CORRESPONDENCE.

ON THE SIGNIFICANCE OF THE EXPRESSION $\frac{1}{1+a}-(1-v)$.

To the Editor of the Assurance Magazine.

SIR,—It is a trite remark, that we know, or think we know, many things, while we may be very far from apprehending their full significance. I have just met with an instance of this, which appears to me to be not devoid of interest.

We have all been long familiar with the expression, first deduced by Mr. Milne, for the annual premium for assurance—namely, $\frac{1}{1+a}-(1-v)$. This, when attention is called to it, must be admitted to be a remarkable

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expression; and yet there has not, so far as I know, any attempt been made to interpret it. Circumstances have led me to attend to it; and I now, in consequence, propose the following

THEOREM.

If a' be the present value of an annuity due of £1, payable during the continuance of any status, then will the annual premium, π, for a benefit of £1, receivable at the end of the year in which that status fails, be determinable by the equation—

$$\pi = \frac{1}{a'} - (1-v).$$

For, if the arrangement were that the £1 benefit should be receivable *now*, instead of at the end of the year in which the status fails, the equitable premium would obviously be $\frac{1}{a'}$, that is, the annuity due which that £1 would purchase; but inasmuch as the £1 is not receivable now, each payment of the above premium made while the £1 is withheld will have to be diminished by the *discount* (not the *interest*) on £1. Hence the foregoing equation holds.

Attention to the terms of the proposition will, of course, enable us to distinguish the cases in which it is applicable from those in which it is not.

1. It is applicable to assurances on single lives, and on joint lives, of whatsoever number the combination may consist, when the assurance is, in the one case for the whole of life, and in the other for the joint duration. This is well known; but it is not applicable in either of these cases if the assurance is only for a term, inasmuch as the status will not *necessarily* fail, and the benefit consequently will not *necessarily* become payable, during or at the end of the term.

2. It is applicable to an assurance and endowment on either a single life or a combination of any number of joint lives; for here the benefit will necessarily become payable, either during the term, by the failure of the status, or at its termination if the status continue to subsist.

To test this:—We know that the annual premium for this benefit is

$$\frac{M_x - M_{x+n} - D_x}{N_{x-1} - N_{x+n-1}},$$

which, if we substitute in it, for M_x and M_{x+n}, their values in terms of D and N, by the formula $M_x = D_x - (1-v)N_{x-1}$, reduces to

$$\frac{D_x}{N_{x-1} - N_{x+n-1}} - (1-v).$$

And this is of the desiderated form, for the first term is the annuity due, for n years, on (x), that £1 paid now will purchase.

This formula was given (first, I believe) by your correspondents Mr. Sprague and "H. A. S.," on pp. 112 and 117 of your eighth volume; and it was consideration of it, with its analogy to the whole life premium for assurance, that led me into the present inquiry.

3. The property enunciated is applicable to assurances on last survivors if the premium is payable till the benefit becomes due, but not otherwise.

4. It is not applicable to survivorship assurances, for here the benefit does not *necessarily* become due by the failure of the status.

5. It is applicable to the case of a sum payable at the end of a term, and subject to no contingency.

Thus, the sum payable in n years being £1, its present value is v^n. And the present value of an annuity of £1 for n years being $\frac{1-v^n}{r}$, anticipating all the payments a year, we have for that of the same annuity due, $\frac{1-v^n}{vr}$, or $\frac{1-v^n}{1-v}$. Now let π be the premium required—

$$\text{Hence } \frac{\pi(1-v^n)}{1-v}=v^n, \text{ and } \therefore \pi=\frac{(1-v)v^n}{1-v^n}.$$

Adding and subtracting $(1-v)$, we get—

$$\frac{(1-v)v^n}{1-v^n}+(1-v)-(1-v)=\frac{(1-v)v^n+(1-v)(1-v^n)}{1-v^n}-(1-v)$$

$$=\frac{1-v}{1-v^n}-(1-v),$$

which verifies the property, since $\frac{1-v}{1-v^n}$ is, as above, the annuity due of n payments that £1 will purchase.

I am, Sir,

Your most obedient servant,

P. GRAY.

7, St. Paul's Villas, Camden Town,
Dec. 10*th*, 1861.

P.S.—Since the foregoing was written, the following additional application of the principle employed has occurred to me:—

THEOREM.

If a' denote the present value of an annuity due of £1, on the continuance of a specified status, then will the present value of £1, receivable at the end of the year in which that status shall fail, be denoted by

$$1-(1-v)a'.$$

£1, payable now, is worth, of course, £1. To defer its payment one year, the equitable consideration would be, the disbursement by the debtor of $1-v$; and, obviously, the deferring of the payment to the failure of the status would be compensated by the disbursement of $1-v$ at the commencement of each year during the subsistence of the status. Hence, the present value of all these disbursements being $(1-v)a'$, that of £1, payable at the end of the year in which the status shall fail, is

$$1-(1-v)a'.$$

We know the truth of this in regard to contingent assurances. But it is also true in the case in which no contingency is involved, that is, when the status is simply a term of years, n. For, in this case, $a'=\frac{1-v^n}{1-v}$. Hence, by substitution—

$$1-(1-v)a'=(1-v)\frac{1-v^n}{1-v}=1-(1-v^n)=v^n.$$

P. G.

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