

PROBLEMS FOR SOLUTION

P 141. Let $v_i = (\alpha_{i1}, \dots, \alpha_{in})$, $i = 1, \dots, m$, be vectors, where α_{ij} are integers such that the greatest common divisor of all the α_{ij} is 1. Prove that there exist integers k_i such that the greatest common divisor of the components of $v = k_1 v_1 + \dots + k_m v_m$ is 1.

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P 142. Let A be a commutative noetherian ring, S a multiplicatively closed subset of A , M an A -module, $s \in S$ and $m \in M$, so $m/s \in M_S$. If ann denotes annihilator in A , prove that $\text{ann}(m/s) = \text{ann}(s'm)$ for some $s' \in S$. (This is used implicitly in Lang, Algebra, p. 151, Proposition 10).

K. Taylor, McGill University

P 143. Find all metric spaces which have no infinite compact sets.

J. Marsden, Princeton University

P 144. Prove that a normed linear space X is an inner product space if and only if for each set $S \subset X$ and $z \in S$, S_z is convex where

$$S_z = \{x: \|x-z\| = \inf_{y \in S} \|x-y\|\}.$$

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P 145. If two disjoint subsets of a metric space have the property that every function lipschitz on each is lipschitz on their union, then every function continuous on each is continuous on their union. Prove this and give an example to show that this is false if the sets are not disjoint.

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