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ALMOST DISJOINT FAMILIES OF REPRESENTING SETS

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The thesis is concerned with a number of problems in Combinatorial Set Theory. The Generalized Continuum Hypothesis is assumed.

Suppose λ and κ are non-zero cardinals. By successively identifying κ with pairwise disjoint sets of power κ, a function $f : \lambda \to \kappa$ can be viewed as a transversal of a pairwise disjoint (λ, κ) family A. Questions about families of functions in ${}^{\lambda}\kappa$ can thus be thought of as referring to families of transversals of A. We wish to consider generalizations of such questions to almost disjoint families; in particular we are interested in extensions of the following two problems:

(i) What is the 'maximum' cardinality of an almost disjoint family of functions each mapping λ into κ ?

(ii) Describe the cardinalities of maximal almost disjoint families of functions each mapping λ into κ.

For this purpose we introduce the notion of a choice set of an almost disjoint family of sets. In almost all cases, a choice set of an almost disjoint family of sets is a 'large' set which intersects each member of the family non-trivially. This notion, it turns out, is an appropriate generalization to almost disjoint families of the notion of a transversal.

The thesis is divided into three parts; the first deals with families of choice sets.

We first consider the problem of determining the 'maximum'

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cardinality, say $CS(A)$, of an almost disjoint family of choice sets of a given almost disjoint (λ, κ) family A. If $\lambda \le \kappa$ then A always possesses a 'large' almost disjoint family of choice sets. When $\lambda > \kappa$, however, A need not even possess an almost disjoint pair of choice sets and the problem becomes to determine the sequence of cardinals ζ for which there exists an almost disjoint (λ, κ) family A with $CS(A) = \zeta$. For this it is necessary to distinguish two cases. If A is an almost disjoint (λ, κ) decomposition of a cardinal Σ and $\lambda > \kappa$ then either

(i) $\lambda = \Sigma^+$ (this is possible only if $\Sigma' = \kappa'$), or

(ii) $\lambda = \Sigma$.

For case (i) we show that if $\Sigma \ge \kappa$ and $\Sigma' = \kappa'$ then there is an almost disjoint (Σ^+, κ) decomposition A of Σ with $CS(A) = \zeta$ if and only if $1 \le \zeta \le \kappa^+$ or $\zeta = \Sigma^+$. A similar result holds in case (ii) and we make substantial progress towards establishing this. Some unresolved problems, however, remain.

We also consider the problem of describing the cardinalities of families of choice sets of A that are maximal with respect to almost disjointness. These are described fully in the case when $\lambda \le \kappa$.

Part 2 deals with problems (i), (ii) applied to families of partial functions and families of partial choice sets. It is a major theme that once the functions or choice sets are required only to be 'partial' then 'large' almost disjoint families of such functions and choice sets can be found.

Chapter 4 deals with families of partial functions. A *μ-partial function* of λ into κ is a function which maps some μ-sized subset of λ into κ. We show there is always a 'large' almost disjoint family of μ-partial functions of λ into κ. The cardinalities of maximal almost disjoint families of μ-partial functions of λ into κ are also described.

Chapter 5 considers families of μ-partial choice sets of a given almost disjoint (λ, κ) family A. A *μ-partial choice set* of A is defined to be a μ-sized choice set of some μ-sized subfamily of A. We show there is always a 'large' almost disjoint family of μ-partial choice sets of A and describe the cardinalities of maximal almost disjoint families of μ-partial choice sets.

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Part 3 deals with a problem of a totally different nature. We consider the following question about families of sets: to what extent is it possible to specify the 'maximum' cardinalities of subfamilies with small degrees of disjunction? In Chapter 6 we introduce the notion of a disjunction sequence of a family of sets and present some results about such sequences. These include some sufficient conditions for a sequence of cardinals to be the disjunction sequence of an almost disjoint family of sets.

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