Non-Separable Preferences in the Statistical Analysis of Roll Call Votes

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Abstract

Conventional multidimensional statistical models of roll call votes assume that legislators' preferences are additively separable over dimensions. In this article, we introduce an item response model of roll call votes that allows for non-separability over latent dimensions. Conceptually, non-separability matters if outcomes over dimensions are related rather than independent in legislators' decisions. Monte Carlo simulations highlight that separable item response models of roll call votes capture non-separability via correlated ideal points and higher salience of a primary dimension. We apply our model to the U.S. Senate and the European Parliament. In both settings, we find that legislators' preferences over two basic dimensions are non-separable. These results have general implications for our understanding of legislative decision-making, as well as for empirical descriptions of preferences in legislatures.

Keywords: roll call voting, item response models, legislator behavior, U.S. Senate, European Parliament

1 Introduction

In political science, legislators' votes are generally explained by legislators' preferences and the expected policy outcomes of proposals (Hare and Poole 2015). Ideal point models build on a spatial representation of these two concepts—ideal points of legislators and outcomes of proposals are represented in a low-dimensional Euclidean space. Statistical models make assumptions about the decision-making process to infer these latent parameters based on observed behavior.[1] A central assumption is that preferences over multiple dimensions are *additively separable*. This implies that the dimensions defining the space have "nothing to do with each other" (Ordeshook 1986) in legislators' decisions and it is present in prominent statistical models of roll call votes (Clinton, Jackman, and Rivers 2004; Poole and Rosenthal 1985). However, the original multidimensional spatial voting model explicitly allows for the possibility of non-separability over dimensions (Davis, Hinich, and Ordeshook 1970; Hinich and Munger 1997).

Non-separability implies that outcomes over dimensions are related in actors' decision-making process. While classical work discusses non-separability as a consequence of budgetary constraints (Hinich and Munger 1997), others have argued that non-separability is a more general feature of decision-making in multiple dimensions (Milyo 2000).[2] Formally, non-separability implies that actors consider the direction of deviations between their unconditional ideal points and outcomes over dimensions jointly when deciding between options. This means that both the overall *distance* and the overall *direction* of deviations matter in an actor's evaluation of proposals.

Political Analysis (2023)
vol. 31: 352–365
DOI: 10.1017/pan.2022.11

Published
24 May 2022

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Edited by
Jeff Gill

1 Recent publications discuss the implications of relaxing certain assumptions, such as the dimensionality of the space (Aldrich *et al.* 2014), the shape of the utility function (Carroll et al. 2013), heterogeneity in stochastic error (Lauderdale 2010), and other parametric assumptions (Tahk 2018).

2 Indeed, non-separability has been considered in distinct areas such as legislative voting on multiple issues (see, e.g., Kadane 1972), voting in simultaneous elections or referenda (see, e.g., Lacy and Niou 2000), survey responses (see, e.g., Lacy 2001b), EU council bargaining (see, e.g., Finke and Fleig 2013), and electoral decisions (see, e.g., Stoetzer and Zittlau 2015).

Building on the derivation of the IDEAL estimator by Clinton *et al.* (2004), we introduce an item response model for roll call votes in legislatures that incorporates non-separability via a weight matrix that captures non-separability across dimensions (i.e., the interaction of deviations across dimensions) and dimensional salience (i.e., the weight attributed to deviations on each dimension). A Monte Carlo simulation reveals that a separable specification estimates correlated ideal points and higher salience of the primary dimension under non-separability, thereby distorting the spatial representation of legislators and their behavior.

We apply the model to roll call votes in the U.S. Senate and the European Parliament (EP). In line with prior research, we assume that legislative behavior in both settings can be characterized by two basic dimensions (e.g., Hix, Noury, and Roland 2006; Poole and Rosenthal 1991). In the Senate, we find substantial levels of non-separability between an interparty dimension (characterized by conflict on economic issues) and an intraparty dimension (characterized by conflict on racial/social issues) in the majority of Congresses since 1945. The differences between Senators' ideal points from the non-separable and the separable specification mirror those from Monte Carlo simulations, and have implications for empirical descriptions of political conflict in the Senate: accounting for non-separability changes dimensional salience (interparty conflict was more important earlier on) and orthogonalizes the political landscape (ideal points are less correlated across dimensions) compared to results from the separable specification. We also find that dimensions in the EP—a left/right dimension and an EU integration dimension—generally have a non-separable relationship in legislators' roll call votes.

These findings have substantial implications for our understanding and description of politics in legislatures. As we highlight, non-separability figures in the decision-making processes of legislators in the U.S. Senate and the EP. Substantially, this implies that proposals bundling specific combinations of outcomes over dimensions are more likely to pass than others. Non-separability therefore may partially explain the importance and the composition of package deals in legislatures (Kadane 1972; Kardasheva 2013). Descriptively, estimates from the separable specification are distorted in the presence of non-separability. These results suggest that one reason for correlated ideal points estimated via established scaling techniques, such as W-NOMINATE (Poole and Rosenthal 1985) and IDEAL (Clinton *et al.* 2004), is non-separability. Explicitly incorporating non-separability in the statistical analysis of roll call votes aids in painting a clearer picture of the latent dimensional structure of political decision-making.

2 A Statistical Model for Roll Call Analysis with Non-Separable Preferences

In this section, we describe our statistical model for roll call votes with non-separable preferences. The theoretical foundation of ideal point estimation are spatial voting models that define actors' utility functions with stochastic disturbances and functional forms for estimation (see, e.g., Clinton *et al.* 2004). Statistical models of roll call analysis assume that legislators' preferences across dimensions are *additively separable*, so that an actor's utility is given by the (weighted) sum of deviations along dimensions. Non-separability also allows for the interaction of deviations across dimensions and is part of the canonical weighted Euclidean distance model of spatial voting (Davis *et al.* 1970; Hinich and Munger 1997).

Budget negotiations are the classical example where non-separability matters in multidimensional decision-making (Hinich and Munger 1997). Because a single budget limits total spending, preferences over individual issues are rendered non-separable: more spending in one area must be offset by less spending in another area. Overarching budgetary constraints induce *quid pro quo* trade-offs across issues.

It is likely that non-separability also matters in other settings of multidimensional decision-making (Milyo 2000). In models of roll call votes, proposals can have implications on multiple dimensions representing conflict in distinct policy areas (e.g., as a consequence of issue bundles across areas; Kadane 1972; Kardasheva 2013). Therefore, legislators generally take deviations from their unconditional ideal point across dimensions into account when casting their votes. The question is whether these deviations enter actors' utility functions solely in an additively separable manner or whether joint cross-dimensional interactions of deviations (as implied by non-separability) also figure in their utility functions.

What could induce such a relationship across dimensions? In an electoral choice example, Stoetzer and Zittlau (2020) argue that distinct issue dimensions are related at a broader ideological level in voters' decisions (see also Stoetzer and Zittlau 2015). Therefore, voters may favor a party representing a specific combination of deviations from their unconditional ideal point to an equally distant party representing a different combination of deviations. Similarly, Finke (2009) argues that preferences over distinct areas are non-separable in political negotiations because the final outcome is an aggregated result of competition between negotiators over each area. Negotiators favor certain aggregate directions of an outcome to others as they give and take across areas, resembling the *quid pro quo* of budgetary negotiations. Comparable rationales could apply to decision-making in legislatures when actors consider the aggregate direction implied by the combination of deviations.

Hence, when proposals have implications on dimensions and these are related at an aggregate level in actors' considerations, it is theoretically plausible to consider non-separability in their decision-making processes. To incorporate non-separability in ideal point estimation, we assume that the utility of actors follows a weighted quadratic Euclidean distance model (Davis *et al.* 1970; Hinich and Munger 1997). The spatial utility function for legislator i with ideal point $\boldsymbol{\theta}_i$ regarding a proposal j with outcome $\mathbf{p}_j$ in a D-dimensional space (both $\boldsymbol{\theta}_i, \mathbf{p}_j \in \mathbb{R}^D$) is assumed to be

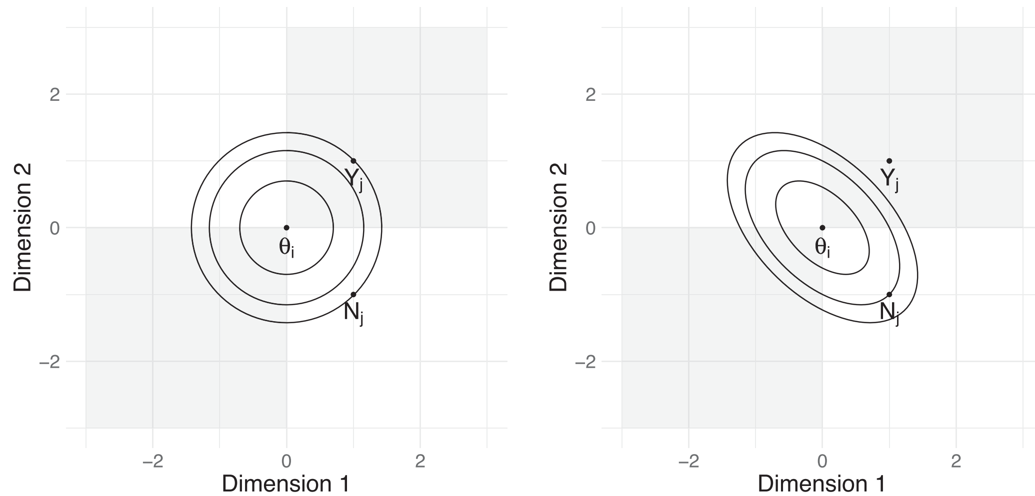
$$v_i\left(\mathbf{p}_j\right) = -[\mathbf{p}_j - \boldsymbol{\theta}_i]'\mathbf{A}[\mathbf{p}_j - \boldsymbol{\theta}_i]. \tag{1}$$

$\mathbf{A}$ is a $D \times D$ weighting matrix that is assumed to be symmetric positive definite. The diagonal entries of $\mathbf{A}$ are dimensional salience weights that reflect the relevance of distances on a particular dimension in a legislator's utility function. The off-diagonal entries of $\mathbf{A}$ contain non-separability parameters and reflect whether and how distances on one dimension are related to distances on any other dimension in the utility function. If an off-diagonal entry equals zero, preferences across the corresponding pair of dimensions are separable, otherwise they are non-separable.

The derivation of a statistical model for roll call votes from this spatial utility function assumes that legislators compare the utility of a Yea-vote to the utility of a Nay-vote and vote accordingly (Clinton *et al.* 2004). We illustrate the implications of non-separability with a two-dimensional example that shows the unconditional ideal point of a legislator $\boldsymbol{\theta}_i$ and her indifference contours in Figure 1. We denote $\mathbf{Y}_j$ as the Yea position and $\mathbf{N}_j$ as the Nay position of a bill in $\mathbb{R}^D$. In both panels, $\mathbf{Y}_j$ and $\mathbf{N}_j$ are equally distant from $\boldsymbol{\theta}_i$. While $\mathbf{Y}_j$ represents deviations in the same direction across dimensions, $\mathbf{N}_j$ combines deviations in opposing directions.

In the left panel of Figure 1, the legislator's preferences are separable and she is indifferent between $\mathbf{Y}_j$ and $\mathbf{N}_j$ because her utility equally decreases in all directions away from her ideal point. In the right panel, her preferences are non-separable and the off-diagonal elements of $\mathbf{A}$ are positive. Therefore, her utility decreases faster when deviations in the same direction across dimensions are combined than when deviations in opposing directions are combined in a substitutionary manner. $\mathbf{N}_j$ is favored over $\mathbf{Y}_j$ because of the aggregate direction implied by the combination of deviations across dimensions. Finally, if non-separability were negative, the contours would be flipped across either axis. Then, $\mathbf{Y}_j$ would be preferred to $\mathbf{N}_j$ as it combines deviations in equal directions across dimensions in a complementary manner.

While this illustration clarifies the mathematical working of non-separability in utility functions, its substantial interpretation depends on the meaning and rotation of the underlying dimensions.



(a) Separable $\mathbf{A} = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$ (b) Non-separable $\mathbf{A} = \left(\begin{smallmatrix} 1 & .5 \\ .5 & 1 \end{smallmatrix}\right)$

Figure 1. Illustration in two dimensions for a proposal with $\mathbf{Y}_j = [1, 1]$ and $\mathbf{N}_j = [1, -1]$.

For example, the two dimensions may represent conflict on economic and social issues with lower values indicating more left-wing outcomes and higher values indicating more right-wing outcomes on both dimensions. Then, a substitutionary relationship would imply that actors favor outcomes combining more left-wing deviations on one issue dimension with more right-wing deviations on the other dimension, while a complementary relationship favors deviations in the same direction across dimensions. If either the meaning or the rotation of a dimension were altered, the same non-separability parameter would represent an altered combination of favored deviations. Crucially, non-separability can only be meaningfully interpreted relative to the latent dimensions.

Stochastic aspects influence legislators' decisions beside the systematic component given by the utilities $v_i\left(\mathbf{Y}_j\right)$ and $v_i\left(\mathbf{N}_j\right)$. Following Clinton *et al.* (2004), we assume that errors for the Yea utility η_{ij} and the Nay utility ν_{ij} are jointly normal distributed with $E[\eta_{ij}] = E[\nu_{ij}] = 0$. We further assume that $Var[\eta_{ij} - \nu_{ij}] = 1$. Given that the overall utility is $u_i\left(\mathbf{Y}_j\right) = v_i\left(\mathbf{Y}_j\right) + \eta_{ij}$ and $u_i\left(\mathbf{N}_j\right) = v_i\left(\mathbf{N}_j\right) + \nu_{ij}$, legislator i votes for a proposal $y_{ij} = 1$ when $u_i(Y_j) > u_i(N_j)$ and opposes a proposal $y_{ij} = 0$ otherwise. This results in a probabilistic choice model in which $\Phi(.)$ is the cumulative density function of the standard normal distribution:

$$\begin{aligned} P(y_{ij} = 1) &= P\left(v_i\left(\mathbf{Y}_j\right) + \eta_{ij} > v_i\left(\mathbf{N}_j\right) + \nu_{ij}\right) \\ &= P\left(v_i(\mathbf{Y}_j) - v_i(\mathbf{N}_j) > \nu_{ij} - \eta_{ij}\right) \\ &= \Phi\left(v_i(\mathbf{Y}_j) - v_i(\mathbf{N}_j)\right) \\ &= \Phi\left(-[\mathbf{Y}_j - \boldsymbol{\theta}_i]'\mathbf{A}[\mathbf{Y}_j - \boldsymbol{\theta}_i] + [\mathbf{N}_j - \boldsymbol{\theta}_i]'\mathbf{A}[\mathbf{N}_j - \boldsymbol{\theta}_i]\right) \\ &= \Phi\left(2(\mathbf{Y}_j - \mathbf{N}_j)'\mathbf{A}\boldsymbol{\theta}_i - \mathbf{Y}_j'\mathbf{A}\mathbf{Y}_j + \mathbf{N}_j'\mathbf{A}\mathbf{N}_j\right). \end{aligned} \tag{2}$$

We can define a multidimensional item response theory (IRT) model based on this specification. The first term of the equation is a function of the Yea and Nay positions of the proposal, the weight matrix, and the legislator's ideal points. Collecting the proposal-specific terms gives a vector of *discrimination parameters* $\boldsymbol{\beta}_j = 2(\mathbf{Y}_j - \mathbf{N}_j)$. The second part of the equation depends on the proposal-specific parameters and the weight matrix, which we denote as the *difficulty parameter* $\alpha_j = -\mathbf{Y}_j'\mathbf{A}\mathbf{Y}_j + \mathbf{N}_j'\mathbf{A}\mathbf{N}_j$, resulting in an IRT model:

$$P(y_{ij} = 1) = \Phi\left(\boldsymbol{\beta}_j'\mathbf{A}\boldsymbol{\theta}_i + \alpha_j\right). \tag{3}$$

In this formulation as an IRT model, non-separability can influence the systematic component via the product of a legislator's ideal point on one dimension and a proposal's discrimination parameter on another dimension. Preferences are additively separable if the off-diagonal elements of the weight matrix are zero, so that the cross-dimensional product equals zero too. Then, the systematic component is determined by the (weighted) sum of products between ideal points and discrimination parameters of each dimension.[3] If off-diagonal elements are not equal to zero, then this analysis is altered: cross-dimensional interactions representing non-separable preferences can influence the systematic component determining legislators' decisions.

2.1 Identification and Estimation

Multidimensional latent variable models such as the IRT model described above are not identified without further constraints regarding the issues of location, scale, and rotation (Bafumi et al. 2005; Jackman 2001; Rivers 2003). The locational issue arises because a constant can be added and subtracted from either term of the model without altering the result: $\beta_j'\mathbf{A}\boldsymbol{\theta}_i + \alpha_j = (\beta_j'\mathbf{A}\boldsymbol{\theta}_i - a) + (\alpha_j + a)$. The scalar issue addresses that any element of the interaction term can be multiplied by a constant if another element is divided by the same constant: $\beta_j'\mathbf{A}\boldsymbol{\theta}_i = (\beta_j/b)'\mathbf{A}(b\boldsymbol{\theta}_i)$. Finally, $\mathbf{A}$ can be decomposed and multiplied with both sides of the interaction, leading to rotational invariance: $\beta_j'\mathbf{A}\boldsymbol{\theta}_i = \beta_j'\mathbf{R}'\mathbf{R}\boldsymbol{\theta}_i = (\mathbf{R}\beta_j)'(\mathbf{R}\boldsymbol{\theta}_i) = \tilde{\beta}_j'\tilde{\boldsymbol{\theta}}_i$.[4] Therefore, we employ standard constraints on the means and variances of the ideal points and the discrimination parameters to address the issues of location and scale, while we use a hierarchical modeling approach for the ideal points to address the rotational issue (similar to Bafumi et al. 2005).[5]

This identification approach comes with a set of advantages. Importantly, it disentangles distinct quantities of interest: variation in non-separability and dimensional salience in legislative decision-making processes is captured by the weight matrix, whereas variation in the substantive meaning of conflict along dimensions is captured by the modeling of the ideal points. The confirmatory modeling approach regarding the latent dimensions is based on prior knowledge and allows us to assess additional quantities of interest regarding them. The estimated coefficients relating covariates to the ideal points tell us how strongly each covariate influences the position of legislators along each dimension. Comparing the predicted values of $\widehat{\boldsymbol{\theta}}$ based on the systematic component of their parametrization to the estimated values of $\boldsymbol{\theta}$, we can assess how inductive the covariates are overall for the spatial dispersion of legislators via the R^2 statistic (Gelman et al. 2019).

We estimate the model using Bayesian inference. Given the assumption of independence of the observed roll call votes for bills and legislators conditional on parameters, the item response model has the following likelihood:

$$L(\boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{A}, \boldsymbol{\theta} \mid \mathbf{Y}) = \prod_{i=1}^{N} \prod_{j=1}^{J} P(y_{ij} = 1)^{y_{ij}} (1 - P(y_{ij} = 1))^{(1-y_{ij})}, \quad (4)$$

where $\boldsymbol{\beta}$ is a $J \times D$ matrix that collects the discrimination parameters β_j of J proposals in rows, $\boldsymbol{\alpha}$ is a vector capturing proposals' difficulty parameters, $\mathbf{A}$ is the weight matrix, $\boldsymbol{\theta}$ is an $I \times D$ matrix

3 Clinton *et al.* (2004) is a special case of this where the weight matrix is an identity matrix, $\mathbf{A} = \mathbf{I}$. Then, the expression reduces to the standard multidimensional IRT model: $P(y_{ij} = 1) = \Phi\left(\beta_j'\boldsymbol{\theta}_i + \alpha_j\right)$.

4 This rotational issue also arises when $\mathbf{A} = \mathbf{I}$, if $\mathbf{R}$ is an orthogonal rotation matrix so that $\mathbf{R}'\mathbf{R} = \mathbf{I}$.

5 Following Rivers (2003), a standard D-dimensional model can be identified by imposing $k = D(D+1)$ independent constraints. In the two-dimensional settings discussed in the subsequent applications ($k = 6$), these constraints are imposed by fixing the means and variances of the ideal points $\theta_{.d}$ to 0 and 1 on both dimensions ($d \in (1, \dots, D)$; four constraints) and modeling their dimensionwise distribution based on legislator-specific covariates (two constraints; Bafumi et al. 2005) to address rotational invariance. An additional source of multiplicative indeterminacy is introduced compared to the standard model by allowing dimensional salience weights to vary. We address this by fixing the variance of the discrimination parameters $\beta_{.d}$ to 1 on both dimensions ($d \in (1, \dots, D)$; two constraints).

of ideal points $\boldsymbol{\theta}_i$ of I legislators in rows, and $\mathbf{Y}$ is an $I \times J$ matrix of binary recorded votes, with y_{ij} at position (i, j).

We specify priors for the item parameters β and α, the weight matrix $\mathbf{A}$, and the ideal points $\boldsymbol{\theta}$. Item parameters are assigned standard normal distributions as prior distributions, so that $\beta_{jd} \sim \mathcal{N}(0, 1)$ and $\alpha_j \sim \mathcal{N}(0, 1)$. We decompose $\mathbf{A}$ in a manner similar to a variance-covariance matrix composed of a correlation matrix and scaling parameters ($A = \tau\Omega\tau$; see Barnard, McCulloch, and Meng 2000).[6] This decomposition assures that $\mathbf{A}$ is symmetric positive definite. The correlation matrix component Ω of the weight matrix $\mathbf{A}$ is drawn from a prior distribution defined in Lewandowski, Kurowicka, and Joe (2009) with $\nu = 1$ and the scale parameters τ from a truncated normal distribution $\tau \sim \mathcal{N}_+(0, 1)$. Finally, the ideal point of legislator i on dimension $d \in (1, \ldots, D)$, θ_{id} is drawn from a normal distribution with legislator-specific mean $\theta_{id} \sim \mathcal{N}(\widehat{\theta}_{id}, 1)$. The parametrization of $\widehat{\theta}_{id}$ is application-specific, and we discuss it in more detail below. We approximate the posterior distribution of our model using Markov Chain Monte Carlo simulations, relying on the No-U-Turn Sampler as implemented in Stan (for code, see Section 1 of the Supplementary Material; Carpenter *et al.* 2017), and assess convergence across chains via the $\widehat{R}$ statistic (Gelman and Rubin 1992).

3 Monte Carlo Simulations

In this section, we compare the estimates from a separable and non-separable model using Monte Carlo simulations.[7] We focus on two parameters: (i) the dimensional salience weights in estimated matrices $\mathbf{A}$ and (ii) estimated ideal points $\boldsymbol{\theta}$. Both are of central interest to applied researchers, as they speak to the relevance and substance of dimensions, and the spatial positioning of legislators. More details of the data-generating process and additional results (including results across scenarios, further parameters, and model comparisons) are provided in Section 2 of the Supplementary Material.

Throughout, we specify that 100 legislators consider deviations from their ideal points along two dimensions and vote on 250 roll call votes. We simulate a total of 1,000 legislatures for which we vary the extent of non-separability, the relative salience of the two dimensions in the weight matrix $\mathbf{A}$, and the correlation between ideal points $\boldsymbol{\theta}$ to accomodate a realistic range of potential scenarios. In line with our confirmatory identification approach, ideal points $\theta_{.d}$ are modeled via $\widehat{\theta}_{.d} = \gamma_d z_d$, where $\gamma_d = 1$ and $z_d \in \{-1, 0, 1\}$. z_d resembles a group membership index, specifying which group of legislators should be positioned toward which end of the latent scale. All other parameters (α, β) are sampled from standard normal distributions.

A separable specification overestimates the salience of a primary dimension in the presence of non-separability, thereby distorting estimates of dimensional salience in legislators' roll call votes. Figure 2 displays the deviation in estimated dimensional salience in weight matrices $\mathbf{A}$ across varying levels of non-separability. If preferences are separable and non-separability equals 0, both specifications return equal estimates. However, the separable specification—represented by the dashed line—systematically overestimates the salience of the primary dimension and underestimates that of the secondary dimension at higher absolute levels of non-separability. The non-separable specification—the solid line—does not exhibit this pattern. The non-separable model estimates the true non-separability value accurately (see Figure 2 in the Supplementary Material).

The estimated ideal points $\boldsymbol{\theta}$ from a separable specification less accurately reflect the original latent parameters if non-separability matters. Figure 3a displays the correlation between true and estimated ideal points across dimensions. Overall, the correlation decreases as non-separability increases. However, this pattern is more pronounced for the separable specification than for

6 Throughout, results concerning non-separability are based on off-diagonal elements of Ω, while those concerning dimensional salience are based on the relative weights of τ.

7 Replication code for this article is available at Binding and Stoetzer (2022) at https://doi.org/10.7910/DVN/XLY5AJ.

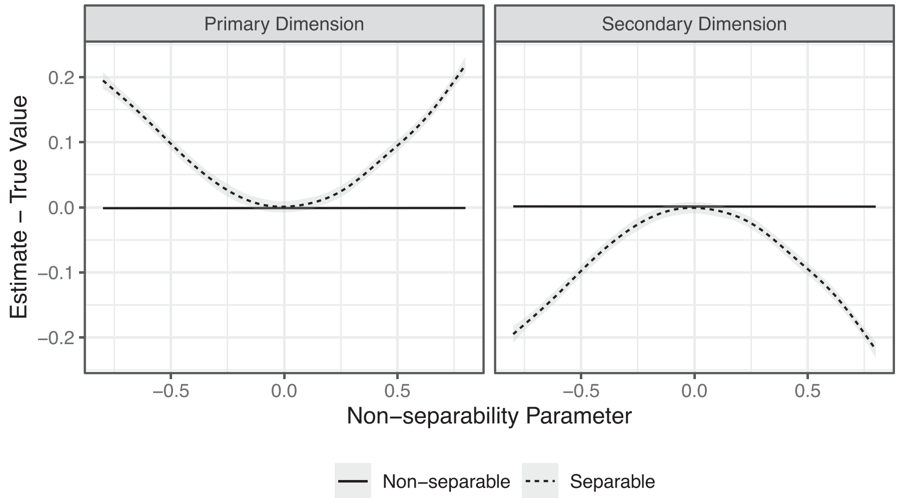
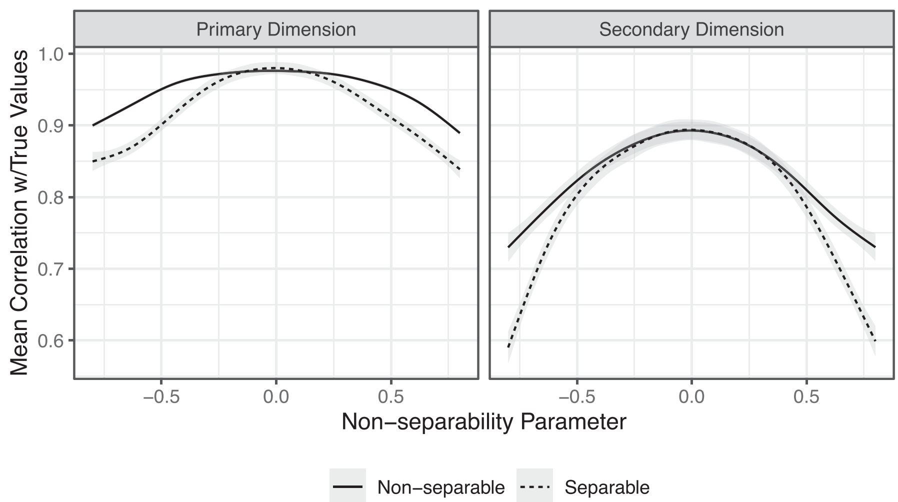
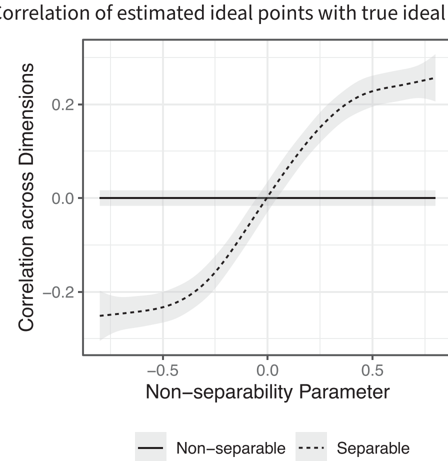


Figure 2. Deviation in estimated dimensional salience in weight matrices **A**.



(a) Correlation of estimated ideal points with true ideal points.



(b) Correlation of estimated ideal points across dimensions.

Figure 3. Estimated ideal points $\boldsymbol{\theta}$.

https://doi.org/10.1017/pan.2022.11 Published online by Cambridge University Press

the non-separable specification. The separable specification estimates an increasingly distorted representation of legislators' preferences at higher absolute values of non-separability compared to the non-separable specification.

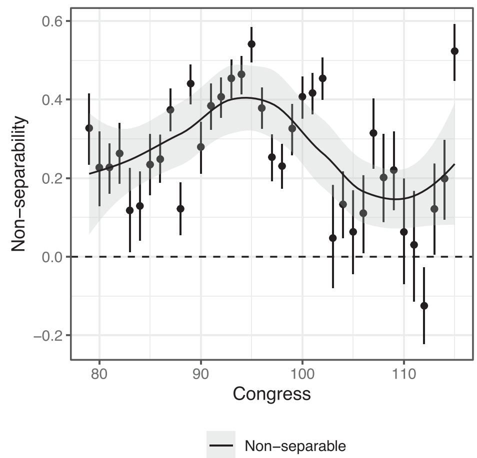
These distortions—in dimensional salience and ideal points—are attributable to the way by which the separable specification accommodates non-separability: the estimated ideal points become increasingly related across dimensions. This is visible in Figure 3b, which displays the correlation of ideal points across dimensions. Across simulations, the expected correlation equals zero. After estimation, ideal points from the separable specification are correlated in a manner consistent with the direction and size of non-separability. Similar to the other quantities focused on in our discussion, the non-separable specification does not exhibit similar patterns.

These results concentrate on distortions relative to latent parameters that are known in simulations, but unknown otherwise. Therefore, we also consider model fit relative to the data. Out-of-sample fit approximated by the leave-one-out cross-validation (LOO) information criteria shows that the non-separable specification generally performs better (see Figure 7 in the Supplementary Material; Vehtari, Gelman, and Gabry 2017), while in-sample predictions show little difference between the two models (see Figure 8 in the Supplementary Material). This is in line with the observation that the separable specification estimates alternative sets of latent parameters to explain the same variance in observed roll call votes. At increasing levels of non-separability, the effective dimensionality of the latent space decreases: if two dimensions are perfect substitutes or complements, indifference contours collapse to a line representing a mixture of the original latent dimensions. In such a scenario, estimates from the separable model resemble an increasingly unidimensional scenario (via deviations in dimensional salience) in which the dominant dimension also reflects variation on the secondary dimension (via correlated ideal points) as it cannot accomodate such dimensionality reduction via non-separability. These patterns are more accentuated when both dimensions are similarly salient. The non-separable specification more accurately portrays legislators' decision-making processes in such scenarios as characterized by distinct, but related latent dimensions.

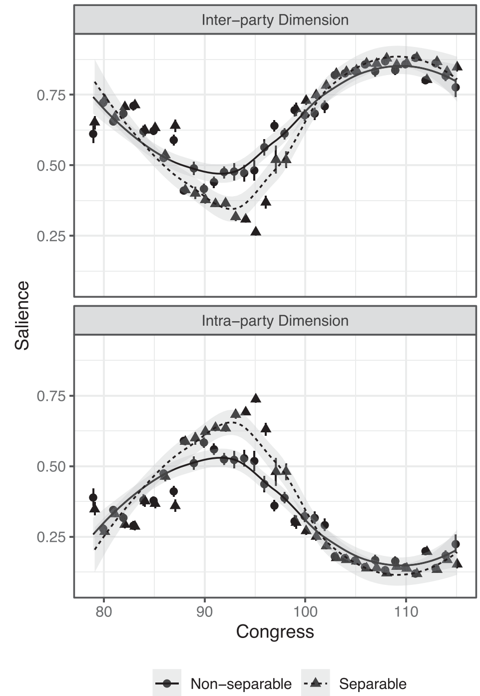
4 Applications

We focus on two applications, the U.S. Senate (37 Congresses between 1945 and 2019; Lewis *et al.* 2020) and the EP (five sessions between 1979 and 2004; Hix *et al.* 2006). We compare the results from the non-separable and separable model for each Congress and EP session separately, focusing on the same parameters as in the Monte Carlo discussion: (i) non-separability and dimensional salience as elements of the weight matrix $\mathbf{A}$ and (ii) legislators' ideal points $\boldsymbol{\theta}$. More details and results are provided in Sections 3 and 4 in the Supplementary Material for the Senate and the EP, respectively.

In line with prior research, we assume that roll call votes in both settings are determined by two latent dimensions whose salience and substantive meaning may vary over time. In the U.S. Senate, these dimensions reflect interparty conflict between Democrats and Republicans as well as intraparty conflict within parties (e.g., Clausen and Cheney 1970; Nye 1991; Poole and Rosenthal 1991, 2001, 2007). While the former conflict revolves around economic issues, the substance of the latter varies more over time: in earlier Congresses, intraparty conflict was more strongly associated with racial/social issues. But as party lines became the increasingly important determinant of legislative behavior in the U.S. Senate overall, the substantive meaning of this dimension has come to vary more as it accommodates residual variation in votes not due to partisanship (Poole and Rosenthal 1991, 2001, 2007). The meaning of the two dimensions in the EP is more consistent over time: while one dimension can be characterized as a left-right divide, a second dimension reflects conflict regarding EU integration (Brack 2013; Hix *et al.* 2006).



(a) Estimated non-separability parameter across Congresses.



(b) Estimated dimensional salience parameters across Congresses and dimensions.

Figure 4. U.S. Senate—estimated weight matrices **A**.

We parametrize the distribution of legislators' ideal points along dimensions accordingly. In the United States, Democrats are pit against Republicans to identify the interparty dimension, while Senators from racially liberal coastal states are juxtaposed with those from racially conservative states of the former Confederacy to identify the intraparty dimension (lower values on dimensions represent more liberal positions; Acharya, Blackwell, and Sen 2016; Key Jr. 1949). In the EP, members of left-wing and of right-wing party groups stand at opposing ends of the left–right dimension, while members of EU-favorable and EU-sceptic party groups occupy the poles of the EU dimension (lower values represent more left-wing or EU-favorable positions).

Additionally, we leverage that many legislators are incumbents to aid in the identification of the dimensions over time. In the U.S. Senate, around 85% of Senators in one Congress will also be present in the next Congress, while this is the case for around 45% of the members of parliament in a session of the EP. We exploit this by adding the residual of a legislator's ideal point from her group-level mean in the last legislative session as a individual-level predictor of her ideal point in the current session on the same dimension (akin to a random walk in dynamic models; e.g., Lo 2018).[8]

4.1 U.S. Senate

We begin our discussion with the U.S. Senate. Elements of the estimated weight matrices (**A**) of each Congress after 1945 are shown in Figure 4. The estimated non-separability parameters from the non-separable specification are shown in Figure 4a (mean and 95% credible interval). The

8 In other words, the systematic component of a legislator's ideal point on dimension $d\theta_{idt}$ in session t is $\widehat{\theta}_{idt} = \gamma_d z_{id} + \delta_{\theta_{idt-1}}$, where the vector z_d contains a group-level indicator for the dimension ($z_d \in \{-1, 0, 1\}$), and $\delta_{\theta_{idt-1}}$ is a legislator's residual from his or her group-level mean on the corresponding dimension in the last session. If a legislator was not present at $t-1$, $\delta_{\theta_{idt-1}} = 0$.

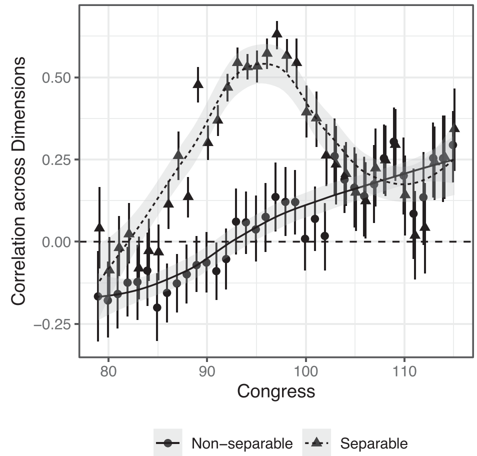


Figure 5. U.S. Senate estimated ideal points $\boldsymbol{\theta}$—correlation of ideal points across dimensions.

resulting estimates indicate a substantial degree of non-separability across dimensions in most Congresses. Cross-dimensional considerations prominently figure in legislative behavior in the U.S. Senate, and the underlying general dimensions are not independent in Senators' roll call votes.

The nonzero non-separability parameters have implications for the estimates of dimensional salience in the decision-making process of Senators as shown in Figure 4b. In line with the Monte Carlo simulations, the separable specification (indicated by the dashed line) overestimates the salience of a dominant dimension and underestimates the weight of the secondary dimension compared to the non-separable model. This pattern is especially visible in the period between the 89th Congress (1965–1967) and the 98th Congress (1983–1985). This period is characterized both by comparatively high levels of non-separability and similarly salient dimensions. Whereas the non-separable specification estimates roughly equal salience weights for both dimensions during this period (indicated by the solid line), the separable specification attributes more weight to the intraparty dimension than to the interparty dimension. Since then, the salience of the interparty dimension has increased and that of the intraparty dimension has decreased in both specifications (in line with, e.g., Poole and Rosenthal 2007).

The influence of non-separability is also visible in the correlation of ideal points across dimensions in Figure 5. Similar to the findings of the Monte Carlo simulations, the separable specification accommodates non-separability by estimating correlated ideal points. From the 86th Congress (1959–1961) to the 102nd Congress (1991–1993), the separable specification estimates a much higher degree of positive correlation across dimensions than the non-separable specification. Accordingly, the correlation between the estimated ideal points across models on the secondary intraparty dimension drops to values of around .6 to .8 in this period (see Figure 10 in the Supplementary Material).

A clarification in the substantive meaning of the intraparty dimension by explicitly incorporating non-separability is visible if we compare the ideal points of Senators across specifications. In Figure 6, the top 10 Senators in terms of their spatial shifts to the extremes of this dimension are shown for the period between the 89th Congress and the 99th Congress, just after the Civil Rights Era.[9] Substantially, the intraparty dimension is more strongly determined by racial/social issues in the non-separable specification: while liberal members of the Republican Party such as Jacob Javits, John Chaffee, and Charles Percy shift downward toward a more liberal position on this

9 We focus on Senators who were present in at least four Congresses in this period (a total of 137 Senators), calculate their mean ideal point in either specification, and calculate the distance between the non-separable and the separable models.

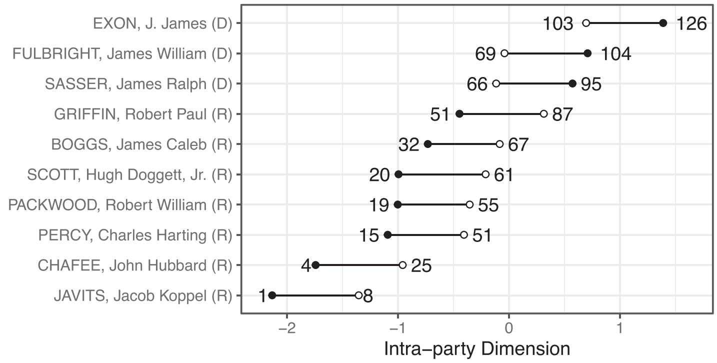


Figure 6. U.S. Senate—shifts in estimated ideal points θ on the intraparty dimension. ∘ = separable specification, • = non-separable specification. The rank of each Senator (out of 137) is shown beside each ideal point.

dimension, conservative Democrats such as J. James Exon, James Fulbright, and James Sasser shift upward toward a more conservative position. We also find that Senators are more strongly polarized according to whether they stem from racially/socially conservative or liberal states on this dimension in the non-separable rather than the separable specification in this period (see Figures 11 and 12 in the Supplementary Material).

Taken together, the positive non-separability estimate in the period just after the Civil Rights Era indicates that Senators favored outcomes combining deviations in opposite directions across dimensions (rather than the same direction). This means that proposals bundling more liberal outcomes on one dimension with more conservative outcomes on the other dimension were preferred to those combining more liberal or conservative outcomes on both (because lower values are associated with more liberal preferences and higher values are associated with more conservative preferences on both dimensions in this time). The non-separable specification reflects this *quid pro quo* between economic and racial/social issues in legislators' decisions, highlights the presence of distinct, but related dimensions, and clarifies the meaning of the intraparty dimension in this period.

4.2 European Parliament

Non-separability also figures in legislative behavior in the EP. Figure 7 displays the recovered non-separability parameter in the five sessions of the EP. In four sessions, the estimated parameter is positive and the 95% credible intervals do not include zero. Similar to the post–Civil Rights Era in the U.S. Senate, the two dimensions act as substitutes: legislators favor proposals representing deviations in opposite directions on the left/right dimension and the EU integration dimension. Because of the polarity of these dimensions, this means that proposals combining more right-wing outcomes with more EU integration (or more left-wing outcomes with less integration) are preferred to those representing deviations in the same direction. The degree of non-separability has increased in the fourth and fifth sessions, potentially indicating that the association between dimensions has increased in EP roll call voting.

The comparatively low absolute level of non-separability limits its impact on other estimated parameters. The two sets of estimates are very similar across specifications. This concerns estimated salience parameters (Figure 21 in the Supplementary Material), the correlation between estimated ideal points across specifications (Figure 22 in the Supplementary Material), the estimated coefficients of covariates structuring the ideal points (Figure 26 in the Supplementary Material), as well as the extent to which the ideal points are structured by the systematic component of their parametrization (Figure 25 in the Supplementary Material).

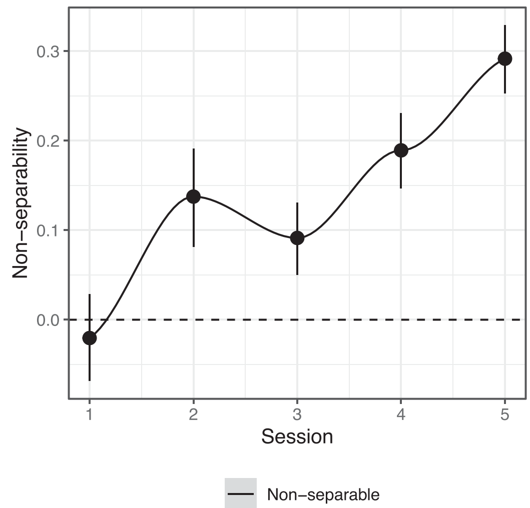


Figure 7. EP—estimated non-separability parameter across sessions.

5 Discussion

A long and fruitful tradition in political science analyzes legislators' behavior in terms of spatial models (e.g. Clinton *et al.* 2004; Hare and Poole 2015; Poole and Rosenthal 1991, 2007). In this paper, we introduce a statistical model that permits legislators' preferences over dimensions to be non-separable. This means that legislators consider both the overall *distance* and the overall *direction* of deviations between their ideal point and proposals. Monte Carlo simulations highlight that our model captures the degree of non-separability, while standard separable models capture non-separability via correlated ideal points and higher salience of a primary dimension. We find substantial levels of non-separability in both the post-war U.S. Senate and in the EP—non-separability is the norm rather than the exception.

While our model integrates long-standing theoretical considerations in line with the original multidimensional spatial voting model, it also relies on assumptions that have been debated in the literature. For example, we assume that two dimensions matter in our applications. While this is a common assumption (e.g., Poole and Rosenthal 2007), it has not passed without criticism (Aldrich, Montgomery, and Sparks 2014). In general, our approach can be extended to scenarios characterized by more than two dimensions (Kim, Londregan, and Ratkovic 2018). We rely on functional form assumptions about the utility functions and error distributions that have been relaxed elsewhere (Carroll et al. 2013; Tahk 2018). We believe, however, that relying on these assumptions while relaxing that of separability allows us to isolate the impact of non-separability as an oftentimes overlooked theoretical consideration in the statistical study of legislative decision-making.

On a more substantial note, research (see, e.g., Finke and Fleig 2013; Lacy 2001a; Stoetzer and Zittlau 2020) highlights that non-separability also matters in other areas beyond legislative behavior. This paper adds to this body of research in general, and introduces a novel model which allows for an explicit test of non-separability in future applications. For example, judicial behavior (Martin and Quinn 2002) or public opinion (Treier and Hillygus 2009) may more aptly be described as structured by multiple non-separable dimensions than either a single dimension or multiple independent dimensions. In general, the description of the dimensionality of the political space in the presence of non-separability requires further attention. Non-separability induces an overall reduction of the effective dimensionality of a political space (Stoetzer and Zittlau 2020)—if two dimensions are perfect substitutes or complements (as in budgetary negotiations), the effective dimensionality reduces to one (composed of a mixture of the original two distinct dimensions). In this respect, it would be useful to develop descriptions and distinctions for the evaluation of the effective dimensionality of political spaces in future research.

Acknowledgments
We would like to thank Marco Steenbergen, Lucas Leemann, Guy Whitten, Marc Ratkovic, Yunkyu Sohn, Martin Elff, Lanny Martin, and Susumu Shikano for valuable comments on the manuscript. Earlier versions of the paper have been presented at EPSA 2018 in Vienna, APSA 2018 in Boston, the Standing Group Conference of ECPR 2019 on Advances in the Empirical and Theoretical Study of Parliaments in Konstanz, and the publication seminar of the IPZ in Zurich.

Data Availability Statement
Replication code for this article is available at Binding and Stoetzer (2022) at https://doi.org/10.7910/DVN/XLY5AJ.

Supplementary Material
For supplementary material accompanying this paper, please visit https://doi.org/10.1017/pan.2022.11.

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