CHARACTERISATION OF SPATIAL STRUCTURE IN MOLECULAR CLOUDS

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Abstract. Two topological tools for studying the global structure of molecular clouds, the genus and the contour-crossing statistic, are discussed. Preliminary results for the Taurus molecular cloud complex are presented.

1. Introduction

Molecular clouds display structure on all observed length scales. If the complexities of this structure can be characterised by a finite number of statistical and/or topological measures, these measures could be used in two ways. Firstly, they might be correlated with cloud properties such as the star-forming efficiency or the local stellar density. Secondly, they could be used to determine which physically motivated models of cloud structure best match with the observations. Realisations of such a model can then be used to investigate other properties of real clouds which are not directly observable, such as the extent to which the interstellar radiation field penetrates into the inner regions of a molecular cloud, or the extent to which the density structure determines the efficiency of energy transport within the molecular cloud.

Several techniques have been investigated to quantify the properties of observed distributions of objects in astrophysics. The autocorrelation function (Peebles 1980) has proved useful in quantifying the large-scale distribution of galaxies, and has been applied to the structure of molecular clouds (Houlahan and Scalo 1990). Fractal analysis has also been used (e.g. Dickman *et al.* 1990). We are currently investigating two topological measures of structure which have been developed to study the galaxy distribution. These are the genus (Gott, Melott and Dickinson 1986) and the contour crossing statistic (Ryden 1988, Ryden *et al.* 1989).

2. Topological Characterisation

We present preliminary results for the topology of the Taurus molecular cloud as mapped in the ^{12}CO J $= 1-0$ transition (Ungerechts and Thaddeus 1985). Here, we

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use two statistics to describe surfaces (1-dimensional, hence contours) of constant integrated emission. If the emission is optically thin, such surfaces trace surfaces of constant column density.

2. 1. GENUS

The genus, $\mathcal{G}$, of a surface is an invariant measure of its connectedness. Formally, it equals the number of topologically distinct closed curves which can be drawn on the surface without cutting it into two unconnected pieces :

$$\mathcal{G} = 1 + \text{Number of Holes} - \text{Number of Disconnected Segments}$$

Thus $\mathcal{G}$ of a surface of constant emission can be used to characterise the connectedness of the surface in terms of either the value on the surface or alternatively, the average emission interior to the surface. A large and negative value of $\mathcal{G}$ implies that the emission is distributed in many isolated clumps. On the other hand, a large positive value of $\mathcal{G}$ suggests that the emission occurs in a medium which permeates the cloud.

For the Taurus data, the run of genus with integrated emission is negative (figure 1), suggesting that the structure consists of isolated clumps at most levels of emission : most of the emission arises from small disconnected regions.

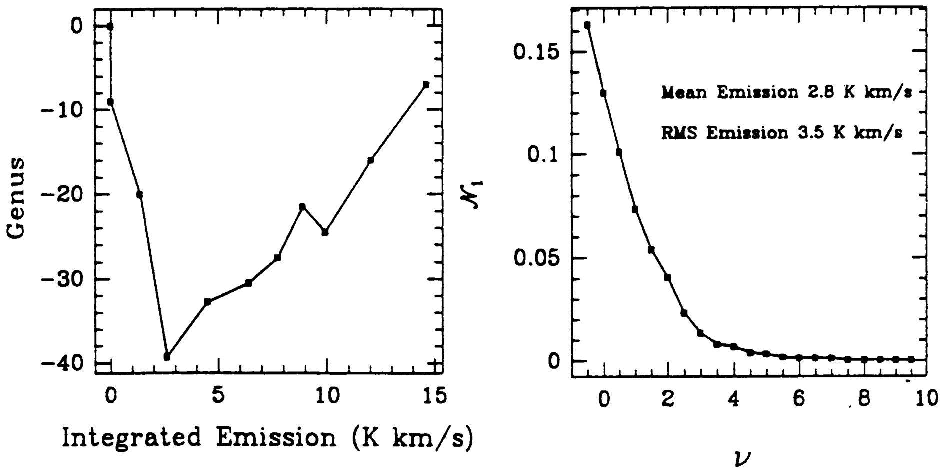

Fig. 1. The genus (left) and contour crossing statistic, $\mathcal{N}_1$ (right) for the integrated $J = 1 - 0$ ^{12}CO emission from the Taurus molecular cloud.

2. 2. CONTOUR-CROSSING STATISTIC

The genus statistic cannot distinguish between topologically identical but commonsensically distinct objects (such as a banana and an orange). However, physical processes such as the energy or mass flux across a surface enclosing some given volume certainly depend on the shape of the surface as well as its topology. The one dimensional contour-crossing statistic, $\mathcal{N}_1$, is complementary to the genus statistic in this respect, and is defined as the mean number of times a randomly directed line crosses the surface per unit length of the line. In higher dimensions $\mathcal{N}_2$ and $\mathcal{N}_3$ are respectively the contour-length statistic which is the mean length of the intersection of the surface with a plane, and the contour-area statistic which is the mean surface area per unit volume. If the distribution of emission is known, the three statistics are related.

We calculated $\mathcal{N}_1$ for isoemission surfaces by counting the mean number of crossings of the surfaces per pixel within the emitting region (figure 1). If the emission was distributed as a Gaussian random field, we would expect $\ln(\mathcal{N}_1) \propto -\nu^2$, where ν is the deviation of the emission from the mean in units of the root-mean-square emission. Instead we find that $\ln(\mathcal{N}_1) \propto -\nu$. The emission is probably not well modelled by a Gaussian distribution, which is consistent with the results from the genus.

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