

# SOME RECENT STUDIES OF NUTATION IN THE U. S. S. R.

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**RÉSUMÉ.** — L'auteur compare les amplitudes des principaux termes de la nutation selon diverses théories avec leurs valeurs tirées des observations. Il rend compte également de la découverte, par Popov, d'un terme de variation des latitudes ayant une période légèrement plus courte qu'un jour sidéral.

**ABSTRACT.** — The author compares amplitudes of the main terms of nutation as derived from various theories with the observed values. He also reports on the discovery, by Popov, of a term in the variation of latitude, with a period slightly shorter than a sidereal day.

**ZUSAMMENFASSUNG.** — Verf. vergleicht die Amplituden der Hauptglieder der Nutation, wie sie aus verschiedenen Theorien hergeleitet wurden, mit den beobachteten Werten. Ausserdem berichtet er über ein von Popov entdecktes Glied in der Breitenschwankung, dessen Periode ein wenig kürzer als ein Sterntag ist.

**Резюме.** — Автор сравнивает амплитуды главных членов нутации согласно различных теорий, пользуясь значениями полученными из наблюдений. Он тоже дает отчет об открытии Поповым, члена колебания широт имеющего период слегка короче чем звездные сутки.

The theory of precession and nutation used until now in the reduction of astronomical observations is based on the assumption that the Earth is a rigid body, though first attempts to take into consideration the effect of a liquid core on the rotation of the Earth were made as far back as by the end of the 19th century. However no data on the actual shape, mass and constitution of the core had been available at that time. Later these data were obtained mainly from the study of propagation of seismic waves. Then it rendered possible to advance to quantitative results the theory of rotation of the Earth with a liquid core.

This has been done by Jeffreys and Vicente for two models of the Earth composed of an elastic shell and a liquid core [1]. In one of these the

core is replaced by a homogeneous incompressible fluid with an extra particle in the centre (model I). In the other it is taken as having a quadratic density distribution, the variation of density being taken as wholly due to compression (model II). Molodensky too has considered two models [2]. One of his models is also based on the assumption that the variation of density in the core is due to adiabatic compression (model III). In the other he has taken into account an inner core with the radius equal to one fifth of that of the Earth. It is assumed that the inner core is homogeneous and incompressible (model IV).

The following values of coefficients of the principal ( $\Omega$ ), fortnightly ( $2\zeta$ ) and semi-annual ( $2\circ$ ) terms of nutation in obliquity and longitude have been obtained for these models.

Model.	Nutation in obliquity.			Nutation in longitude.		
	$\zeta$ .	$2\zeta$ .	$2\circ$ .	$\zeta$ .	$2\zeta$ .	$2\circ$ .
I.....	9".2015	0".0972	0".5734	6".826	0".0896	0".5232
II.....	9.2187	0.0971	0.5403	6.8491	0.0897	0.4883
III.....	9.1963	0.0969	0.5770	6.8325	0.0899	0.5274
IV.....	9.1997	0.0965	0.5745	6.8369	0.0895	0.5255
Observed values....	9.1974	0.0965	0.578	6.8437	0.0934	0.533
Mean errors.....	$\pm 10$	$\pm 11$	—	$\pm 10$	$\pm 11$	—

Observed values represent the results obtained :

- (1) for the principal term, by the author, from observations of the International Latitude Service;
- (2) for the fortnightly term, by the author, from the same observations combined with the values derived by A. J. Orlov from latitude observations at Pulkovo [3], and by Morgan from those at Washington [4];
- (3) for the semi-annual term, by Popov, from observations of two bright zenith stars at Poltava [5].

It should be noticed that in [6] the results of the International Latitude Service were analysed for both the corrections to coefficients and the terms giving phase shifts in the 19-yearly nutation.

However neither the studies of Jeffreys and Vicente, nor those of Molodensky, take into consideration the effect of viscosity of the Earth and tidal friction, and consequently no phase shifts could result from their theoretical calculations. Moreover, Jeffreys considers it extremely unlikely that there are any measurable phase shifts at all. For these reasons we have omitted the terms giving phase shifts in our previous calculations. Then the 19-yearly variation of latitude can be ascribed to some inaccuracy of the adopted values of the major and minor semi-

axes of the nutational ellipse. Let  $\Delta M$  and  $\Delta N$  be corrections to these semi-axes. Then we shall have

$$\Delta\varphi = \Delta M \cos \alpha \sin \Omega - \Delta N \sin \alpha \cos \Omega,$$

where  $\alpha$  is right ascension of the observed star pair.

The observed values given in the table have been derived in just this way. The comparison of these values with theoretical ones shows that they are in the best agreement with model IV. The discrepancy may be partly due to some uncertainty of the temperature coefficients of scale values adopted in the reduction of observations.

Recently one more effect of a liquid core on the rotation of the Earth has been revealed from latitude observations. It is well known that in the case of the elastic Earth the pole would execute a free motion with the period of 14 months. As it has been shown by Sloudsky [7] and Hough [8], in the case of the Earth with a liquid core there must exist one more period of the free polar motion approximately equal to a sidereal day. Recently the exact length of this period has been calculated by Molodensky. For models III and IV he found respectively (in sidereal time).

$$T_0 = 23 \text{ h } 56 \text{ m } 56 \text{ s}, \quad T_0 = 23 \text{ h } 56 \text{ m } 54 \text{ s}.$$

Then Popov succeeded in finding the period of 23 h 56 m 54 s in the variation of latitude. He used 23-yearly series of observations of bright zenith stars  $\alpha$  *Persei* and  $\eta$  *Ur. Maj.* at Potava. The free polar motion with the period  $T_0$  should produce the term

$$l_t = 1.957 a_0 \sin \left( \frac{2\pi}{T_0} t - \alpha_m + \nu_0 \right)$$

in the differences of latitude obtained from observations of these stars. In this expression  $\alpha_m$  is the mean right ascension of the stars,  $t$  time measured in sidereal days. The amplitude  $a_0$  and phase  $\nu_0$  are to be derived from observations. This was done by least square fit of  $a_0$  and  $\nu_0$  to 2437 observed differences of latitude. Thus Popov has obtained

$$\Delta\varphi_0 = 0''.020 \cos \left( \frac{2\pi}{T_0} t - \alpha_m + 99^\circ \right) \begin{matrix} \pm 4 \\ \pm 10 \end{matrix}$$

Then to check this result he has divided the whole series of observations into two sub-series and has analysed each of them separately. For the sub-series 1939 to 1949 and 1950 to 1962 he has found respectively

$$\Delta\varphi_0 = 0''.023 \cos \left( \frac{2\pi}{T_0} t - \alpha_m + 89^\circ \right) \begin{matrix} \pm 8 \\ \pm 18 \end{matrix}$$

$$\Delta\varphi_0 = 0''.018 \cos \left( \frac{2\pi}{T_0} t - \alpha_m + 102^\circ \right) \begin{matrix} \pm 6 \\ \pm 20 \end{matrix}$$

The results indicate that the Earth's pole moves with the period little shorter than a sidereal day as it was predicted by some writers dealing with the dynamical effect of the liquid core.

*Note* : In [3] the correction for the tidal variation of latitude was applied to the fortnightly terms of nutation with the wrong sign. The mistake is corrected in this paper.

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