16. The Distribution of $1/a$ in Photographic Meteor Orbits

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A study is made of the distribution of reciprocal semi-major axis in photographic meteor orbits. A detailed classification of the orbits is made according to quality. The distribution of $1/a$ *in precise orbits is multimodal with two broad maxima approximately centered on 0.05 and 0.40* $(AU)^{-1}$*. Minima in the distribution appear near 0.20 and 0.66* $(AU)^{-1}$ *corresponding to Jupiter's and Mars' position in the* $1/a$ *diagram. Considerable fine structure appears in the* $1/a$ *distribution. Resonance gaps corresponding to commensurabilities with Jupiter are detected. The gaps are similar to the well studied Kirkwood gaps in the asteroid belt.*

DURING THE LAST 20 TO 30 YEARS photographic double-station programs have provided detailed information about radiants and orbits of individual meteors. The number of individual orbits listed in the photographic catalogues is about 3600, or if only precisely reduced orbits are considered about 1800. The number of precisely reduced photographic meteor orbits is thus now comparable to the number of asteroid and comet orbits.

The purpose of the present investigation is to study the frequency distribution of the reciprocal of the semi-major axis of precisely reduced meteor orbits and in particular to look for fine structure in the distribution.

OBSERVATION MATERIAL

The study includes the majority of precisely reduced photographic two station orbits published to date. In all 1822 meteor orbits, comprising 1218 meteors recorded in the Harvard meteor program and 604 recorded in the Dushanbe and Odessa programs, are used.

The methods used in the Harvard program for obtaining the meteor path, velocity and orbit from the segmented trials on the photographic plates, are described in detail in Whipple and Jacchia (1957) and Hawkins (1957). Descriptions of the Dushanbe reductional program are given by Katasev (1957) and Babadjanov and Kramer (1963).

The survey included 139 small-camera meteor orbits published by Whipple (1954), 413 Super-Schmidt meteor orbits listed by Jacchia and Whipple (1961), 313 Super-Schmidt meteor orbits listed by Hawkins and Southworth (1961) and 352 Super-Schmidt meteor orbits listed by Posen and McCrosky (1967). Of the published 144 small-camera orbits, five were excluded because of incomplete data. Of the 360 orbits in the Hawkins and Southworth random samples of Super-Schmidt meteors 47 are excluded since they are already in the 413 orbits listed by Jacchia and Whipple. The total number of Harvard orbits used in the analysis was 1218. The fireball orbits published by McCrosky (1967), were not used in this study, since the fireball data represent a selection of extremely bright objects and their orbits (semi-major axis) are

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probably influenced by observational bias (Kresák, 1970).

The survey further included 73 small-camera orbits listed by Katasev (1957), 225 orbits recorded at Odessa and listed by Babadjanov and Kramer (1963, 1967) and 330 orbits recorded at Dushanbe and published by Babadjanov and Kramer (1963, 1967) and Babadjanov et al. (1969). Of the 73 meteors published by Katasev 24 are excluded because of incomplete data. The total number of USSR orbits available in the study was 604.

The data sample used in the present investigation differs from that of a previous study (Lindblad, 1971a) in the following minor respects: (1) a few hyperbolic meteors which were omitted in the first study have now been included; (2) an additional 77 Dushanbe orbits have been included in the analysis; (3) a total of 100 fireball orbits have been excluded.

DATA CLASSIFICATION AND PREPARATION

Classification on the Basis of Quality

Because of the necessity to select orbits of very high precision we have assigned to each meteor orbit a degree of reliability. This index of orbital accuracy was obtained as follows.

Whipple (1954) and Jacchia and Whipple (1961) assign to each individual orbit a quality class. Whipple classified the small-camera meteor orbits as of quality A, A−, B and C. Only orbits marked A and A− were assigned by us to the high accuracy group. Jacchia and Whipple list the Super-Schmidt orbits as of quality 1.0, 1.5, . . ., 4.0. Classes 1.0, 1.5 and 2.0 were assigned to the high accuracy group. Hawkins and Southworth (1961) do not give a quality index but in Hawkins and Southworth (1958) they list the standard deviation Δv_∞ of the extrapolated extra-atmosphere velocity v_∞. This quantity is directly related to the observational error in $1/a$, and was therefore used by us as a quality index. Only orbits for which $\Delta v_\infty \leqslant 0.1$ km/s were assigned to the high precision group. Posen and McCrosky (1967) do not give a quality index but list the number of segments measured on each individual meteor trail. Trails with 20 or more measured segments were assigned by us to the high precision group.

In the Odessa and Dushanbe catalogues no direct quality index is given. The catalogues, however, list the quantity sin Q, where Q is the angle between the apparent great circles of motions as seen from the two observing stations. Only orbits with sin $Q \geq 0.40$ were assigned by us to the high accuracy group. This index of quality is particularly appropriate for the Dushanbe orbits, since a number of these were collected using a very short base line between the two observing stations (Stohl, 1970).

The number of orbits assigned to the high and low accuracy groups are 755 and 1067, respectively. Thus slightly less than half of the data sample (41 percent) falls into the high accuracy group.

Computer Program

Orbital elements and other relevant data for each meteor were available on cards. The reciprocal of the semi-major axis, $1/a$, was computed to 0.0001 (AU)$^{-1}$. The program prepared histograms of $1/a$, in class intervals of 0.0050 (AU)$^{-1}$. The program further prepared a smoothed distribution of the data and then compared this distribution with the observed one in order to obtain a chi-square test of significance.

DISTRIBUTION OF RECIPROCAL SEMI-MAJOR AXIS

The frequency distribution of reciprocal semi-major axis $1/a$ of the high accuracy and low accuracy samples are compared in figure 1. Shower meteors, representing about 50 percent of the data, are included in both diagrams. No correction for observational selection (cosmic weight) has been applied to the data.

Figure 1 shows for the more accurate orbits a multimodal distribution. Two broad maxima, corresponding to the long period and short period orbits, respectively, are approximately centered on 0.05 and 0.40 (AU)$^{-1}$, while a third, less pronounced maximum is centered on about 0.71 (AU)$^{-1}$. The paucity of orbits near $1/a = 0.20$ is clearly related to Jupiter's position in the diagram. We note that this minimum is not very pronounced in the low accuracy sample, indicating that precision orbits are necessary in order to resolve details of the $1/a$ distribution.

The main maximum in the distribution of $1/a$

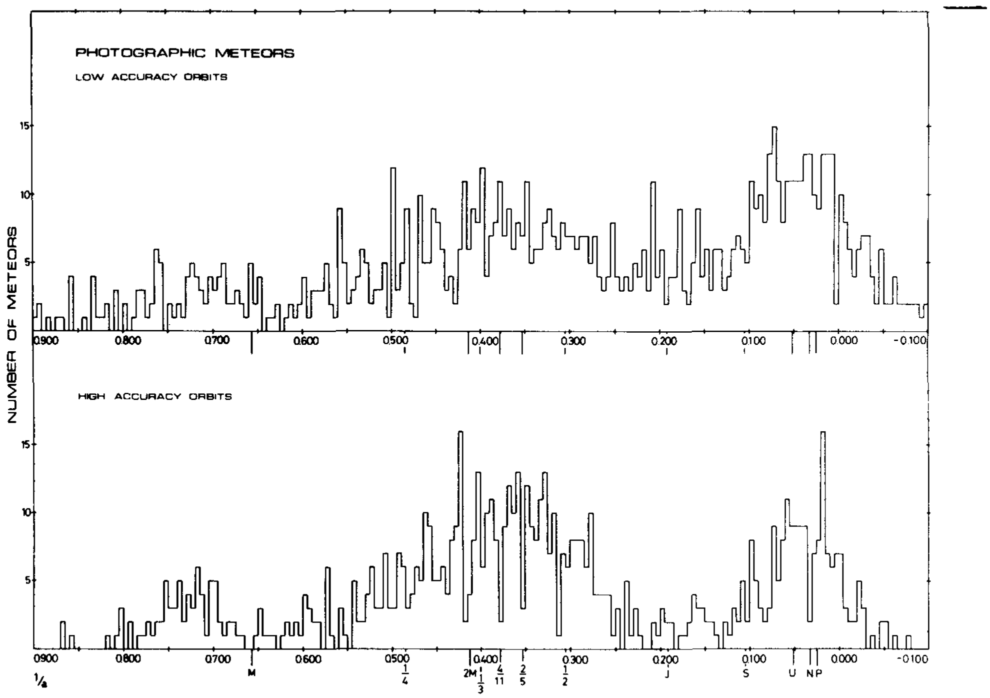

FIGURE 1.—Distribution of photographic meteor orbits in reciprocal semi-major axis in intervals of 0.0050 AU^{-1}.

coincides rather closely with the corresponding distribution for asteroids. If only accurate orbits in the interval 0.20–0.55 $(AU)^{-1}$ are considered the mean reciprocal semi-major axis of short period meteor orbits is 0.404 $(AU)^{-1}$. This value is surprisingly similar to the corresponding value 0.366 $(AU)^{-1}$ deduced from the numbered asteroid population. We interpret this result as indicating that the orbital distribution of minor objects in the solar system is to a large extent determined by the perturbational effects of Jupiter.

The second maximum, corresponding to long period meteor orbits, includes a number of stream meteors in retrograde orbits (Perseids, Orionids, etc.). It, however, also includes a surprisingly large number of sporadic meteors in both direct and retrograde orbits.

About 5 percent of the accurate orbits are hyperbolic. The number of hyperbolic orbits in the accurate Harvard and USSR samples, are 16 and 19 respectively. As may be seen from figure 1 most of these orbits are very near to the hyperbolic limit. In the less accurate data sample 14 percent of the orbits are hyperbolic. (A number of hyperbolic orbits are outside the range of the diagram.) The larger percentage of hyperbolic orbits in the low accuracy sample is evidence that the majority of hyperbolic orbits are the result of measuring errors.

Stream Orbits vs Sporadic Orbits

The above data analysis has been performed regardless of the stream—or sporadic—character of the individual meteor orbits. In a further study the sample was subdivided into a stream and a sporadic component using the D-criterion proposed by Southworth and Hawkins (1963). This computer method provides an impersonal means of separating sporadic and shower meteors. It is known to give very reliable results in photographic data samples (Lindblad, 1971b). The computer search was undertaken at the rejection level $D_s = 0.13$.

The $1/a$ distribution for stream and sporadic meteors, respectively, are compared in figure 2. Only accurate orbits are included in the diagram.

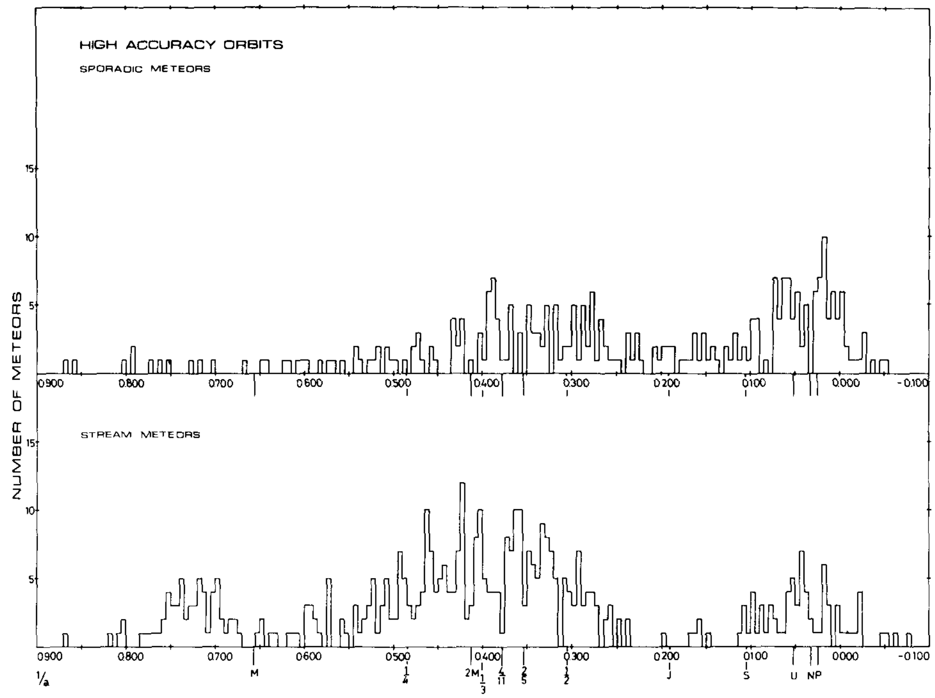

FIGURE 2.—Distribution of meteor orbits in reciprocal semi-major axis in intervals of 0.0050 AU^{-1}. Accurate orbits only.

Noticeable differences exist between the two distributions. It is observed that the separation between short period and long period orbits is more pronounced in the stream component. The grouping of orbits between 0.67 and 0.77 $(AU)^{-1}$ in the stream sample is missing in the sporadic sample. Inspection of the data showed that this maximum was ascribable to the Geminid stream. It should hence not be considered as an intrinsic property of the $1/a$-distribution.

It is interesting to speculate on the different $1/a$ distributions for stream and sporadic meteors. If the Geminids are excluded one could perhaps consider the sporadic meteor histogram as an error-dispersed version of the stream histogram. The sporadic meteors may be of greater age and are thus more gravitationally perturbed than stream meteors. It is, however, difficult to understand how any meteors can exist for any length of time in an orbit with an $1/a$ value near to Jupiter's value.

The large number of long period sporadic orbits could be indicative of a reservoir of non-stream meteors in the outer parts of the solar system. A significant fraction of the sporadic meteors may be of considerable age and may have an evolutionary history different from that of the stream meteors. This question deserves further detailed study.

Direct vs Retrograde Orbits

Direct and retrograde orbits of stream meteors are compared in figure 3. Again, only accurate orbits are included. Inspection of the diagram indicates that two types of stream meteors are predominant in the data samples; stream meteors having $1/a>0.20$ and moving in direct orbits, stream meteors having $1/a<0.20$ and moving in retrograde orbits. It is observed that there are practically no stream meteors moving in short period, retrograde orbits, and only very few moving in long-period, direct orbits.

A similar study of the non-stream meteor group, figure 4, indicated a somewhat more dispersed system. The two groups mentioned above are present in the diagram but in addition

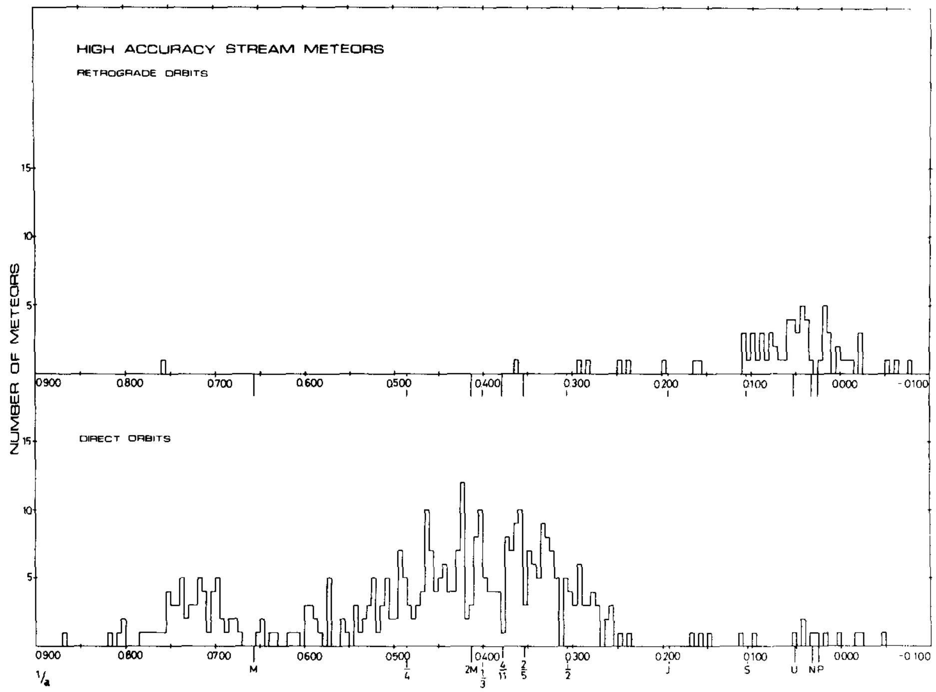

FIGURE 3.—Distribution of meteor orbits in reciprocal semi-major axis in intervals of 0.0050 AU^{-1}. Accurate stream orbits only.

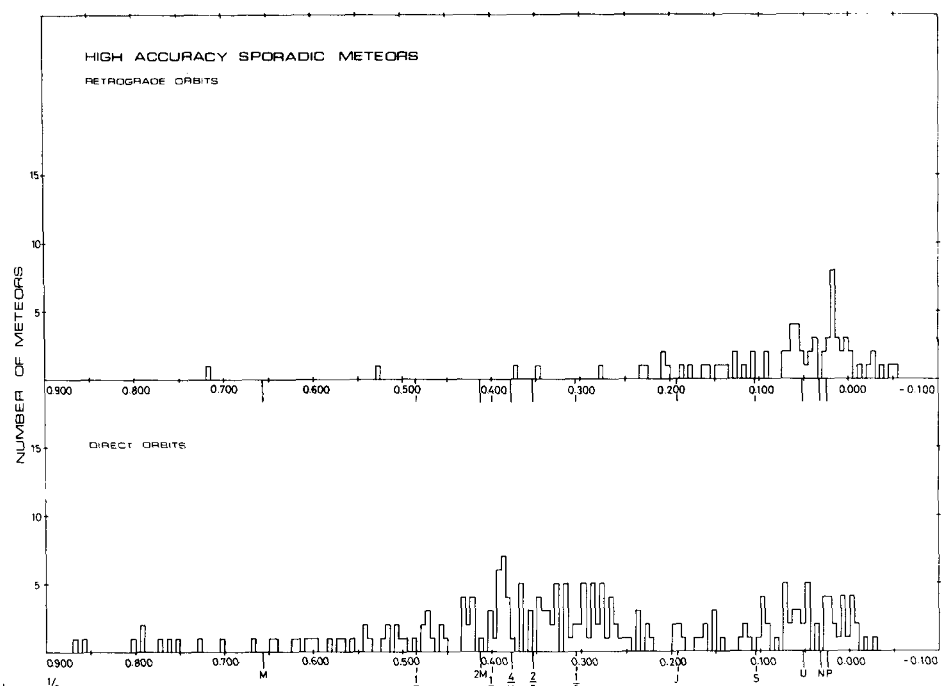

FIGURE 4.—Distribution of meteor orbits in reciprocal semi-major axis in intervals of 0.0050 AU^{-1}. Accurate sporadic orbits only.

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a number of sporadic meteors moving in long period, direct orbits are observed.

If it is assumed that stream meteors are the disintegration products of comets, the two types of stream meteors (fig. 3) can be explained as associated with short period comets in direct orbits and long period comets in retrograde orbits, respectively. The reason why very few stream meteors are observed in direct, long-period orbits is, however, not at all clear. In this category we find streams such as the Lyrids, Ursids, and Monocerotids, all associated with well known comets moving in long-period, direct orbits. It is possible, but not very likely, that these particular streams may be under-represented in the data sample studied.

FINE STRUCTURE OF $1/a$-DISTRIBUTION

The histograms, figures 1 to 4, depict the frequency distribution of reciprocal semi-major axis $1/a$ in intervals of 0.0050 $(AU)^{-1}$. It will be noticed that there is considerable fine structure in the distributions. The question naturally arises whether this structure is accidental, or if it is found to be significant, what is the cause of this structure.

Kirkwood Gaps

In the $1/a$-distribution of accurate orbits, figure 1, deep minima or gaps are located at 0.305 to 0.315, 0.350 to 0.355, 0.375 to 0.380 and 0.395 to 0.400. These minima correspond to orbital periods of ½, ⅖, $4/11$ and ⅓ respectively of Jupiter's orbital period. The ½ gap appears to be slightly displaced. A minimum at 0.410–0.420 corresponds to twice the orbital period of Mars. Additional, but less pronounced minima occur at 0.320 to 0.325, 0.335 to 0.340 and 0.360 to 0.365 corresponding to $5/11$, $3/7$ and $5/13$ of Jupiter's orbital period. Knees at 0.270 to 0.275 and 0.425 to 0.430 could indicate the ⅗ and $3/10$ resonance gaps.

The observed fine structure is thus related to the orbital period of Jupiter, with few meteors having values of $1/a$ corresponding to commensurabilities with Jupiter. The phenomena is thus of a similar nature as the Kirkwood gaps in the asteroidal belt. The discovery of Kirkwood gaps in the meteor population indicates that the time scale necessary to produce resonance effects of this type in the solar system is very short.

The question arises whether the observed gap structure could be produced by several well defined meteor streams of small scatter in $1/a$. An investigation of the major meteor streams in our sample showed that the standard deviation of $1/a$ in a stream is of the order of 0.10 $(AU)^{-1}$. Hence, the internal scatter within a stream is almost an order of magnitude larger than the spacing between the observed maxima and minima. That corresponding minima exist also in the sporadic meteor background may be inferred by direct inspection. From figure 2 it is seen that the ½, ⅖, $4/11$, ⅓ and 2× Mars minima may be easily identified.

Tests of Statistical Significance

The statistical significance of the observed fine structure was tested with the chi-square test. Histograms of $1/a$ in class intervals of 0.0050 $(AU)^{-1}$ were taken as the observed distributions. A smoothed distribution was next constructed by using a running mean of 11 class intervals. This distribution was considered as the hypothetical or "true" distribution. For various parts of the observed distribution the probability that the observed gaps could have occurred by chance was calculated with the help of the chi-square test.

For the interval 0.30 to 0.40 $(AU)^{-1}$ the chi-square test gives a 10 percent probability that the observed gap structure is accidental. This interval includes the ½, ⅖, $4/11$ and ⅓ Jupiter resonance minima. In the interval 0.40 to 0.45 $(AU)^{-1}$ the probability of an accidental occurrence was less than 1 percent. For the entire range 0.30 to 0.45 the probability of an accidental occurrence of the gaps is about 0.5 percent.

It should, however, be observed that the significance problem answered by the chi-square test is one of testing if an observed series of maxima and minima are significant, without regard to the actual positioning of the maxima and minima. In the present problem the positions of the minima may be specified in advance to certain multiples of the orbital period of Jupiter (and Mars). The detection of a series of gaps at the predicted positions is thus of higher statistical significance than indicated by the chi-square test.

Velocity Errors

In order to obtain $1/a$ with an accuracy of 0.005 $(AU)^{-1}$ meteor orbits with an accuracy in heliocentric velocity of 0.06 km/s or about 0.16 percent (for $v_H = 38$ km/s) are needed. This corresponds to an error of about 0.3 percent in semi-major axis. The 1218 Harvard orbits, on which this study is mainly based, are given to 0.01 km/s in geocentric and heliocentric velocity and to 0.001 AU in semi-major axis a. It is evident that the real errors are somewhat larger. For the better half of the data sample the velocity error is probably 0.04–0.05 km/s. The velocity errors are thus just at the limit of what is acceptable for the purpose of the present study.

Sampling Procedure

For about 15 percent of the accurate orbits the semi-major axis a is only given to three digits, in which case spurious gaps in the $1/a$-distribution could appear because of the smallness of the sampling interval. A series of computer runs in synthetic distributions showed that this effect was negligible.

As a further check on the consistency of the data the mean motions μ were computed for all meteors, and histograms of μ were plotted. The same gaps as above were detected and tested for statistical significance. Very nearly the same chi-square probabilities were derived.

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