace it. This makes us shift attention, in Section 6, from formal and conceptual considerations to applications. We consider an argument for the usefulness of NExt's additional generality that turns on the application of grammars to the analysis of natural-language phenomena, specifically deixis. This argument is shown to be without force in Section 7, where we explain that the expressive advantage of NExt_Λ disappears when that grammar is compared to an extensional grammar Ext_{Λ^+} over a suitably expanded lexicon $\Lambda^+ \supseteq \Lambda$. The concluding Section 8 pulls together the results and observations obtained.

§2. The common core.

2.1. Lexicon and well-formed expressions. By a *lexicon* we shall mean a triple $\Lambda = (\mathcal{C}_\iota, \mathcal{C}_\mu, \mathcal{P})$, where:

- $\mathcal{C}_\iota$ is a set of *constant symbols of type e*, or *individual constants* for short,
- $\mathcal{C}_\mu$ is a set of *constant symbols of type* $(et)t$, or *Montagovian constants* for short, and
- $\mathcal{P}$ is a set of *predicate symbols*, each $P \in \mathcal{P}$ having a finite *arity* $\#P \geq 1$.

$\mathcal{P}$ must contain at least one element. We let $\mathcal{P}_n$ be the set of n-ary members of $\mathcal{P}$. Either or both of the sets $\mathcal{C}_\iota$ and $\mathcal{C}_\mu$ may be empty. Individual constants are the constant symbols familiar from elementary logic; Montagovian constants are, as it were, individual constants lifted to quantifier type. Their availability will only become relevant in Section 7.

All grammars we shall consider use for the construction of their well-formed expressions, in addition to the specific vocabulary provided by its lexicon, the following symbols:

- the members of a fixed countably infinite set $\mathcal{V}$ of *(individual) variables* $\mathsf{v}_0, \mathsf{v}_1, \mathsf{v}_2, \ldots$;
- the *Boolean connectives* $\neg$ and $\wedge$; and
- the two *quantifier symbols* $\forall$ (universal) and $\exists$ (existential).

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

We make the usual disjointness assumptions, so that no connective may occur in a lexicon, no quantifier symbol may also be a variable, etc.[6]

The Λ-*formulas* are defined inductively as follows:

1. Whenever $P \in \mathcal{P}_n$ and $t_1, \dots, t_n \in \mathcal{V} \cup \mathcal{C}_\iota$, the result $Pt_1 \dots t_n$ of prefixing the string $t_1 \dots t_n$ with P is a Λ-formula.
2. Whenever ϕ and ψ are Λ-formulas, so are $\neg\phi$ and $(\phi \wedge \psi)$.
3. Whenever ϕ is a Λ-formula, $x \in \mathcal{V}$, and $C \in \mathcal{C}_\mu$, the result $Cx\phi$ of prefixing ϕ with Cx is a Λ-formula.[7]
4. Whenever ϕ is a Λ-formula, $x \in \mathcal{V}$, and $Q \in \{\forall, \exists\}$, the result $Qx\phi$ of prefixing ϕ with Qx is a Λ-formula.

The set of all Λ-formulas is Fml_Λ. By a *well-formed* Λ-*expression* we shall mean a member of $\mathcal{V} \cup \mathcal{C}_\iota \cup \mathcal{C}_\mu \cup \mathcal{P} \cup \{\forall, \exists\} \cup \mathsf{Fml}_\Lambda =: \mathsf{WF}(\Lambda)$.

Where α and β are well-formed Λ-expressions, we define the notion of an *immediate constituent occurrence* (ico) of α in β as follows. (1) If β is not a Λ-formula, there are no ico's of α in β. (2) If β is $Pt_1 \dots t_n$, then o is an ico of α in β if and only if either (a) α is P and o is $*t_1 \dots t_n$, or (b) for some $i \in \{1, \dots, n\}$, α is t_i and o is $Pt_1 \dots t_{i-1} * t_{i+1} \dots t_n$. (3) If β is $\neg\phi$, then o is an ico of α in β if and only if α is ϕ and o is $\neg *$. (4) If β is $(\phi \wedge \psi)$, then o is an ico of α in β if and only if either (a) α is ϕ and o is $(* \wedge \psi)$, or (b) α is ψ and o is $(\phi \wedge *)$. (5) If β is $\Sigma \mathsf{v}_i \phi$ with $\Sigma \in \mathcal{C}_\mu \cup \{\forall, \exists\}$, then o is an ico of α in β if and only if either (a) α is Σ and o is $*\mathsf{v}_i\phi$, or (b) α is ϕ and o is $\Sigma\mathsf{v}_i *$. We say that o is a *constituent occurrence* of α in β if and only if o is $\pi[\rho]$, where (a) ρ is an ico of α in a Λ-formula γ, and (b) β is $\pi[\gamma]$.[8] Where α_0, α_1, and β are Λ-expressions and o is a constituent occurrence of α_0 in β (so that β is $o[\alpha_0]$), α_1 is *legitimately substitutable* for α_0 at o in β if $o[\alpha_1]$ is a well-formed Λ-expression.

The *set of variables having a free occurrence in* a well-formed Λ-expression is defined as follows. For $x \in \mathcal{V}$, $\mathsf{FV}(x) = \{x\}$, and for $\alpha \in \mathcal{C}_\iota \cup \mathcal{C}_\mu \cup \mathcal{P} \cup \{\forall, \exists\}$, $\mathsf{FV}(\alpha) = \varnothing$. For Λ-formulas ϕ, the set $\mathsf{FV}(\phi)$ is defined recursively thus:

(1) $\mathsf{FV}(Pt_1 \dots t_n) = \bigcup\{\mathsf{FV}(t_i) : 1 \leq i \leq n\}$.
(2) $\mathsf{FV}(\neg\phi) = \mathsf{FV}(\phi)$.
(3) $\mathsf{FV}(\phi \wedge \psi) = \mathsf{FV}(\phi) \cup \mathsf{FV}(\psi)$.
(4) $\mathsf{FV}(Cx\phi) = \mathsf{FV}(Qx\phi) = \mathsf{FV}(\phi) \setminus \{x\}$ for $C \in \mathcal{C}_\mu$, $Q \in \{\forall, \exists\}$, and $x \in \mathcal{V}$.

2.2. Recursive semantics. A Λ-*structure* is a pair $\mathfrak{M} = (M, \mathfrak{I})$, where M, the *universe* of $\mathfrak{M}$, is a non-empty set and $\mathfrak{I}$ is a Λ-*interpretation* in M that maps each $\alpha \in \mathcal{C}_\iota \cup \mathcal{C}_\mu$ to an element $\mathfrak{I}(\alpha) \in M$ and each $P \in \mathcal{P}_n$ to a function $\mathfrak{I}(P) : M^n \to \{0, 1\}$ (i.e., to the characteristic function of a subset of M^n). The class of all Λ-structures is $\mathsf{Str}(\Lambda)$.[9]

---

[6] For technical reasons, we assume that the symbol $*$ occurs in no lexicon and is neither a variable, nor a connective, nor a quantifier symbol. We also assume the convention that whenever θ and ζ are strings of symbols, $\theta[\zeta]$ is the result of simultaneously replacing $*$ everywhere in θ by ζ. We call θ an *occurrence* of ζ in β if and only if θ contains $*$ exactly once and $\theta[\zeta]$ is β.

[7] As is obvious from a comparison of this clause with the next, Montagovian constants behave syntactically just like the quantifier symbols $\forall$ and $\exists$.

[8] Note that we're treating the connectives syncategorematically in not according them constituent status; a categorematic treatment could easily be given but would add nothing to the point at issue in this paper.

[9] Whenever "$\mathfrak{M}$" is used to denote a structure, it will be assumed that $\mathfrak{M}$ can also be denoted "$(M, \mathfrak{I})$".

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

A *variable assignment* (or just *assignment*) in a non-empty set M is a function from a finite subset of ω into M.[10] The empty function $\varnothing$ is thus an assignment in any M. The set of all assignments in M is $\mathcal{G}(M)$ or just $\mathcal{G}$ when no confusion is likely; where $\mathfrak{M}$ is a structure, $\mathcal{G}(\mathfrak{M})$ is $\mathcal{G}(M)$. When $g \in \mathcal{G}(M)$ and $i \in \mathsf{dom}(g)$, we sometimes write g_i instead of $g(i)$. For $g, h \in \mathcal{G}(M)$ and $i \in \omega$, we say that h is an *i-permissive extension* of g, in symbols $g \sqsubset_i h$, just in case (a) $\mathsf{dom}(h) = \mathsf{dom}(g) \cup \{i\}$ and (b) for all $j \in \mathsf{dom}(g) \setminus \{i\}$: $g(j) = h(j)$. For $g \in \mathcal{G}(M)$, $i \in \omega$, and $m \in M$, we write g_i^m for the *i*-permissive extension of g that maps i to m.

Where $g \in \mathcal{G}$, we say that a variable $\mathsf{v}_i \in \mathcal{V}$ is *covered by g*, or *g-covered*, if $i \in \mathsf{dom}(g)$. The set $\mathcal{V}_g$ of g-covered variables is $\{\mathsf{v}_i : i \in \mathsf{dom}(g)\}$; hence $\mathcal{V}_\varnothing = \varnothing$. A well-formed Λ-expression α is *g-closed* if $\mathsf{FV}(\alpha) \subseteq \mathcal{V}_g$, that is, if every variable that has a free occurrence in α is g-covered. Thus the members of $\mathcal{C}_\iota$, of $\mathcal{C}_\mu$, of $\mathcal{P}$ and of $\{\forall, \exists\}$ are g-closed for any g; in particular, they are $\varnothing$-closed or, as we will also say, closed *tout court*. A Λ-expression that is not g-closed is *g-open*; so the $\varnothing$-open expressions (i.e., the open expressions *tout court*) are just those expressions α with $\mathsf{FV}(\alpha) \neq \varnothing$.

Definition 1 (Tarskian satisfaction values). *For each* $\alpha \in \mathsf{WF}(\Lambda)$, *$\alpha$'s* Tarskian satisfaction value (*or just* Tarski-value) $[\![\alpha]\!]$ *is a function on* $\mathsf{Str}(\Lambda)$, *where for* $\mathfrak{M} \in \mathsf{Str}(\Lambda)$, $[\![\alpha]\!]_\mathfrak{M} := [\![\alpha]\!](\mathfrak{M})$ *is the* Tarski-value of α in $\mathfrak{M}$. *The definition proceeds by recursion on* α. *For each* α *and* $\mathfrak{M}$, $[\![\alpha]\!]_\mathfrak{M}$ *is to be a function whose domain* $\mathsf{dom}([\![\alpha]\!]_\mathfrak{M})$ *is a (not necessarily proper) subset of* $\mathcal{G}(\mathfrak{M})$.

1. *For variables* $\mathsf{v}_i \in \mathcal{V}$:
   (a) $\mathsf{dom}([\![\mathsf{v}_i]\!]_\mathfrak{M}) = \{g \in \mathcal{G} | i \in \mathsf{dom}(g)\}$,
   (b) *and where* $g \in \mathsf{dom}([\![\mathsf{v}_i]\!]_\mathfrak{M})$, $[\![\mathsf{v}_i]\!]_\mathfrak{M}(g) = g(i)$.
2. *For individual constants* $c \in \mathcal{C}_\iota$:
   (a) $\mathsf{dom}([\![c]\!]_\mathfrak{M}) = \mathcal{G}$,
   (b) *and where* $g \in \mathsf{dom}([\![c]\!]_\mathfrak{M})$, $[\![c]\!]_\mathfrak{M}(g) = \mathfrak{I}(c)$.
3. *For Montagovian constants* $C \in \mathcal{C}_\mu$:
   (a) $\mathsf{dom}([\![C]\!]_\mathfrak{M}) = \mathcal{G}$,
   (b) *and where* $g \in \mathsf{dom}([\![C]\!]_\mathfrak{M})$,
   $[\![C]\!]_\mathfrak{M}(g) : \mathsf{Pow}(M) \to \{0, 1\}$ *with* $[\![C]\!]_\mathfrak{M}(g)(X) = 1$ *iff* $\mathfrak{I}(C) \in X$.
4. *For predicate symbols* $P \in \mathcal{P}_n$:
   (a) $\mathsf{dom}([\![P]\!]_\mathfrak{M}) = \mathcal{G}$,
   (b) *and where* $g \in \mathsf{dom}([\![P]\!]_\mathfrak{M})$, $[\![P]\!]_\mathfrak{M}(g) = \mathfrak{I}(P)$.
5. *For quantifier symbols* $Q \in \{\forall, \exists\}$:
   (a) $\mathsf{dom}([\![Q]\!]_\mathfrak{M}) = \mathcal{G}$,
   (b) *and where* $g \in \mathcal{G}$, $[\![Q]\!]_\mathfrak{M}(g) : \mathsf{Pow}(M) \to \{0, 1\}$ *with* $[\![\forall]\!]_\mathfrak{M}(g)(X) = 1$ *iff* $X = M$ *and* $[\![\exists]\!]_\mathfrak{M}(g)(X) = 1$ *iff* $X \neq \varnothing$.
6. *For predications* $Pt_1 \dots t_n$:
   (a) $\mathsf{dom}([\![Pt_1 \dots t_n]\!]_\mathfrak{M}) = \bigcap_{i=1}^n \mathsf{dom}([\![t_i]\!]_\mathfrak{M})$,

[10] Logic textbooks typically define assignments to be total functions on ω (or equivalently, total functions on the set of all variables), but as far as we are aware, there is no satisfactory conceptual argument for choosing as assignments total functions on ω over partial functions on ω or, as we propose here (and as is customary in formal semantics, e.g., in [6]), *finite* partial functions on ω. In this way we will be able to obtain a *semantic* characterization of what are traditionally called *closed* formulas that is unavailable in the "total" setting, and we will also be able to cast the non-extensionalists' grammars as generalizations of the extensionalists'.

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

(b) *and for* $g \in \mathsf{dom}(\llbracket Pt_1 \dots t_n \rrbracket_{\mathfrak{M}})$:
$\llbracket Pt_1 \dots t_n \rrbracket_{\mathfrak{M}}(g) = \llbracket P \rrbracket_{\mathfrak{M}}(g)(\llbracket t_1 \rrbracket_{\mathfrak{M}}(g), \dots, \llbracket t_n \rrbracket_{\mathfrak{M}}(g))$.

7. *For negations* $\neg\phi$:
   (a) $\mathsf{dom}(\llbracket \neg\phi \rrbracket_{\mathfrak{M}}) = \mathsf{dom}(\llbracket \phi \rrbracket_{\mathfrak{M}})$,
   (b) *and where* $g \in \mathsf{dom}(\llbracket \neg\phi \rrbracket_{\mathfrak{M}})$: $\llbracket \neg\phi \rrbracket_{\mathfrak{M}}(g) = 1 - \llbracket \phi \rrbracket_{\mathfrak{M}}(g)$.
8. *For conjunctions* $(\phi \wedge \psi)$:
   (a) $\mathsf{dom}(\llbracket \phi \wedge \psi \rrbracket_{\mathfrak{M}}) = \mathsf{dom}(\llbracket \phi \rrbracket_{\mathfrak{M}}) \cap \mathsf{dom}(\llbracket \psi \rrbracket_{\mathfrak{M}})$,
   (b) *and where* $g \in \mathsf{dom}(\llbracket \phi \wedge \psi \rrbracket_{\mathfrak{M}})$:
   $\llbracket \phi \wedge \psi \rrbracket_{\mathfrak{M}}(g) = \min\{\llbracket \phi \rrbracket_{\mathfrak{M}}(g), \llbracket \psi \rrbracket_{\mathfrak{M}}(g)\}$.
9. *For formulas* $C\mathsf{v}_i\phi$ *with* $C \in \mathcal{C}_\mu$:
   (a) $\mathsf{dom}(\llbracket C\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}}) = \{g \in \mathcal{G} :$ *for all* $h \in \mathcal{G}$ *with* $g \sqsubset_i h, h \in \mathsf{dom}(\llbracket \phi \rrbracket_{\mathfrak{M}})\}$,
   (b) *and where* $g \in \mathsf{dom}(\llbracket C\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}})$:
   $\llbracket C\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}}(g) = \llbracket C \rrbracket_{\mathfrak{M}}(g)\left(\{m \in M \mid \llbracket \phi \rrbracket_{\mathfrak{M}}(g_i^m) = 1\}\right)$, *which equals* $\llbracket \phi \rrbracket_{\mathfrak{M}}(g_i^{\mathfrak{I}(C)})$.
10. *For formulas* $\forall\mathsf{v}_i\phi$:
   (a) $\mathsf{dom}(\llbracket \forall\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}}) = \{g \in \mathcal{G} :$ *for all* $h \in \mathcal{G}$ *with* $g \sqsubset_i h, h \in \mathsf{dom}(\llbracket \phi \rrbracket_{\mathfrak{M}})\}$,
   (b) *and where* $g \in \mathsf{dom}(\llbracket \forall\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}})$:
   $\llbracket \forall\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}}(g) = \llbracket \forall \rrbracket_{\mathfrak{M}}(g)\left(\{m \in M : \llbracket \phi \rrbracket_{\mathfrak{M}}(g_i^m) = 1\}\right)$.
11. *For formulas* $\exists\mathsf{v}_i\phi$:
   (a) $\mathsf{dom}(\llbracket \exists\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}}) = \{g \in \mathcal{G} :$ *for all* $h \in \mathcal{G}$ *with* $g \sqsubset_i h, h \in \mathsf{dom}(\llbracket \phi \rrbracket_{\mathfrak{M}})\}$,
   (b) *and where* $g \in \mathsf{dom}(\llbracket \exists\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}})$:
   $\llbracket \exists\mathsf{v}_i\phi \rrbracket_{\mathfrak{M}}(g) = \llbracket \exists \rrbracket_{\mathfrak{M}}(g)\left(\{m \in M : \llbracket \phi \rrbracket_{\mathfrak{M}}(g_i^m) = 1\}\right)$.

Note that, if $c \in \mathcal{C}_\iota$, $C \in \mathcal{C}_\mu$, and $\mathfrak{I}(c) = \mathfrak{I}(C)$, the formulas $Cx\phi$ and $\exists x(x = c \wedge \phi)$ have the same Tarski-value in $\mathfrak{M}$ (and so does $\phi_x[c]$, the result of replacing all free occurrences of x in ϕ by c). Montagovian constants are therefore quite innocuous with respect to first-order languages, since they are definable in terms of individual constants, identity, and the existential quantifier (conversely, of course, individual constants are reducible to Montagovian constants as well). As the identity of $\llbracket Cx\phi \rrbracket_{\mathfrak{M}}$, $\llbracket \exists x(x = c \wedge \phi) \rrbracket_{\mathfrak{M}}$, and $\llbracket \phi_x[c] \rrbracket_{\mathfrak{M}}$ suggests, the members of $\mathcal{C}_\mu$ are hybrids between individual constants and quantifiers: Their syntactic behavior is that of quantifiers, and they bind variables just like quantifiers do; on the other hand, their interpretations are single objects, just like those of individual constants, and like individual constants, they are essentially scopeless (cf. [27]).

One readily shows:

LEMMA 1. *Let* $\mathfrak{M}$ *be a* Λ*-structure and* α *a* Λ*-expression. Then*:

1. *For all* $g \in \mathcal{G}$, $g \in \mathsf{dom}(\llbracket \alpha \rrbracket_{\mathfrak{M}})$ *if and only if* α *is* g*-closed.*
2. *If* $g, h \in \mathsf{dom}(\llbracket \alpha \rrbracket_{\mathfrak{M}})$ *agree on the indices of all variables in* $\mathsf{FV}(\alpha)$, $\llbracket \alpha \rrbracket_{\mathfrak{M}}(g) = \llbracket \alpha \rrbracket_{\mathfrak{M}}(h)$.
3. *If* $g \in \mathsf{dom}(\llbracket \alpha \rrbracket_{\mathfrak{M}})$ *and* $h \in \mathcal{G}$ *extends* g, *then* $h \in \mathsf{dom}(\llbracket \alpha \rrbracket_{\mathfrak{M}})$ *and* $\llbracket \alpha \rrbracket_{\mathfrak{M}}(g) = \llbracket \alpha \rrbracket_{\mathfrak{M}}(h)$.

For $g \in \mathcal{G}$, we let $\mathcal{G}_g$ be the set $\{h \in \mathcal{G} \mid \mathsf{dom}(g) \cap \mathsf{dom}(h) = \varnothing\}$. Obviously $\mathcal{G}_\varnothing = \mathcal{G}$. If F is a function whose domain is a subset of $\mathcal{G}$, we let $F[g]$ be the function with domain $\{h \in \mathcal{G}_g : (g \cup h) \in \mathsf{dom}(F)\}$ that maps any assignment h in its domain to $F(g \cup h)$, i.e., $F[g](h) = F(g \cup h)$. Note that $F[\varnothing] = F$. Moreover, $\varnothing \in \mathsf{dom}(F[g])$ if and only if $g \in \mathsf{dom}(F)$. For any set X, let $\mathbf{1}_X$ be the total constant function on X with value 1 and let $\mathbf{0}_X$ be the total constant function on X with value 0.

We can then show, using Lemma 1:

Lemma 2. *Let $\mathfrak{M}$ be a Λ-structure, $g \in \mathcal{G}$, and ϕ a Λ-formula. The following are equivalent*:

(a) ϕ *is* g-*closed*,
(b) $[\![\phi]\!]_{\mathfrak{M}}[g] \in \{\mathbf{1}_{\mathcal{G}_g}, \mathbf{0}_{\mathcal{G}_g}\}$,
(c) $\mathsf{dom}([\![\phi]\!]_{\mathfrak{M}}[g]) = \mathcal{G}_g$,
(d) $\varnothing \in \mathsf{dom}([\![\phi]\!]_{\mathfrak{M}}[g])$.

As an immediate consequence of Lemma 2, we obtain:

Corollary 1. *Let $\mathfrak{M}$ be a Λ-structure and ϕ a Λ-formula. The following are equivalent*:

(a) ϕ *is closed*,
(b) $[\![\phi]\!]_{\mathfrak{M}} \in \{\mathbf{1}_{\mathcal{G}}, \mathbf{0}_{\mathcal{G}}\}$,
(c) $\mathsf{dom}([\![\phi]\!]_{\mathfrak{M}}) = \mathcal{G}$,
(d) $\varnothing \in \mathsf{dom}([\![\phi]\!]_{\mathfrak{M}})$.

§3. The grammar schema Ext.

In this section we define, for each lexicon Λ, a grammar Ext_Λ, by providing the necessary ingredients (G1) through (G7); in Section 4 we'll see that each Ext_Λ is extensional. The function $\Lambda \mapsto \mathsf{Ext}_\Lambda$ will be called the *grammar schema* Ext.

Let Λ be an arbitrary lexicon.

(G1) requires the specification of a lexicon; this is our chosen Λ. The set of well-formed expressions of Ext_Λ is $\mathsf{WF}(\Lambda)$ as defined in Section 2.

(G2) requires the specification of Ext_Λ's logical space or model class $\mathsf{Mod}(\mathsf{Ext}_\Lambda)$. We let the Ext_Λ-models be the Λ-structures as defined in Section 2; so $\mathsf{Mod}(\mathsf{Ext}_\Lambda)$ is $\mathsf{Str}(\Lambda)$.

(G3) requires the specification of a function $\mathsf{Obj}^{\mathsf{Ext}_\Lambda}$ that maps each Λ-structure $\mathfrak{M}$ to a set $\mathsf{Obj}^{\mathsf{Ext}_\Lambda}_{\mathfrak{M}}$ of Ext_Λ-objects$_{\mathfrak{M}}$. For each Λ-structure $\mathfrak{M} = (M, \mathfrak{I})$, let $\mathsf{Obj}^{\mathsf{Ext}_\Lambda}_{\mathfrak{M}}$ be M.

(G4) requires, for each Λ-structure $\mathfrak{M}$, a partition of Ext_Λ's well-formed expressions into Ext_Λ-sentences$_{\mathfrak{M}}$, Ext_Λ-terms$_{\mathfrak{M}}$, and Ext_Λ-predicators$_{\mathfrak{M}}$. In the present case, this partition will in fact be independent of the Ext_Λ-model $\mathfrak{M}$: The Ext_Λ-sentences$_{\mathfrak{M}}$ (in any $\mathfrak{M}$) are the closed Λ-formulas, the Ext_Λ-terms$_{\mathfrak{M}}$ (in any $\mathfrak{M}$) are the members of $\mathcal{V} \cup \mathcal{C}_\iota$, and the Ext_Λ-predicators$_{\mathfrak{M}}$ (in any $\mathfrak{M}$) are the members of $\mathcal{C}_\mu$, the members of $\mathcal{P}$, the quantifier symbols $\forall$ and $\exists$, and the open Λ-formulas.[11] (We will in future simply speak of Ext_Λ-sentences, Ext_Λ-terms, and Ext_Λ-predicators.)

(G5) requires the definition of a truth operator that maps each Λ-structure $\mathfrak{M}$ to a $\{0, 1\}$-valued function $\mathsf{true}^{\mathsf{Ext}_\Lambda}_{\mathfrak{M}}$ on the set of Ext_Λ-sentences (i.e., closed Λ-formulas). Let $\mathsf{true}^{\mathsf{Ext}_\Lambda}_{\mathfrak{M}}$ be the function that maps each closed Λ-formula ϕ to $[\![\phi]\!]_{\mathfrak{M}}(\varnothing)$. Equivalently, a closed Λ-formula ϕ is Ext_Λ-true in $\mathfrak{M}$ if and only if its Tarskian value in $\mathfrak{M}$ is $\mathbf{1}_{\mathcal{G}}$.

11 Note that the classification of open formulas as predicators is an integral part of Quine's conception of coextensiveness: "In defining coextensiveness, I lumped predicates, general terms, and open [formulas] together. They are what can be predicated of objects or sequences of objects, and in that capacity they all three come to the same thing" (Quine [15, p. 215]; Quine uses "open sentence" for what we call "open formula"). We will encounter this passage again, within in its larger context, in Section 4.

(G6) requires the definition of a designation operator that maps each Λ-structure $\mathfrak{M} = (M, \mathfrak{I})$ to a designation function $\mathsf{des}_{\mathfrak{M}}^{\mathsf{Ext}_\Lambda}$. Let $\mathsf{des}_{\mathfrak{M}}^{\mathsf{Ext}_\Lambda}$ be the set that contains, for each individual constant $c \in \mathcal{C}_\iota$, the pair $\langle c, \mathfrak{I}(c)\rangle$, and nothing else (so that, in particular, no variable designates anything in any Ext_Λ-model).

(G7), finally, requires the definition of a predication operator that maps each Λ-structure $\mathfrak{M}$ to a function $\mathsf{true\text{-}of}_{\mathfrak{M}}^{\mathsf{Ext}_\Lambda}$ on the set of Ext_Λ-predicators whose value for a given predicator Π is a $\{0, 1\}$-valued function on some set of arrangements of members of the type hierarchy over M. We let $\mathsf{true\text{-}of}_{\mathfrak{M}}^{\mathsf{Ext}_\Lambda}$ be the function mapping:

(i) each Montagovian constant $C \in \mathcal{C}_\mu$ to the function $[\![C]\!]_{\mathfrak{M}}(\varnothing)$, which is the characteristic function of the unary relation $\{X \subseteq M \mid \mathfrak{I}(C) \in X\}$ on $\mathsf{Pow}(M)$.

(ii) each $P \in \mathcal{P}_n$ to $[\![P]\!]_{\mathfrak{M}}(\varnothing)$, i.e., to $\mathfrak{I}(P)$, which by definition is the characteristic function of an n-ary relation over M.

(iii) each $Q \in \{\forall, \exists\}$ to the function $[\![Q]\!]_{\mathfrak{M}}(\varnothing)$, which in either case is the characteristic function of a unary relation on $\mathsf{Pow}(M)$, viz. of $\{M\}$ or of $\{X \subseteq M \mid X \neq \varnothing\}$, respectively.

(iv) each open Λ-formula ϕ to the function $[\![\phi]\!]_{\mathfrak{M}} : \mathsf{dom}([\![\phi]\!]_{\mathfrak{M}}) \to \{0, 1\}$, which is the characteristic function of the unary relation $\{g \in \mathsf{dom}([\![\phi]\!]_{\mathfrak{M}}) \mid [\![\phi]\!]_{\mathfrak{M}}(g) = 1\}$ on $\mathsf{dom}([\![\phi]\!]_{\mathfrak{M}})$.

With respect to (iv), we take members of $\mathcal{G}$ to be *bona fide* arrangements of objects, just as members of M^n are. In fact it would not be stretching terminology to call both types of arrangement *sequences*: A member $\{\langle 0, k\rangle, \langle 1, l\rangle, \langle 2, m\rangle\}$ of M^3, say, is a sequence $\langle k, l, m, \_, \_, \ldots\rangle$ all of whose gaps occur after an initial gapless segment, and is at the same time an assignment in M covering $\mathsf{v}_0, \mathsf{v}_1, \mathsf{v}_2$; whereas $g = \{\langle 1, k\rangle, \langle 3, l\rangle, \langle 6, m\rangle\}$ is a sequence $\langle \_, k, \_, l, \_, \_, m, \_, \_, \ldots\rangle$ with gaps between entries, and is at the same time an assignment in M, covering $\mathsf{v}_1, \mathsf{v}_3$, and v_6. We thus take both predicate symbols and open formulas to be true and false of arrangements of objects (which, incidentally, we'll need to do within the grammar schema NExt, too).

§4. Extensionality of Ext_Λ. Given Ext_Λ, conditions (DC1) through (DC4) determine the Ext_Λ-coextensiveness relation in each Λ-structure $\mathfrak{M}$. Applied to Ext_Λ, these conditions yield:

($\mathrm{DC1}_{\mathsf{Ext}_\Lambda}$) Closed Λ-formulas ϕ and ψ are Ext_Λ-coextensive in $\mathfrak{M}$ just in case they are either both Ext_Λ-true in $\mathfrak{M}$ or both Ext_Λ-false in $\mathfrak{M}$, i.e., just in case $[\![\phi]\!]_{\mathfrak{M}} = \mathbf{1}_{\mathcal{G}}$ and $[\![\psi]\!]_{\mathfrak{M}} = \mathbf{1}_{\mathcal{G}}$, or $[\![\phi]\!]_{\mathfrak{M}} = \mathbf{0}_{\mathcal{G}}$ and $[\![\psi]\!]_{\mathfrak{M}} = \mathbf{0}_{\mathcal{G}}$. It follows that closed Λ-formulas ϕ and ψ are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![\phi]\!]_{\mathfrak{M}} = [\![\psi]\!]_{\mathfrak{M}}$.

($\mathrm{DC2}_{\mathsf{Ext}_\Lambda}$) Members t_1 and t_2 of $\mathcal{V} \cup \mathcal{C}_\iota$ are Ext_Λ-coextensive in $\mathfrak{M}$ just in case they are both members of $\mathcal{C}_\iota$ and $\mathfrak{I}(t_1) = \mathfrak{I}(t_2)$. Equivalently, distinct members t_1, t_2 of $\mathcal{V} \cup \mathcal{C}_\iota$ are Ext_Λ-coextensive in $\mathfrak{M}$ just in case t_1 and t_2 are both members of $\mathcal{C}_\iota$ and $[\![t_1]\!]_{\mathfrak{M}} = [\![t_2]\!]_{\mathfrak{M}}$. It follows that distinct members t_1, t_2 of $\mathcal{V} \cup \mathcal{C}_\iota$ are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![t_1]\!]_{\mathfrak{M}} = [\![t_2]\!]_{\mathfrak{M}}$.[12]

---

[12] This is because no two variables have the same Tarski-value in any $\mathfrak{M}$ (the Tarski-values of distinct variables have distinct domains), and no variable has the same Tarski-value in any

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

($\mathrm{DC3}_{\mathsf{Ext}_\Lambda}$) Where each of α_0 and α_1 is either a member of $\mathcal{C}_\mu$, a member of $\mathcal{P}$, a member of $\{\forall, \exists\}$, or an open Λ-formula, α_0 and α_1 are Ext_Λ-coextensive in $\mathfrak{M}$ just in case they are Ext_Λ-true and Ext_Λ-false in $\mathfrak{M}$ of the same arrangements of members of the type hierarchy over M. It follows that:

(a) If $n \geq 1$ and α_0 and α_1 are both members of $\mathcal{P}_n$, they are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![\alpha_0]\!]_{\mathfrak{M}}(\varnothing) = [\![\alpha_1]\!]_{\mathfrak{M}}(\varnothing)$, i.e., just in case $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$ (since each $[\![\alpha_i]\!]_{\mathfrak{M}}$ is a total constant function on $\mathcal{G}$).

(b) If α_0 and α_1 are both members of $\mathcal{C}_\mu \cup \{\forall, \exists\}$, they are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![\alpha_0]\!]_{\mathfrak{M}}(\varnothing) = [\![\alpha_1]\!]_{\mathfrak{M}}(\varnothing)$, i.e., just in case $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$ (again since each $[\![\alpha_i]\!]_{\mathfrak{M}}$ is a total constant function on $\mathcal{G}$).

(c) If α_0 and α_1 are both open Λ-formulas, they are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$.

($\mathrm{DC4}_{\mathsf{Ext}_\Lambda}$) Well-formed Λ-expressions belonging to distinct Ext_Λ-categories are not Ext_Λ-coextensive in $\mathfrak{M}$.

What will matter for the extensionality of Ext_Λ in $\mathfrak{M}$ is to what extent distinct Ext_Λ-equicategorial Λ-expressions α_0 and α_1 that are legitimately substitutable for each other are Ext_Λ-coextensive in $\mathfrak{M}$. Among Ext_Λ-equicategorial expressions, there are five cases of legitimate substitutability:

(GS1) A member of $\mathcal{V} \cup \mathcal{C}_\iota$ may be legitimately replaced by another member of $\mathcal{V} \cup \mathcal{C}_\iota$.

(GS2) A member of any $\mathcal{P}_n$ may be legitimately replaced by another member of $\mathcal{P}_n$.

(GS3) A member of $\mathcal{C}_\mu \cup \{\forall, \exists\}$ may be legitimately replaced by a member of $\mathcal{C}_\mu \cup \{\forall, \exists\}$.

(GS4) An open Λ-formula may be legitimately replaced by another open Λ-formula.

(GS5) A closed Λ-formula may be legitimately replaced by another closed Λ-formula.

In case (GS1), we already know that distinct members α_0 and α_1 of $\mathcal{V} \cup \mathcal{C}_\iota$ are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$. In case (GS2), we likewise already know that distinct members α_0 and α_1 of $\mathcal{P}_n$ are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$, and again in case (GS3), we've already seen that distinct members α_0 and α_1 of $\mathcal{C}_\mu \cup \{\forall, \exists\}$ are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$. In cases (GS4) and (GS5), we already know that two open (respectively, two closed) formulas α_0 and α_1 are Ext_Λ-coextensive in $\mathfrak{M}$ just in case $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$. Thus:

LEMMA 3. *Λ-expressions α_0 and α_1 that are Ext_Λ-equicategorial and legitimately intersubstitutable are Ext_Λ-coextensive in $\mathfrak{M}$ iff $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$.*

A Quinean digression. We note that ($\mathrm{DC1}_{\mathsf{Ext}_\Lambda}$) through ($\mathrm{DC4}_{\mathsf{Ext}_\Lambda}$) accord with Quine's notion of coextensiveness (if we take into account that, unlike us, Quine treats quantifier expressions syncategorematically, and assume that he leaves ($\mathrm{DC4}_{\mathsf{Ext}_\Lambda}$)

$\mathfrak{M}$ as any individual constant (Tarski-values of constant symbols have domain $\mathcal{G}$ while those of variables do not).

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

implicit). Of particular relevance here is ($DC3_{Ext_\Lambda}$)(c), the treatment of open formulas, which is the critical difference between our two grammar schemas. This comes out most clearly in the following passage from "Confessions of a Confirmed Extensionalist" [15], where we've indicated in boldface which of ($DC1_{Ext_\Lambda}$) through ($DC3_{Ext_\Lambda}$) we take Quine to be referring to.[13]

> **($\mathbf{DC1}_{Ext_\Lambda}$)** I shall call two closed [formulas] *coextensive* if they are both true or both false. **($\mathbf{DC3}_{Ext_\Lambda}$)** Two predicates or general terms or open [formulas] are coextensive, of course, if they are true of just the same objects or sequences of objects. **($\mathbf{DC2}_{Ext_\Lambda}$)** Two singular terms are coextensive if they designate the same object. [...] In defining coextensiveness, I lumped predicates, general terms, and open [formulas] together. They are what can be predicated of objects or sequences of objects, and in that capacity they all three come to the same thing. [...] **($\mathbf{DC3}_{Ext_\Lambda}$)(c)** Two open [formulas] are coextensive if they have the same free variables and agree with each other in truth-value for all values of those variables. [15, p. 215]

Note that two open formulas are Ext_Λ-coextensive in $\mathfrak{M}$ (i.e., have the same Tarski-value in $\mathfrak{M}$) if and only if they have the same free variables and have the same truth value in $\mathfrak{M}$ for all assignments covering those variables, just as Quine requires in the final sentence of the quoted passage. *End of Quinean digression.*

We are now in a position to show that Ext_Λ is extensional in every Λ-structure $\mathfrak{M}$. This requires showing that, for any $\mathfrak{M}$ and any Λ-formula ϕ, if we construct a new Λ-formula ϕ' from ϕ by legitimately replacing some constituent α_0 of ϕ by some α_1 that is Ext_Λ-equicategorial and Ext_Λ-coextensive in $\mathfrak{M}$ with α_0, the formula ϕ' is Ext_Λ-coextensive in $\mathfrak{M}$ with ϕ.

As a first step, we observe that this holds if we replace "constituent" with "*immediate* constituent". Call this property *immediate-constituent substitutivity*.[14]

This follows by Lemma 3 from inspection of clauses 6 through 11 in Definition 1. For suppose α_0 and α_1 are legitimately intersubstitutable, Ext_Λ-equicategorial, and Ext_Λ-coextensive in $\mathfrak{M}$. Then $[\![\alpha_0]\!]_\mathfrak{M} = [\![\alpha_1]\!]_\mathfrak{M}$ by Lemma 3. Now if ϕ is $Pt_1 \dots t_n$, as long as $[\![P]\!]_\mathfrak{M} = [\![R]\!]_\mathfrak{M}$ and $[\![t_i]\!]_\mathfrak{M} = [\![s_i]\!]_\mathfrak{M}$ for each $1 \leq i \leq n$, we have $[\![Pt_1 \dots t_n]\!]_\mathfrak{M} = [\![Rs_1 \dots s_n]\!]_\mathfrak{M}$ by clause 6 of Definition 1, and so $[\![\phi]\!]_\mathfrak{M} = [\![\phi']\!]_\mathfrak{M}$. Similarly, by clause 7, if ϕ is $\neg\alpha_0$, as long as $[\![\alpha_0]\!]_\mathfrak{M} = [\![\alpha_1]\!]_\mathfrak{M}$, we have $[\![\neg\alpha_0]\!]_\mathfrak{M} = [\![\neg\alpha_1]\!]_\mathfrak{M}$, and thus $[\![\phi]\!]_\mathfrak{M} = [\![\phi']\!]_\mathfrak{M}$. The case of conjunction works analogously. Finally, by clauses 9–11, for any $\alpha_0, \alpha_1 \in \mathcal{C}_\mu \cup \{\forall, \exists\}$ and Λ-formula ψ, if ϕ is $\alpha_0 \mathsf{v}_i \psi$, as long as $[\![\alpha_0]\!]_\mathfrak{M} = [\![\alpha_1]\!]_\mathfrak{M}$, we will have $[\![\alpha_0 \mathsf{v}_i \phi]\!]_\mathfrak{M} = [\![\alpha_1 \mathsf{v}_i \psi]\!]_\mathfrak{M}$ and thus $[\![\phi]\!]_\mathfrak{M} = [\![\phi']\!]_\mathfrak{M}$; and for any $Q \in \mathcal{C}_\mu \cup \{\forall, \exists\}$ and Λ-formulas α_0, α_1, if ϕ is $Q\mathsf{v}_i\alpha_0$, as long as $[\![\alpha_0]\!]_\mathfrak{M} = [\![\alpha_1]\!]_\mathfrak{M}$, $[\![Q\mathsf{v}_i\alpha_0]\!]_\mathfrak{M} = [\![Q\mathsf{v}_i\alpha_1]\!]_\mathfrak{M}$ and hence $[\![\phi]\!]_\mathfrak{M} = [\![\phi']\!]_\mathfrak{M}$. Thus in all cases $[\![\phi]\!]_\mathfrak{M} = [\![\phi']\!]_\mathfrak{M}$. Therefore ϕ and ϕ' are either both closed or both open Λ-formulas (otherwise $\mathrm{dom}([\![\phi]\!]_\mathfrak{M}) \neq \mathrm{dom}([\![\phi']\!]_\mathfrak{M})$), hence Ext_Λ-equicategorial and Ext_Λ-legitimately substitutable in $\mathfrak{M}$ and so, by Lemma 3, Ext_Λ-coextensive in $\mathfrak{M}$.

---

13 In order to avoid terminological confusion we have replaced "sentence" by "formula" in the quotation throughout, since Quine uses the former to mean what we've been calling the latter.

14 This is, of course, essentially the compositionality of Tarskian semantics.

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

This observation generalizes easily to arbitrary-constituent substitutivity.[15]

OBSERVATION 1 (General substitutivity). *Let $\mathfrak{M}$ be a Λ-structure and let α_0 be be legitimately replaceable by α_1 at a constituent occurrence* o *in* ϕ. *Let α_0 and α_1 be* Ext_Λ-*equicategorial and* Ext_Λ-*coextensive in* $\mathfrak{M}$. *Let ϕ' be the result of replacing α_0 at* o *by α_1. Then ϕ and ϕ' are* Ext_Λ-*coextensive in* $\mathfrak{M}$.

Proof. Since α_0 and α_1 are Ext_Λ-equicategorial and legitimately substitutable, they are either both in $\mathcal{C}_\iota \cup \mathcal{V}$, both in $\mathcal{P}_n$ for the same n, both in $\mathcal{C}_\mu \cup \{\forall, \exists\}$, both open Λ-formulas, or both closed Λ-formulas. Since α_0 and α_1 are Ext_Λ-coextensive in $\mathfrak{M}$, $[\![\alpha_0]\!]_{\mathfrak{M}} = [\![\alpha_1]\!]_{\mathfrak{M}}$. We show by induction on ϕ that $[\![\phi]\!]_{\mathfrak{M}} = [\![\phi']\!]_{\mathfrak{M}}$ (from which Ext_Λ-coextensiveness of α_0 and α_1 follows as above). For the induction basis, note that any constituent occurrence of α_0 in an atomic Λ-formula ϕ is an immediate constituent occurrence, and hence the desired result follows from immediate-constituent substitutivity.

If ϕ is $\neg\chi$ and α_0 occurs once in ϕ, α_0 is either χ and thus an immediate constituent (so we're done by immediate-constituent substitutivity), or α_0's occurrence is within χ. In the latter case, by induction hypothesis $[\![\chi]\!]_{\mathfrak{M}} = [\![\chi']\!]_{\mathfrak{M}}$ and so $[\![\phi]\!]_{\mathfrak{M}} = [\![\neg\chi]\!]_{\mathfrak{M}} = [\![\neg\chi']\!]_{\mathfrak{M}} = [\![\phi']\!]_{\mathfrak{M}}$ by immediate-constituent substitutivity. The case of conjunctions is quite analogous.

If finally ϕ is $Q\mathsf{v}_i\chi$ with $Q \in \mathcal{C}_\mu \cup \{\forall, \exists\}$ and α_0 has a single occurrence in ϕ, α_0 occurs either as (i) one of Q, v_i, and χ and thus as an immediate constituent (so the desired result follows from immediate-constituent substitutivity), or else (ii) within χ. In the latter case, by induction hypothesis $[\![\chi]\!]_{\mathfrak{M}} = [\![\chi']\!]_{\mathfrak{M}}$ and again, by immediate-constituent substitutivity, $[\![\phi]\!]_{\mathfrak{M}} = [\![Q\mathsf{v}_i\chi]\!]_{\mathfrak{M}} = [\![Q\mathsf{v}_i\chi']\!]_{\mathfrak{M}} = [\![\phi']\!]_{\mathfrak{M}}$, as required. □

COROLLARY 2. *For any Λ-structure $\mathfrak{M}$,* Ext_Λ *is extensional in* $\mathfrak{M}$.

COROLLARY 3. *For any lexicon Λ,* Ext_Λ *is extensional.*

§5. The grammar schema NExt**.** We now define a grammar schema NExt, i.e., a function $\Lambda \mapsto \mathsf{NExt}_\Lambda$ mapping lexicons Λ to grammars NExt_Λ, that will be able to underwrite Salmon's claim regarding the non-extensionality of quantification. As with Ext_Λ, we need to provide, for each lexicon Λ, the ingredients (G1) through (G7) required for the specification of a grammar. We begin by picking an arbitrary lexicon $\Lambda = (\mathcal{C}_\iota, \mathcal{C}_\mu, \mathcal{P})$.

(G1) requires the specification of a lexicon; this is our chosen Λ. The set of well-formed expressions of NExt_Λ is $\mathsf{WF}(\Lambda)$. Thus Ext_Λ and NExt_Λ agree on (G1).

(G2) requires the specification of NExt_Λ's model class. We let the NExt_Λ-models be pairs $\mathfrak{M}^g := (\mathfrak{M}, g)$ consisting of a Λ-structure $\mathfrak{M}$ and an $\mathfrak{M}$-assignment g. We will call such pairs Λ-*context systems*, g being the *context* and $\mathfrak{M}$ the *underlying structure* of the context system $\mathfrak{M}^g$. (By a *minimal context system* we will mean a context system $\mathfrak{M}^\varnothing$.) The logical space $\mathsf{Mod}(\mathsf{NExt}_\Lambda)$ of NExt_Λ is thus the class of all Λ-context systems. Thus every NExt_Λ-model $\mathfrak{M}^g$ contains an Ext_Λ-model $\mathfrak{M}$ as its underlying structure.

15 That immediate-constituent substitutivity implies arbitrary-constituent substitutivity under very general conditions is well known, see, e.g., (Pagin and Westerståhl [11, note 15]). We include a proof for our special case since the argument is brief and the claim central to our discussion.

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

(G3) requires the specification of a function $\mathsf{Obj}^{\mathsf{NExt}_\Lambda}$ that maps each Λ-context system $\mathfrak{M}^g$ to a set $\mathsf{Obj}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ of NExt_Λ-objects$_{\mathfrak{M}^g}$. Where the structure $\mathfrak{M}$ underlying the context system $\mathfrak{M}^g$ is $(M, \mathfrak{I})$, we let $\mathsf{Obj}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ be M. Thus Ext_Λ and NExt_Λ essentially agree, modulo the difference in their notions of model, on (G3).

(G4) requires, for each Λ-context system $\mathfrak{M}^g$, a partition of NExt_Λ's well-formed expressions into NExt_Λ-sentences$_{\mathfrak{M}^g}$, NExt_Λ-terms$_{\mathfrak{M}^g}$, and NExt_Λ-predicators$_{\mathfrak{M}^g}$. The NExt_Λ-sentences$_{\mathfrak{M}^g}$ are the g-closed Λ-formulas, the NExt_Λ-terms$_{\mathfrak{M}^g}$ are the members of $\mathcal{V} \cup \mathcal{C}$, and the NExt_Λ-predicators$_{\mathfrak{M}^g}$ are the members of $\mathcal{C}_\mu$, the members of $\mathcal{P}$, the members of $\{\forall, \exists\}$, and the g-open Λ-formulas. Thus the NExt_Λ-categories are model-dependent, unlike the Ext_Λ-categories. Note that relative to *minimal* context systems, the NExt_Λ-notions of sentence, term, and predicator are just those of Ext_Λ.

(G5) requires the definition of a truth operator that maps each Λ-context system $\mathfrak{M}^g$ to the set $\mathsf{true}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ of NExt_Λ-sentences$_{\mathfrak{M}^g}$ (i.e., g-closed Λ-formulas) that are NExt_Λ-true in $\mathfrak{M}^g$. Let $\mathsf{true}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ be the set of all g-closed Λ-formulas ϕ for which $[\![\phi]\!]_{\mathfrak{M}}[g](\varnothing) = 1$ (equivalently, for which $[\![\phi]\!]_{\mathfrak{M}}[g] = \mathbf{1}_{\mathcal{G}_g}$). Note that by Lemma 2, a g-closed formula ϕ that is not in $\mathsf{true}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ is such that $[\![\phi]\!]_{\mathfrak{M}}[g](\varnothing) = 0$, or equivalently, $[\![\phi]\!]_{\mathfrak{M}}[g] = \mathbf{0}_{\mathcal{G}_g}$. Also, since $[\![\phi]\!]_{\mathfrak{M}}[\varnothing] = [\![\phi]\!]_{\mathfrak{M}}$, $\mathsf{true}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^\varnothing} = \mathsf{true}^{\mathsf{Ext}_\Lambda}_{\mathfrak{M}}$.

(G6) requires the definition of a designation operator that maps each Λ-context system $\mathfrak{M}^g$ to a function $\mathsf{des}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ whose domain consists of NExt_Λ-terms$_{\mathfrak{M}^g}$ (i.e., variables and constant symbols) and whose values are NExt_Λ-objects$_{\mathfrak{M}^g}$ (i.e., members of M). Let $\mathsf{des}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ be the set that contains, for each individual constant $c \in \mathcal{C}_\iota$, the pair $\langle c, \mathfrak{I}(c)\rangle$, and for each g-covered variable v_i, the pair $\langle \mathsf{v}_i, g(i)\rangle$; and nothing else (so that variables not covered by g do not designate anything). Thus $\mathsf{des}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^\varnothing} = \mathsf{des}^{\mathsf{Ext}_\Lambda}_{\mathfrak{M}}$.

(G7), finally, requires the definition of a predication operator that maps each Λ-context system $\mathfrak{M}^g$ to a function $\mathsf{true\text{-}of}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ mapping each NExt_Λ-predicator$_{\mathfrak{M}^g}$ (i.e., member of $\mathcal{C}_\mu$, member of $\mathcal{P}$, member of $\{\forall, \exists\}$, or g-open Λ-formula) to a $\{0, 1\}$-valued function on a set of arrangements of members of the type hierarchy over M. We let $\mathsf{true\text{-}of}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^g}$ be the function mapping:

- (i) each Montagovian constant $C \in \mathcal{C}_\mu$ to the function $[\![C]\!]_{\mathfrak{M}}[g](\varnothing)$, which is the characteristic function of the subset $\{X \subseteq M \mid \mathfrak{I}(C) \in X\}$ of $\mathsf{Pow}(M)$.
- (ii) each $P \in \mathcal{P}_n$ to $[\![P]\!]_{\mathfrak{M}}[g](\varnothing)$, i.e., to $\mathfrak{I}(P)$, which by definition is the characteristic function of a subset of M^n.
- (iii) each $Q \in \{\forall, \exists\}$ to the function $[\![Q]\!]_{\mathfrak{M}}[g](\varnothing)$, which in either case is the characteristic function of a subset of $\mathsf{Pow}(M)$, viz. of $\{M\}$ or of $\{X \subseteq M \mid x \neq \varnothing\}$, respectively.
- (iv) each g-open Λ-formula ϕ to the function $[\![\phi]\!]_{\mathfrak{M}}[g] : \mathsf{dom}([\![\phi]\!]_{\mathfrak{M}}[g]) \to \{0, 1\}$, which is obviously the characteristic function of some subset of the domain of $[\![\phi]\!]_{\mathfrak{M}}[g] \subseteq \mathcal{G}_g$.

Note that $\mathsf{true\text{-}of}^{\mathsf{NExt}_\Lambda}_{\mathfrak{M}^\varnothing} = \mathsf{true\text{-}of}^{\mathsf{Ext}_\Lambda}_{\mathfrak{M}}$.

Obviously, modulo the identification of minimal Λ-context systems $(\mathfrak{M}, \varnothing)$ with their underlying Λ-structures $\mathfrak{M}$, the grammar Ext_Λ is simply the result of restricting the logical space of NExt_Λ to minimal context systems, i.e., context systems with an empty context. In this precise sense, then, NExt_Λ is a generalization of Ext_Λ. It also follows that Ext_Λ-coextensiveness in $\mathfrak{M}$ and NExt_Λ-coextensiveness in $\mathfrak{M}^\varnothing$ coincide, so any violation of extensionality in NExt_Λ will have to occur in non-minimal context systems.

The coextensiveness condition (DC1) plays out as follows in NExt_Λ:[16]

($\mathrm{DC1}_{\mathsf{NExt}_\Lambda}$) g-closed Λ-formulas ϕ and ψ are NExt_Λ-coextensive in $\mathfrak{M}^g$ just in case they are either both NExt_Λ-true in $\mathfrak{M}^g$ or both NExt_Λ-false in $\mathfrak{M}^g$, i.e., just in case $[\![\phi]\!][g]_{\mathfrak{M}}(\varnothing) = 1$ and $[\![\psi]\!][g]_{\mathfrak{M}}(\varnothing) = 1$, or $[\![\phi]\!][g]_{\mathfrak{M}}(\varnothing) = 0$ and $[\![\psi]\!][g]_{\mathfrak{M}}(\varnothing) = 0$. Since, for g-closed formulas χ, $[\![\chi]\!]_{\mathfrak{M}}[g](\varnothing) = [\![\chi]\!]_{\mathfrak{M}}(g)$, we have that g-closed Λ-formulas ϕ and ψ are NExt_Λ-coextensive in $\mathfrak{M}^g$ just in case $[\![\phi]\!]_{\mathfrak{M}}(g) = [\![\psi]\!]_{\mathfrak{M}}(g)$.

Violations of extensionality occur within NExt_Λ regardless of the nature of Λ: By definition $\mathcal{P} \neq \varnothing$. Suppose without loss of generality that $\mathcal{P}$ contains a one-place predicate symbol P. Let $\mathfrak{M}$ be such that $\mathfrak{I}(P)(a) = 1$ and $\mathfrak{I}(P)(b) = 0$. Let $g(0) = g(1) = a$. Then both $P\mathsf{v}_0$ and $P\mathsf{v}_1$ are NExt_Λ-true in $\mathfrak{M}^g$, yet $\forall \mathsf{v}_0 P\mathsf{v}_0$ will be NExt_Λ-false and $\forall \mathsf{v}_0 P\mathsf{v}_1$, NExt_Λ-true in $\mathfrak{M}^g$. (Clearly a variant of this strategy can be used for a lexicon containing a predicate symbol of an arity $n > 1$.)[17,18] And since atomic formulas can be shown to be NExt_Λ-extensional in any Λ-context system, it is indeed quantification that is responsible for this failure of extensionality.[19] Thus:

OBSERVATION 2. NExt_Λ *is never extensional.*

§6. Context systems and deixis. In Ext and NExt, we have two grammar schemas that build on the same generating syntax and the same recursive semantics but that nevertheless differ with respect to the extensionality of their generated grammars. Since grammars of either kind are, in an intuitive but clear sense, grammars of quantification, we are led to conclude that the question whether quantification is extensional is, unless further qualified, ill-posed, because it is answerable only relative to an antecedently chosen grammar.

It is nevertheless conceivable that one of our two grammar schemas should, all things considered, be preferable to the other, and if this were the case, it might indirectly support the answer to the extensionality question given by the camp endorsing the

16 It is routine to spell out (DC2) through (DC4) in analogous terms, but we won't need to invoke them.

17 For a counterexample that doesn't involve vacuous quantification, let $P, Q \in \mathcal{P}_1$, let $\mathfrak{M} = (M, \mathfrak{I})$ be a model, let $m \in M$, let $\mathfrak{I}(P)$ the constant function on M with value 1, and let $\mathfrak{I}(Q) : M \to \{0, 1\}$ be non-constant with $\mathfrak{I}(Q)(m) = 1$. Let g be the function with domain $\{0\}$ for which $g(0) = m$. Then $P\mathsf{v}_0$ and $Q\mathsf{v}_0$ are both NExt_Λ-true sentences in $\mathfrak{M}^g$. But $\forall \mathsf{v}_0 P\mathsf{v}_0$ is NExt_Λ-true in $\mathfrak{M}^g$ while $\forall \mathsf{v}_0 Q\mathsf{v}_0$ is NExt_Λ-false in $\mathfrak{M}^g$. Thus switching the constituent $P\mathsf{v}_0$ of $\forall \mathsf{v}_0 P\mathsf{v}_0$ for $Q\mathsf{v}_0$, with which it is NExt_Λ-coextensive in $\mathfrak{M}^g$, results in an expression, namely $\forall \mathsf{v}_0 Q\mathsf{v}_0$, that is *not* NExt_Λ-coextensive in $\mathfrak{M}^g$ with the original $\forall \mathsf{v}_0 P\mathsf{v}_0$.

18 The reader is encouraged to consult Salmon's [17, pp. 415–416] counterexample to extensionality, which uses a binary predicate symbol and identity. It is, we admit, much wittier than ours.

19 The root cause for the non-extensionality of NExt-grammars is actually variable-binding rather than quantification in the narrow sense of universal and existential quantification. One way to see this is by noting that Montagovian constants also generate extensionality failures in NExt-grammars. For suppose that $g(\mathsf{v}_0) = a$ and $\mathfrak{I}(C) = b$, where $\mathfrak{I}(P)(a) = \mathfrak{I}(P)(b) = 1$ while $\mathfrak{I}(Q)(a) = 1$ and $\mathfrak{I}(Q)(b) = 0$. Then $P\mathsf{v}_0$ and $Q\mathsf{v}_0$ are both NExt_Λ-true in $\mathfrak{M}^g$ while $C\mathsf{v}_0 P\mathsf{v}_0$ is NExt_Λ-true and $C\mathsf{v}_0 P\mathsf{v}_0$ is NExt_Λ-false in $\mathfrak{M}^g$.

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

superior grammar schema. Are there arguments, then, that might be advanced in favor of Ext over NExt or *vice versa*?

For a start, as we've seen in the preceding section, there is a precise sense in which NExt generalizes Ext (if we identify a Λ-structure $\mathfrak{M}$ with the minimal Λ-context system $\mathfrak{M}^{\varnothing}$ that it underlies). It is thus tempting to argue that it is the more general schema, i.e., NExt, that should win out; adherents of Ext, according to this argument, take too narrow a view of logical space by considering only minimal context systems, and are misled into adopting a grammar schema that, because of its myopia, makes quantification look extensional.

One might counter on behalf of the extensionalist that not every generalization is an *appropriate* generalization. Perhaps the generalization NExt of Ext is a spurious one, one that accidentally destroys extensionality but that otherwise has no *point*. If that is right, it should be Ext that rules; after all, its grammars are not only more parsimonious[20] than NExt's, but also extensional; and as Lewis [9, p. 256] notes, "extensionality itself is generally thought to be an important dimension of simplicity." Thus, if there's no point to adopting NExt, we should embrace Ext on grounds of theoretical simplicity.

It seems, however, that this argument of the extensionalist's won't fly. Granted, for the purpose of formalizing mathematics there is perhaps little point in replacing simple structures with arbitrary context systems. But within philosophy of language and linguistic semantics, context systems are routinely appealed to in the analysis of deictic utterances involving third-person singular pronouns, and thus certainly appear to *have* a point.

The idea that references for deictic pronouns are to be furnished by a contextual variable assignment goes back at least to Montague [10], is prominent in Kaplan [8], and has gained wide currency through the influential textbook by Heim and Kratzer [6].[21] In brief, the proposal is that the context g is the result of various demonstrative acts on the part of the speaker, and that the objects demonstrated can be designated *via* variables co-indexed with the objects' places in g.

To take a simple example, suppose we wanted to formalize the following utterance:

(A) *Pointing at Venkatraman Ramakrishnan:* He is a genius.

According to the view under discussion, the speaker's pointing gesture places Ramakrishnan into a suitable position in the context g, and it is assumed that the deictic pronoun (tacitly, as it were) carries the index that corresponds to that position. The utterance is therefore to be represented more precisely as

(A_i) *Pointing at Venkatraman Ramakrishnan:* He_i is a genius,

where the context g generated by the pointing gesture is such that $g_i =$ Ramakrishnan.

Now (A_i) can be formalized, in any grammar NExt_Λ whose lexicon contains the one-place predicate symbol `Genius`, as

(B) $\mathtt{Genius}(\mathsf{v}_i)$.

---

20 In that, for instance, its models are mere structures rather than context systems, and its syntactic categories do not depend on models.

21 The analysis by Del Prete and Zucchi [3] uses precisely the kind of context-system apparatus we built into NExt.

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

In any context system $\mathfrak{M}^h$ whose context h covers v_i, formula (B) is NExt_Λ-true just in case h_i belongs to $\mathfrak{I}(\mathtt{Genius})$. So in the *intended* context system where $\mathfrak{I}(\mathtt{Genius})$ is the set of geniuses and g is determined by the speaker's pointing gesture as assumed above, (B) will be true just in case g_i, i.e., Venki Ramakrishnan, is a genius, which is precisely what is required of an adequate formalization of (A).[22]

An analysis of deixis of this nature is unavailable to adherents of Ext. No grammar Ext_Λ counts the open formula (B) as a sentence and thus neither its truth nor its falsity can even be entertained—after all, no Ext_Λ-model, i.e., no Λ-structure $\mathfrak{M}$ (equivalently, no minimal Λ-context system $\mathfrak{M}^\varnothing$) supplies a value for the variable v_i. The grammars Ext_Λ therefore seem severely limited in their applicability to natural-language analysis in that they cannot handle deixis—surely a serious weakness.[23]

It thus looks like we have a substantive argument, based on the logical analysis of a natural-language phenomenon, in favor of the more general grammars NExt_Λ, and thus, indirectly, in favor of the non-extensionality of quantification.

§7. An extensionalist approach to deixis. What might an adherent of Ext do in the face of utterances like (A)? More precisely, how might they treat the pronoun *he* as it occurs in (A)? Given the available lexical resources, and given that they cannot model *he* as a free variable, there would seem to be two options: The pronoun might correspond to an individual constant, or it might correspond to a *bound* rather than a free variable. Let us consider these in turn.

If we want to render deictic pronouns as individual constants, we need to think of the lexicon as being expanded, at the time of a deictic utterance like (A), by a new individual constant d corresponding to the deictic pronoun *he*, and to think of a demonstrative gesture not as assigning an object m to a certain hitherto valueless variable v_i that thereby becomes covered by the context (as a NExt-theorist would), but as introducing, and fixing m as the interpretation of, the new individual constant d.

Put somewhat differently, where the received view regards a deictic utterance like (A), or rather (A_i), as transforming an initial context system $(\mathfrak{M}, g)$ via the gesture pointing at Venki Ramakrishnan, into a richer context system $(\mathfrak{M}, g_i^{\mathrm{Venki}})$, Venki thus becoming

22 There are a great many details about this story that would need to be explained, such as the rule that determines the position in the context into which a demonstrated object is written, the ability of hearers to identify the right variable index to retrieve the appropriate object from the context, etc. We will assume, for the sake of argument, that this can be made to work.

23 Something like this observation appears to underlie Szabó and Thomason's [22] complaints about Tarski's reticence towards counting open formulas as truth-apt: After decrying "a strong tradition in logic of treating free variables and 'open formulas' as second-class citizens" [22, pp. 172–173], they go on to say: "[…] Tarski was forced to assign temporary values to variables in order to produce a compositional definition of generalized truth, or satisfaction, but […] he refused to give first-class status to open formulas. […] [O]vercoming this prejudice was crucial to securing a viable semantic theory of indexicals." It is presumably their ineligibility for truth and falsity that makes open formulas "second-class citizens" in grammars like Ext_Λ. But note that Szabó and Thomason may be overshooting their mark if they mean to suggest that *all* open formulas should count as sentences. After all, given a context system $\mathfrak{M}^g$, among the $\varnothing$-open formulas it is only the *g-closed* ones that are true or false. This problem only goes away if one makes context assignments total functions on ω, and it is hard to see how this could be justified vis-à-vis linguistic applications in particular.

the value of the variable v_i, Ext-grammarians rather think of (A) as transforming an initial Λ-structure $(M, \mathfrak{I})$ via the pointing gesture into a richer Λ_d-structure $(M, \mathfrak{I}_d^{\text{Venki}})$, where Λ_d is the expansion $(\mathcal{C}_\iota \cup \{d\}, \mathcal{C}_\mu, \mathcal{P})$ of Λ by the previously uninterpreted individual constant d, and $\mathfrak{I}_d^{\text{Venki}}$ is $\mathfrak{I} \cup \{\langle d, \text{Venki}\rangle\}$.

It should be clear that, if the extensionalist follows this route, they have no expressiveness problems in dealing with deixis. After all, in order to say what is expressed in NExt_Λ by $\phi(\mathsf{v}_{i_0}, \dots, \mathsf{v}_{i_n})$, with the free variables $\mathsf{v}_{i_0}, \dots, \mathsf{v}_{i_n}$ having been assigned values through pointing, our extensionalist can use $\phi(d_0, \dots, d_n)$, where each d_j is *interpreted* by their $\mathsf{Ext}_{\Lambda_{d_0,\dots,d_n}}$-model as the object that the NExt-theorist would *assign* to the variable v_{i_j} as its contextual value.

So it looks as if the pendulum has swung back towards Ext: the logical analysis of deixis does not, after all, favor NExt over Ext, if we're willing to expand the lexicon by individual constants for deictic pronouns. But then simplicity considerations would seem to push us back towards Ext.[24]

Friends of NExt_Λ may not be willing to admit defeat, though. While they can hardly object to the extensionalist's claim of being able to formalize deictic utterances in a truth-conditionally adequate manner, they can point to an implausible feature of the proposed representation. The issue comes out quite clearly if we compare:

(A) *Pointing at Venkatraman Ramakrishnan:* He is a genius

with

(C) Every living US President is such that he is a genius.

According to the current extensionalist proposal, the formalization of (A) would be

(B′) $\mathtt{Genius}(d)$

whereas the uncontentious[25] formalization of (C) is

(D) $\forall \mathsf{v}_j \left(\mathtt{Pres}(\mathsf{v}_j) \to \mathtt{Genius}(\mathsf{v}_j)\right)$.

Both (A) and (C) contain the pronoun *he*, but the formalization (B′) of (A) renders it as d while the formalization (D) of (C) renders the very same pronoun as v_j. Granted, in (A) *he* is used deictically while in (C) it is used like a bound variable, but the pronoun itself is morphologically the same in both cases; indeed this seems to be the situation

24 One might object that each individual NExt_Λ actually has an expressive advantage over its counterpart Ext_Λ, since the latter must expand its lexicon in order to achieve expressive parity. But this is ignoring the fact that NExt_Λ needs to do something quite similar in order to achieve the expressive power that it has: It must shift a variable that previously wasn't context-covered into the cover of the context, which is in a sense to lexicalize that variable. That this doesn't involve a change of grammar is only due to the possibility of making the grammatical categories (of sentence, term, and predicator) relative to a model, which we built into the definition of a grammar precisely in order to accommodate this need of NExt; Ext does not require such flexibility. We could have alternatively made NExt a function not just of a lexicon Λ but also of a finite set A of variables, in such a way that the grammar $\mathsf{NExt}_{\Lambda, A}$ has as its models precisely those Λ-context systems whose contexts cover all and only the variables in A. Under that definition, a move from Ext_Λ to Ext_{Λ_d} would correspond to a move from $\mathsf{NExt}_{\Lambda, A}$ to some $\mathsf{NExt}_{\Lambda, A \cup \{\mathsf{v}_i\}}$.

25 Assuming a prohibition against vacuous binding, (C) only has the formalization according to which *he* is bound by *every living US President*. Of course any other variable than v_j would do just as well in (D).

in a wide range of human languages. Formalizing the two occurrences of the same pronoun by distinct symbols, i.e., in effect positing an ambiguity, for the sole purpose of safeguarding extensionality, therefore seems difficult to justify.[26]

Now a similar charge of ambiguity may perhaps be leveled against NExt. Recall that the NExt_Λ-formalization of (A), or rather of (A_i), is

(B) $\mathtt{Genius}(\mathsf{v}_i)$,

in which *he* corresponds to a variable (here v_i), just as it does in

(D) $\forall \mathsf{v}_j\ (\mathtt{Pres}(\mathsf{v}_j) \to \mathtt{Genius}(\mathsf{v}_j))$,

where it corresponds to v_j. So it is certainly true that the pronoun is represented as a variable in both cases, which is not the case in the Ext-proposal we just examined. They are nevertheless distinct variables—unless one happens to choose $j = i$ for the formalization (D) of (C), for which there is no particular reason (after all, the choice of index for bound variables is largely irrelevant). What is disconcerting is that the NExt-strategy requires the choice of a *particular* index i for the formalization of the deictic pronoun, i.e., a decision that (A) is really (A_i) rather than, say, (A_k), without an obvious story as to how this might work; a requirement of choice that has no counterpart in our Ext-strategy. Stojnić [18, chap. 3] argues that this commits the received analysis of deixis, i.e., the one appealed to by the NExt-theorist, to a substantive ambiguity problem. We are sympathetic to this objection but will set it aside here.[27]

All this said, the extensionalist should, we believe, concede that her strategy of expanding the lexicon by new *individual constants*, while able to establish expressive parity with NExt_Λ, is nevertheless problematic as an analysis of deixis.

We've left one potential Ext-strategy for the formalization of deixis unexplored, however, namely the option of rendering the deictic pronoun as a bound rather than a free variable. Suppose that, instead of introducing the new *individual* constant d into the lexicon, with the understanding that its interpretation is to be the demonstrated object m (which, for the NExt-theorist, would be the new value of v_i), we introduce a new *Montagovian* constant D into the lexicon, with the understanding that the interpretation of D is the object m. We then have that $[\![D\mathsf{v}_j\phi]\!]_{\mathfrak{M}}(h) = [\![\phi]\!]_{\mathfrak{M}}(h_j^m)$ and thus in particular that, for any $h \in \mathcal{G}$, $[\![D\mathsf{v}_j\, \mathtt{Genius}(\mathsf{v}_j)]\!]_{\mathfrak{M}}(h) = 1$ if and only if $[\![\mathtt{Genius}(\mathsf{v}_j)]\!]_{\mathfrak{M}}(h_j^m) = 1$, if and only if $m \in \mathfrak{I}(\mathtt{Genius})$. Accordingly, under this proposal, the Ext_Λ-theorist may formalize (A) as

(B″) $D\mathsf{v}_j\, \mathtt{Genius}(\mathsf{v}_j)$

provided that Venkatraman Ramakrishnan is the interpretation of D.

Thus, where the previous extensionalist proposal was to think of (A) as transforming an initial Λ-structure $(M, \mathfrak{I})$ via the pointing gesture into the richer Λ_d-structure $(M, \mathfrak{I}_d^{\text{Venki}})$, the new proposal is to think of (A) as transforming $(M, \mathfrak{I})$ into the richer Λ_D-structure $(M, \mathfrak{I}_D^{\text{Venki}})$. It should be clear that this, too, solves the extensionalist's expressiveness problem, for they can now do with a closed formula $D_1x_1 \dots D_nx_n\, \phi(x_1, \dots, x_n)$ whatever the non-extensionalist can do with the open

[26] See, e.g., Zimmermann [26, p. 204], Jacobson [7, p. 62], and Del Prete and Zucchi [3, sec. 1], who all express suspicion vis-à-vis such an ambiguity treatment of deictic and bound pronouns.

[27] Thanks to an anonymous referee for drawing our attention to this issue.

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

formula $\phi(\mathsf{v}_{i_1}, \dots, \mathsf{v}_{i_n})$.[28] Consequently, $\mathsf{Ext}_{\Lambda_{D_1,\dots,D_n}}$ can formalize any deictic utterance formalizable in NExt_Λ.

But do these new *Montagovian* constants solve the representational problem of unwanted deictic–bound ambiguity in pronouns? Well, yes. For where the extensionalist's first attempt at sprucing up the expressive power of Ext_Λ was unsatisfactory in that it formalized:

(A) *Pointing at Venkatraman Ramakrishnan:* He is a genius

as

(B′) $\texttt{Genius}(d)$

with an individual constant d rather than a variable, as in NExt_Λ's formalization

(B) $\texttt{Genius}(\mathsf{v}_i)$,

she can now, in Ext_{Λ_D}, render (A) as

(B″) $D\mathsf{v}_j\,\texttt{Genius}(\mathsf{v}_j)$,

with a *variable* v_j in the argument place of G, as desired.[29] Taking that variable to be the formal equivalent of the deictic pronoun *he* in (A), she thus avoids the spurious ambiguity of pronouns that was objectionable about her initial idea: bound and deictic pronouns alike now correspond to variables.[30]

But wait—if the deictic pronoun in (A) corresponds to the bound variable occurrence in the argument place of Genius in (B″), what in (A) corresponds to the Montagovian constant D in (B″)? In other words, what is the justification for including $D\mathsf{v}_j$ in the logical form of (A)?

A comparison of (A) to its proposed Ext_{Λ_D}-formalization (B″) suggests an answer:

(A) [*Pointing at Venkatraman Ramakrishnan:*] He is a genius.

(B″) [$D\mathsf{v}_j$] $\texttt{Genius}(\mathsf{v}_j)$

What the extensionalist can say is that the predicate symbol Genius represents the verb phrase *is a genius*, the variable v_j occurring in its argument place represents the pronoun *he*, and the Montagovian constant D represents the *pointing gesture*, aimed at Ramakrishnan, that accompanies the speaker's production of the pronoun. Thus the pointing gesture itself, the extensionalist will maintain, functions as a Montagovian constant and is represented as such at the level of logical form.

While we don't have the space to enter into much detail here, we want to point out that the recent philosophical and linguistic literature has seen a sophisticated, comprehensive proposal [18–21] for the semantic analysis of deixis which centrally includes the requirement that demonstrative gestures be represented at the level of

[28] The variables x_i can be chosen arbitrarily (provided they are all distinct, as we're assuming the v_{i_k} are).

[29] Again, the choice of index is immaterial, since the variable is bound (by D). This contrasts with the situation in NExt_Λ, where the variable's index is crucial.

[30] The "bound" vs. "deictic" terminology then seems unfortunate, since deictic pronouns are bound, too.

https://doi.org/10.1017/S1755020324000066 Published online by Cambridge University Press

logical form, and indeed in the form of variable binders, much as we have just suggested.[31]

The formal analysis proposed by Stojnić [18] is, setting aside the facts that her overall framework is a dynamic semantics rather than the "static" one with which we are concerned in this paper, and that she aims to provide a mechanism for pronoun resolution as well, very similar to the one at which we have arrived on behalf of the extensionalist: Stojnić's demonstration operator $\langle \pi k d \rangle$ is essentially our $D\mathsf{v}_k$, if we follow the convention that D is the Montagovian constant corresponding to the individual constant d; in other words, a formula $\langle \pi k d \rangle\, \phi$ in Stojnić's formalism is essentially equivalent to the gloss $\exists \mathsf{v}_k (\mathsf{v}_k = d \wedge \phi)$ we gave for $D\mathsf{v}_k\phi$ under the assumption that $\Im(d) = \Im(D)$.[32]

Deictic pronoun use does of course also occur without an accompanying demonstrative gesture. Does this fact complicate the position of the Ext-theorist?[33] We are inclined to think not. Pointing, the extensionalist will acknowledge, is just one way of introducing a (Montagovian) constant into the language, a way that makes the mechanism responsible for the identification of a particular individual as its interpretation particularly transparent. But there are other rule-based mechanisms that can serve the same purpose, as Stojnić et al. [19] have shown in some detail, using the theory of coherence relations. And such cases of deixis without ostension, too, Stojnić's proposed logical-form correlate of introducing into the discourse an object made salient by coherence relations is essentially a Montagovian constant that binds the variable representing the deictic pronoun.

§8. Conclusion. In order better to understand the disagreement between Quine and Salmon over the extensionality of quantification that we saw in Section 1, we developed (or perhaps better: made explicit) two schemas for constructing quantificational grammars. Both schemas base their syntax and semantics on the Tarskian techniques reviewed in Section 2, but the first, Ext, builds an *extensional* grammar Ext_Λ from any lexicon Λ (Sections 3 and 4), while the second, NExt, builds a *non-extensional* grammar NExt_Λ (in which quantification is responsible for the failure of extensionality) from any Λ (Section 5). There is thus a sense in which both Quine and Salmon are right: depending on the ambient grammar, quantification can turn out extensional, as Quine claims, or non-extensional, as Salmon has it. At the same time, there is a sense in which they are both wrong—quantification *per se* is neither extensional nor non-extensional; it is only relative to an entire grammar environment that it turns out one way or another.

In Section 6, we imagined our two camps not being content with this annoyingly philosophical outcome, and wondered what arguments there might be to favor the grammar schema Ext over NExt or *vice versa*, arguments that would then indirectly

31 The central insight of this proposal, as we see it, is that in the case of physical ostension, the identification of the relevant object can be construed as an entirely rule-based, grammaticalized mechanism rather than a more or less free-wheeling attempt to guess the speaker's referential intention.

32 Basically the same analysis is independently suggested by Ebert et al. [4]. They, too, work in a dynamic framework; the projection of their analysis of deictic utterances into our setting gives essentially $\exists \mathsf{v}_i\, (\mathsf{v}_i = d \wedge \phi)$ rather than our $D\mathsf{v}_i\, \phi$, where again it is assumed that $\Im(d) = \Im(D)$ (cf. [4, p. 167]).

33 Thanks to two anonymous reviewers for insisting on this point.

support either the extensionality or the non-extensionality of quantification. In terms of purely conceptual arguments, we ended up in something of a stalemate: On the one hand, Ext's grammars are *simpler* than NExt's in that they are extensional and their internal machinery is more parsimonious; on the other hand, NExt's grammars seem to *generalize* Ext's. At this point in the dialectic, we had the extensionalist challenge their opponent to demonstrate that the ostensibly greater generality of NExt's grammars wasn't just a pointless complication. The non-extensionalist came back with a well-established application of NExt-grammars in the logical analysis of natural language, namely to deixis. According to a widely held view, Ext-type grammars are inadequate for the modeling of deixis since deictic pronouns are to be rendered as free variables, and consequently, deictic utterances as open formulas; within the grammars Ext_Λ, however, open formulas are never truth-eligible, and thus the formalization of deixis appears to fall outside the range of applicability of the extensionalist's grammar schema.

The final dialectical move we considered was the extensionalist's attempt to refute their opponent's contention that Ext is unable to handle natural-language deixis. To this end, we examined two strategies available within Ext, namely, on the one hand, rendering deictic pronouns as individual constants, and on the other, rendering them as variables bound by a Montagovian constant. While the first strategy was found to be problematic in that it postulates a morphologically unattested ambiguity in natural-language pronouns between deictic occurrences (which would correspond to individual constants) and bound occurrences (which would correspond to variables), the second strategy was seen to avoid this problem, but is committed to loading additional material into the logical forms of deictic utterances, to wit, Montagovian constants. We suggested on behalf of the Ext-theorist that there is a natural interpretation of this additional material, namely that it is the logical-form counterpart of the demonstrative gesture. Indeed, as it happens, recent proposals for the linguistic treatment of deixis by Stojnić and by Ebert (and respective collaborators) *require* just such material in the relevant logical forms, so the presence of Montagovian constants in the Ext-formalizations of deictic utterances appears to be an independently motivated feature of the grammar of deixis.

Where does this leave us? It is still the case that, technically speaking, depending on the kind of grammar that controls, quantification may present as extensional or as non-extensional. But there now seems to be little reason to adopt a grammar schema that makes quantification non-extensional, given that extensional grammars are not only simpler but, by all appearances, also no less expressive than their non-extensional generalizations. In the field of quantification theory, extensionalism seems to be the most attractive option.

Acknowledgment. I presented versions of this paper to the Philosophy Colloquium at Utrecht University, the Logic and Epistemology Colloquium at Bochum University, the Carl Friedrich von Weizsäcker Colloquium at Tübingen University, the Logic Seminar of the Center for the Advancement of Logic, its Philosophy, History, and Applications (C–ALPHA) at UC Irvine, the First Nancy–Irvine Workshop on Formalization in Logic, Language, and Mathematics, a joint colloquium of the Cologne Center for Language, Information and Philosophy (CLIP) and the Linguistics Circle at Cologne University, the UC Davis Philosophy Colloquium, and the South by Semantics Workshop at the University of Texas at Austin. Many thanks to the participants for numerous helpful questions and comments, including Andy Arana, Ray Buchanan, Rohan French, Jeremy Heis, Klaus von Heusinger, Fabio Lampert, Greg Lauro, Harvey Lederman, Robert May, Toby Meadows, Dolf Rami, Manuel Rebuschi, Helge Rückert, Peter Schroeder-Heister, David Sosa, Will Stafford, Sean Walsh, Heinrich Wansing, and Ede Zimmermann. I have benefited from generous comments on earlier versions of this paper by Michael Glanzberg, Lloyd Humberstone, Hans Kamp, Bryan Pickel, Brian Rabern, Luis Rosa, and Greg Scontras, to all of whom I am deeply grateful. Thank you, finally, to two anonymous reviewers for this journal, who provided exceptionally useful advice.

BIBLIOGRAPHY

[1] Belnap, N. (2005). Under Carnap's lamp: Flat pre-semantics. *Studia Logica*, **80**(1), 1–28.

[2] Carnap, R. (1947). *Meaning and Necessity*. Chicago: Chicago University Press.

[3] Del Prete, F., & Zucchi, S. (2017). A unified non monstrous semantics for third person pronouns. *Semantics and Pragmatics*, **10**(10), 1–55. https://doi.org/10.3765/sp.10.10.

[4] Ebert, C., Ebert, C., & Hörnig, R. (2020). Demonstratives as dimension shifters. *Proceedings of Sinn Und Bedeutung*, **24**(1), 161–178.

[5] Glanzberg, M., & King, J. (2020). Binding, compositionality, and semantic values. *Philosophers' Imprint*, **20**(2), 1–29.

[6] Heim, I., & Kratzer, A. (1998). *Semantics in Generative Grammar*. Blackwell, Oxford.

[7] Jacobson, P. (2003). Binding without pronouns (and pronouns without binding). In Oehrle, R., and Kruiff, G.-J., editors. *Resource-Sensitivity, Binding, and Anaphora*. Dordrecht: Kluwer, 57–96.

[8] Kaplan, D. (1989). Demonstratives. In Almog, J., Perry, J., and Wettstein, H., editors. *Themes from Kaplan*. Oxford: Oxford University Press, pp. 481–563.

[9] Lewis, D. (1974). Tensions. In Munitz, M., and Unger, P., editors. *Semantics and Philosophy*, New York University Press, pp. 49–61. Reprinted in and cited according to: David Lewis, *Philosophical Papers*, volume I. Oxford: Oxford University Press, pp. 250–260.

[10] Montague, R. (1970). Universal grammar. *Theoria*, **36**(3), 373–398.

[11] Pagin, P., & Westerståhl, D. (2010). Compositionality I: Definitions and variants. *Philosophy Compass*, **5**(3), 250–264.

[12] Pickel, B., & Rabern, B. (2021). The myth of occurrence-based semantics. *Linguistics and Philosophy*, **44**, 813–837.

[13] ———. (2023). Against Fregean quantification. *Ergo*, **9**(37), 971–1007.

[14] Quine, W. V. O. (1957). The scope and language of science. *British Journal for the Philosophy of Science* VIII (29), 1–17.

[15] ———. (2001). Confessions of a confirmed extensionalist. In Floyd, J., and Shieh, S., editors. *Future Pasts: The Analytic Tradition in Twentieth-Century Philosophy*. Oxford: Oxford University Press, pp. 215–221.

[16] Rabern, B. (2013). Monsters in Kaplan's logic of demonstratives. *Philosophical Studies*, **164**(2), 393–404.

[17] Salmon, N. (2006). A theory of bondage. *Philosophical Review*, **115**(4), 415–448.

[18] Stojnić, U. (2021). *Context and Coherence*. Oxford: Oxford University Press.

[19] Stojnić, U., Stone, M., & Lepore, E. (2013). Deixis (even without pointing). *Philosophical Perspectives*, **27** (1), 502–525.

[20] ———. (2017). Discourse and logical form: Pronouns, attention and coherence. *Linguistics and Philosophy*, **40**(5), 519–547.

[21] ———. (2020). Pointing things out: In defense of attention and coherence. *Linguistics and Philosophy*, **43**, 139–148.

[22] Szabó, Z. G., & Thomason, R. H. (2019). *Philosophy of Language*. Cambridge: CUP.

[23] Tarski, A. (1935). Der Wahrheitsbegriff in den formalisierten Sprachen. *Studia Philosophica*, **I**, 261–405. Translated by W. H. Woodger as 'The Concept of Truth in Formalized Languages,' pp. 152–278 in A. Tarski, *Logic, Semantics, Metamathematics*, ed. J. Corcoran, Oxford: Oxford University Press, 1956.

[24] Wehmeier, K. F. (2018). The proper treatment of variables in predicate logic. *Linguistics and Philosophy*, **41**(2), 209–249.

[25] ———. (2021). Are quantifiers intensional operators? *Inquiry* **64**(5–6), 511–532. Reprinted in M. Kölbel and D. Rey (eds), *Operators vs Quantifiers*, Routledge, Abingdon 2024, 5–26.

[26] Zimmermann, T. E. (1991). Kontextabhängigkeit. In von Stechow, A., and Wunderlich, D., editors. *Semantik/Semantics*. Berlin–New York: de Gruyter, pp. 156–229.

[27] ———. (1993). Scopeless quantifiers and operators. *Journal of Philosophical Logic*, **22**, 545–561.

DEPARTMENT OF LOGIC AND PHILOSOPHY OF SCIENCE
SCHOOL OF SOCIAL SCIENCES
UNIVERSITY OF CALIFORNIA, IRVINE
IRVINE, CA 92697-5100, USA
E-mail: wehmeier@uci.edu