An addendum

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(*Received* 20*th June* 1934. *Read* 2*nd November*, 1934.)

A generalised lemma used in the second of two papers[1] enables us, as was suggested there, to extend results of the first. Thus, among others, we easily get the following:

Let t_n *be determined by the relation*

$$t_n = c_n^1 \, t_{n-1} + c_n^2 \, t_{n-2} + \ldots . + c_n^m \, t_{n-m} + e_n \, y_n$$
where $\lim y_n = 0$.

Then $t_n \to 0$, *if, denoting* $| c_n^r |$ *by* d_n^r *everywhere,*

(i) $d_n^1 < 1$ *for all* n

(ii) $\prod_{p=1}^{n} d_p^1 \to 0$ *as* $n \to \infty$

(iii) $| e_n | \leqslant K \, (1 - d_n^1)$, *where* K *is fixed*

(iv) $\prod_{n=1}^{\infty} \left(1 + \frac{d_n^2}{d_n^1 \, d_{n-1}^1} + \frac{d_n^3}{d_n^1 \, d_{n-1}^1 \, d_{n-2}^1} + \ldots . + \frac{d_n^m}{d_n^1 d_{n-1}^1 \ldots . d_{n-m+1}^1} \right)$ *is convergent.*

Condition (iv) will be satisfied if the series

$$\sum^{n=\infty} \frac{d_n^r}{d_n^1 \, d_{n-1}^1 \ldots . \, d_{n-r+1}^1}$$ converge, where $r = 2, 3, \ldots m$.

Making the substitutions

$$c_n^r = \frac{a_n^r}{1 + a_n^1}, \qquad e_n = \frac{1}{1 + a_n^1}$$

a theorem recently proved by Sunouchi[2] follows, namely :

Let $(1 + a_n^1) \, t_n = a_n^1 \, t_{n-1} + a_n^2 \, t_{n-2} + \ldots . + a_n^m \, t_{n-m} + y_n$ *where* $\lim y_n = 0$.

Then $t_n \to 0$ *if*

(i) $a_n^1 > 0$ *for all* n,

(ii) $\sum_{n=1}^{\infty} \frac{1}{a_n^1}$ *diverges,*

(iii) $\sum_{n=r}^{\infty} \frac{| a_n^r |}{a_n^1} \prod_{s=1}^{r-1} \left(1 + \frac{1}{a_{n-s}^1} \right)$ *converges, where* $r = 2, 3, \ldots . m$.

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[1] *Proc. Edinburgh Math. Soc.*, (2) 3 (1932), 147-150 and 220-222.

[2] G. Sunouchi, "Theorems on limits of recurrent sequences," *Proc. Imperial Academy*, X (1934) 4-7.