"SUBJECT GENERALITY" AND DISTRIBUTION IN MEDIEVAL ARABIC SYLLOGISTIC

KHALED EL-ROUAYHEB

Harvard University, Cambridge (Mass.), USA
Department of Near Eastern Languages & Civilizations
Email: kel@fas.harvard.edu

Abstract. A relatively well-known medieval Latin innovation is the doctrine of distributive supposition. This notion came to be used in syllogistic theory in the late medieval and early modern periods, as Latin logicians sought to establish general rules for syllogistic productivity across the various figures. It is much less well-known that some logicians in the medieval Arabic tradition also attempted to establish general rules for the syllogism, appealing to what they called "subject generality." In the present article, I introduce this use of "subject generality" in some influential Arabic works on logic from the thirteenth century to the sixteenth, specifically *Al-ǧumal* by Afḍal al-Dīn al-Ḫūnaǧī (d. 1248) and *Tahḏīb al-manṭiq* by Saʿd al-Dīn al-Taftāzānī (d. 1390) and some of their commentators. I also compare this concept of "subject generality" to the Latin concept of "distribution."

Résumé. Une innovation latine médiévale relativement bien connue est la doctrine de la supposition distributive. Cette notion est venue à être utilisée dans la théorie syllogistique à la fin du Moyen Âge et au début de la période moderne, alors que les logiciens latins cherchaient à établir des règles générales pour la productivité syllogistique à travers les différentes figures. Il est beaucoup moins connu que certains logiciens de la tradition arabe médiévale ont également tenté d'établir des règles générales pour le syllogisme, faisant appel à ce qu'ils appelaient la «généralité du sujet». Dans le présent article, j'introduis cette utilisation de la «généralité du sujet» dans certains ouvrages arabes influents en logique du xiiie au xvie siècle, en particulier *Al-ǧumal* par Afḍal al-Dīn al-Ḫūnaǧī (mort en 1248) et *Tahḏīb al-manṭiq* par Saʿd al-Dīn al-Taftāzānī (mort en 1390) et certains de leurs commentateurs. Je compare également ce concept de «généralité du sujet» au concept latin de «distribution».

The medieval Latin and Arabic traditions of logic had common roots in the Greek Aristotelian tradition but developed in distinct ways. A relatively well-known medieval Latin innovation is the doctrine of distributive supposition. This notion came to be used in syllogistic theory in the late medieval and early modern periods, as logicians sought to establish general rules for syllogistic productivity across the various figures. In the 1960s, Peter Geach launched a spirited attack on this use of distribution in syllogistic, though in more recent decades his criticisms have been countered or qualified.[1]

It is much less well-known that some logicians in the medieval Arabic tradition also attempted to establish general rules for the syllogism, appealing to what they called "subject generality." In the present article, my aim is to introduce this use of "subject generality" in some influential Arabic works on logic from the thirteenth century to the sixteenth. I will first discuss what seems to be the first appeal to this notion in the discussion of syllogistic productivity, namely in a short handbook on logic entitled *Al-ǧumal* by Afḍal al-Dīn al-Ḫūnaǧī (d. 646/1248). I will then briefly compare Ḫūnaǧī's use of "subject generality" to the mainstream Latin notion of "distribution." I will then discuss a more ambitious employment of "subject generality" by the Timurid scholar Saᶜd al-Dīn al-Taftāzānī (d. 792/1390) to capture the conditions for the productivity of both categorical and modal syllogisms.

1. SUBJECT-GENERALITY IN ḪŪNAǦĪ'S *ǦUMAL*

In his short and dense handbook *Al-ǧumal*, after introducing the four figures of the syllogism, Ḫūnaǧī went on to discuss their conditions of productivity. Unusually, he did not proceed figure by figure, but instead gave a general rule. He wrote:

> The general rule (ḍābiṭ) for productivity is: That the middle is subject of the two extremes – actually or potentially – while being subject generally (ᶜumūm waḍᶜihi) to one of them and to the minor affirmatively, or that the middle is true of all the major term and denied of the minor.

Ḫūnaǧī's student Ibn Wāṣil (d. 697/1298), in his commentary on the

Ǧumal, wrote that Ḫūnaǧī had been the first to propose such a general condition. He wrote:

> The author has innovated wondrously in this general rule, for – despite its succinctness – it makes known all the productive and sterile moods of the four figures, and the conditions of productivity with respect to quantity and quality.[2]

Ibn Wāṣil then proceeded to unpack Ḫūnaǧī's general rule.[3] The overall form of the rule is a disjunction of two complex conditions. A syllogistic mood that meets one of the two conditions is productive, and a mood that does not is sterile. The conditions are as follows:

1. The first is a conjunction of three subconditions:
   (a) The middle term is actually or potentially subject to both extremes. Being a subject potentially means that it can become subject if the premise is converted. In third-figure syllogisms, the middle term is actually subject to both the minor and the major term. In the other figures, it is potentially subject in both insofar as the premises convert.
   (b) The middle term is subject generally to one of the extremes, either actually or potentially. By "subject generality" is meant being a subject in a universal proposition. The subject of a universal proposition (Every J is B; No J is B) has subject generality actually, whereas the predicate of a universal-negative proposition (No J is B) has subject-generality potentially, for the negative-universal proposition converts to a universal-negative (No B is J).
   (c) The middle term is subject affirmatively to the minor term, either actually or potentially.
2. The second condition is a conjunction of two subconditions:
   (a) The middle term is true of all the major term.
   (b) The middle term is denied of the minor term.

We may consider the four syllogistic figures in turn:

The first figure has the following form (with "J" the minor term, "B" the middle term, and "A" the major term):

---

[2] Ibn Wāṣil, *Commentary on the Jumal on Logic*, ed. Khaled El-Rouayheb (Leiden: Brill, 2022), p. 117.
[3] Ibn Wāṣil, *Commentary*, p. 117–123.

$$J - B$$
$$B - A$$
$$\overline{}$$
$$J - A$$

It is productive if the minor premise is affirmative and the major premise universal. In such cases, the first condition mentioned by Ḫūnağī is met. If the minor premise is affirmative then it converts, meaning that the middle term is potentially subject in both premises (1a). If the major premise is universal then the middle term is subject generally to one of the extremes (1b). And if the minor premise is affirmative then it converts, meaning that the middle term is potentially subject – affirmatively – to the minor term (1c).

The second figure has the following form:

$$J - B$$
$$A - B$$
$$\overline{}$$
$$J - A$$

This is productive if the two premises are of different quality (one is affirmative and the other negative) and the major premise is universal. Of its productive moods, two fulfill the first of Ḫūnağī's two conditions:

(CESARE)	Every J is B	(FESTINO)	Some J is B
	No A is B		No A is B
	No J is A		Some J is not A

In these moods, both premises convert, so that the middle term is potentially subject in both premises (1a); the major premises convert simply (to "No B is A"), so that the middle term is potentially the general subject of one of the extremes (1b); and the minor premises convert to an affirmative proposition, so that the middle term is potentially subject – affirmatively – to the minor term (1c).

The two other productive moods are:

(CAMESTRES)	No J is B	(BAROCO)	Some J is not B
	Every A is B		Every A is B
	No J is A		Some J is not A

Both moods satisfy the second of Ḫūnağī's conditions. The middle term B is universally predicated of the major term A (2a), and the middle term is denied of the minor term J (2b).

The third figure has the following form:

$$B - J$$
$$B - A$$
$$\overline{}$$
$$J - A$$

The figure is productive if the minor premise is affirmative and one of the premises is universal. In such cases, the first condition outlined by Ḥūnaǧī is met: In the third figure, the middle term is actually subject in both premises (1a); if one of the premises is universal then the middle term is subject generally to one of the extremes (1b); and the middle term is subject affirmatively to the minor term (1c).

The fourth figure has the following form:

$$B - J$$
$$A - B$$
$$\overline{}$$
$$J - A$$

The figure is productive when negation and particularity are not both present in the premises, with the single exception of the minor premise being particular-affirmative and the major premise universal-negative. This yields five productive moods. Of these, four meet the first of Ḥūnaǧī's conditions:

(BAMALIP)	Every B is J	(DIMATIS)	Every B is J
	Every A is B		Some A is B
	———————		———————
	Some J is A		Some J is B
(FESAPO)	Every B is J	(FRESISON)	Some B is J
	No A is B		No A is B
	———————		———————
	Some J is not A		Some J is not A

In all four moods, the major premise converts, so the middle term is actually subject in one premise and potentially subject in the other (1a); the middle term is actually subject generally to one of the extremes in three moods (BAMALIP, DIMATIS, FESAPO) and potentially subject generally in one mood (FRESISON) (1b); and the middle term is subject affirmatively to the minor term (1c).

A fifth mood of the fourth figure meets the second of Ḥūnaǧī's conditions:

(CALEMES)	No B is J
	Every A is B
	——————
	No J is A

The middle term B is universally predicated of the major term A (2a), and – given that the minor premise converts simply – the middle term B is potentially denied of the minor term J (2b).

The next lemma in Ḫūnaǧī's *Ǧumal* adds:

> The universality of the conclusion depends on the subject generality (*ʿumūm mawḍūʿiyyat*) of the minor term and the universality of the major premise.[4]

There are five moods that entail a universal conclusion:
(BARBARA) Every J is B & Every B is A ⇒ Every J is A
(CELARENT) Every J is B & No B is A ⇒ No J is A
(CESARE) Every J is B & No A is B ⇒ No J is A
(CAMESTRES) No J is B & Every A is B ⇒ No J is A
(CALEMES) No B is J & Every A is B ⇒ No J is A

In all these moods, the two conditions mentioned by Ḫūnaǧī in the just-quoted lemma are met. In all five moods, the major premise is universal. And in all five, the minor term J has "subject generality" actually or potentially, for J is the actual subject of a universally quantified proposition ("Every J is B" or "No J is B") in the first four moods, and it is the potential subject of a universally quantified proposition in the last mood (given that "No B is J" converts simply to "No J is B").

2. SUBJECT GENERALITY AND DISTRIBUTION

Ḫūnaǧī did not explicitly define "subject generality." But in light of Ibn Wāṣil's commentary, it is clear that a term has "subject generality" if it is the subject of a universally quantified proposition. This applies to the subject of a universal-affirmative proposition (Every J is B) and the subject of a universal-negative proposition (No J is B). A term has subject generality potentially if, by conversion, it becomes the subject of a universally quantified proposition. This applies to the predicate of a universal-negative proposition (No J is B) given that, by conversion, it becomes the subject of a universal-negative proposition (No B is J).

It may be helpful to compare this notion of "subject generality" to the medieval and early-modern Latin notion of "distribution." There is, to be sure, not a single doctrine of distribution throughout medieval and post-medieval times. But one influential account is that a term has distributive supposition when it allows "descent" from the general term to

---

[4] Ibn Wāṣil, *Commentary*, p. 123.

each particular falling under it.[5] In the proposition "Every J is B," the subject-term J has distributive supposition, for it asserts that j1 is B, j2 is B, j3 is B, j4 is B, and so on, for all subjects falling under the term "J." The predicate term, by contrast, does not have distributive supposition, for the original proposition "Every J is B" does not assert that Every J is b1 & b2 & b3 & b4, etc. The subject and predicate of universal-negative propositions are also distributed in this sense: "No J is B" allows descent of the subject (j1 & j2 & j3 & j4, etc. are not B) and of the predicate (Every J is not b1 & not b2 & not b3 & not b4, etc.).

So far, distributed terms can be said to have subject generality in Ḫūnaǧī's sense. Yet, the notions of "distribution" and "subject generality" are not extensionally equivalent. The main difference is that the predicate of a particular-negative proposition was typically thought of as distributed: In the assertion "Some J is not B," the predicate term B is distributed, for it allows descent: "Some J is not b1 & not b2 & not b3 & not b4, etc." But the predicate term of a particular-negative proposition does not have subject generality even potentially.

So, while the medieval Arabic notion of "subject generality" overlaps with the medieval Latin notion of "distribution," there is also a significant difference. This difference affects the use of the notions in the syllogistic. The two rules in the Latin syllogistic tradition that involve the notion of distribution are:

(D1) No term may be distributed in the conclusion unless it is distributed in the premises.

(D2) The middle term must be distributed in at least one premise.

If we were to insert "subject generality" instead of "distribution" then we would get:

(SG1) No term may have subject generality in the conclusion unless it has subject generality in the premises.

(SG2) The middle term must have subject generality in at least one premise.

(SG1) holds. There is no productive mood in which a term has subject generality in the conclusion but not in the premises. But (SG2) fails. In

---

[5] Parsons, "The Doctrine of Distribution," p. 63–65; E. Jennifer Ashworth, *Language and Logic in the Post-Medieval Period* (Dordrecht: Reidel Publishing Company, 1974), p. 207–213; Alexander Broadie, *Introduction to Medieval Logic* (Oxford: Clarendon Press, 1993), p. 36–48; Christoph Kann, "Supposition and Properties of Terms," in Catarina Dutilh-Novaes & Stephen Read (eds.), *The Cambridge Companion to Medieval Logic* (Cambridge: Cambridge University Press, 2016), p. 220–244, at 228–232.

the second-figure mood BAROCO, the middle term does not have subject generality in either premise:

Some J is not B
Every A is B
―――――――――
Some J is not A

It is therefore difficult to give a succinct statement of the conditions for syllogistic productivity in term of subject generality. Ḫūnaǧī's attempt at giving general conditions across syllogistic figures and moods yielded something much more complex than (D1) and (D2). This may be one reason why "subject generality" did not become as central a term in medieval Arabic syllogistic as "distribution" became in medieval and early-modern Latin syllogistic. Certainly, "subject generality" never became part of introductory Arabic surveys of the syllogism. Neither the *Ǧumal* nor the few later handbooks that were influenced by it were introductory-level works.

3. THE GENERAL RULE OF TAFTĀZĀNĪ

In the Eastern Islamic lands, a widely studied handbook of logic in later centuries was *Tahḏīb al-manṭiq* by the Timurid scholar Saʿd al-Dīn al-Taftāzānī (d. 792/1390). After introducing the four figures of the syllogism, and each figure's conditions of productivity, Taftāzānī added:

> The general rule (*ḍābiṭat*) of the conditions of the four [figures] is that there must be either a subject generality of the middle term and that it is actually and affirmatively connected to (*mulāqātihi li-*) the minor term or predicated of the major term, or a subject generality of the major term together with a difference in quality and an incompatibility between the relation of the description (*waṣf*) of the middle term to the description (*waṣf*) of the major term and its relation to the substance (*ḏāt*) of the minor.[6]

Again, the general rule takes the form of a disjunction with complex disjuncts. The two conditions set forth are as follows:

1. The first condition is a conjunction of two subconditions:

―――――――――

[6] Mullā ʿAbdullāh Yazdī, *Šarḥ Tahḏīb al-manṭiq*, ed. ʿAbd al-Ḥamīd al-Turkmānī (Amman: Dār al-Nūr al-Mubīn, 2018), p. 312.

(a) The middle term has subject generality.

(b) Either the middle is connected to the minor in actuality or the middle is predicated of the major.[7]

2. The second condition, too, is a conjunction of subconditions:

(a) The major term has subject generality.

(b) The two premises differ in quality.

(c) There is opposition between (i) the relation of the description of the middle to the description of the major and (ii) the relation of the description of the middle to the substance of the minor.

Later commentators did not expand on the similarities and differences between Taftāzānī's "general rule" in *Tahḏīb al-manṭiq* and Ḫūnaǧī's "general rule" in *Al-ǧumal*. This is partly because premodern commentators on logic tended not to be interested in historical matters, and partly because Ḫūnaǧī's *Ǧumal* and Taftāzānī's *Tahḏīb* came to be studied in different parts of the Islamic world, the former in the Maghreb and the latter in the Islamic East. It is reasonable to suspect that Ḫūnaǧī's *Ǧumal* served as a model for Taftāzānī, as indicated by the use of the term "general rule" (*ḍābiṭ*), the appeal to "subject generality" (*ʿumūm al-mawḍūʿiyya*), and the very idea of trying to summarize the conditions of syllogistic productivity across figures and moods. But there are also some striking differences. These differences can be explained by the fact that Taftāzānī attempted to incorporate the conditions of the modal syllogistic into his "general rule." Ḫūnaǧī only presented the conditions of modal syllogisms *after* he had presented the "general rule." Taftāzānī, by contrast, presented the conditions of modal syllogisms *before* his "general rule," clearly implying that his general rule is meant to capture these. In the preceding section of *Tahḏīb al-manṭiq*, he had outlined the four figures of the categorical syllogism and their conditions of productivity, as well as the conditions for productivity in the modal syllogisms of the first three figures. For the fourth figure, his presentation is more confusing. On the one hand, he only mentioned the non-modal conditions for productivity in that figure. On the other hand, he still maintained that the fourth figure has eight productive moods. The three extra moods had been recognized by some thirteenth-century Ara-

---

[7] The condition that the middle term be "connected to" (*mulāqātihi li-*) the minor term in actually (*bi-l-fiʿl*) is explained by commentators to mean that the middle term is a subject and the minor term a predicate, or vice-versa, in either case in a premise that is modally stronger than mere possibility.

bic logicians, but only when the premises are certain complex modality propositions.[8] The combination of leaving unmentioned the modal conditions of the fourth figure while still presenting eight moods as productive is, of course, misleading.

The three extra "productive" moods of the fourth figure are:

(IV, 6) Some B is not J
         Every A is B
         ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯
         Some J is not A

(IV, 7) Every B is J
         Some A is not B
         ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯
         Some J is not A

(IV, 8) No B is J
         Some A is B
         ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯
         Some J is not A

Taftāzānī's general rule is meant to capture these three further "productive" moods. It is also meant to capture the conditions for the productivity of modal syllogisms in the first three figures. The modal syllogistic in later Arabic logic is a challenging topic and cannot be pursued in depth in the present context. But some basic overview cannot be avoided in light of the discussion that follows.[9]

The post-Avicennan tradition accepts more than a dozen distinct modalities, but for present purposes the following eight should suffice:

1. *Absolute necessity*: The predicate is necessarily true of the subject as long as the subject exists, for example "Every human is necessarily an animal."

2. *Descriptional necessity*: The predicate is necessarily true of the subject as long as the subject is described in a certain way, for example "Every bachelor is necessarily unmarried as long as he is a bachelor."

3. *Specific descriptional necessity*: The predicate is necessarily true of the subject as long as the subject is described in a certain way but not always as long as it exists, for example "Every sleeper is necessarily asleep but not always."

4. *Absolute perpetuity*: The predicate is always true of the subject as long as the subject exists.

5. *Descriptional perpetuity*: The predicate is always true of the subject as long as the subject is described in a certain way.

---

[8] See Quṭb al-Dīn al-Rāzī al-Taḥtānī, *Taḥrīr al-qawāᶜid al-manṭiqiyya fī šarḥ Al-risāla al-šamsiyya* (Cairo: Muṣṭafá al-Bābī al-Ḥalabī, 1948), p. 149.

[9] For a more detailed overview, see Riccardo Strobino & Paul Thom, "The Logic of Modality," in Catarina Dutilh-Novaes & Stephen Read (eds.), *The Cambridge Companion to Medieval Logic* (Cambridge: Cambridge University Press, 2016), p. 342–369, at 343–359.

6. *Specific descriptional perpetuity*: The predicate is always true of the subject as long as the subject is described in a certain way but not always as long as it exists.

7. *Possibility*: The predicate is possibly true of the subject. The negative possibility proposition is the contradictory of the affirmative absolute-necessity proposition.

8. *Absolute*: The predicate is at some time true of the subject. The negative absolute proposition is the contradictory of the affirmative absolute-perpetuity proposition.

For the first and third syllogistic figures, the conditions given by Taftāzānī are straightforward. According to received doctrine, the minor premise in the first and third figure has to be an actuality proposition, i.e., not merely a possibility proposition. The conditions of the second figure are more complex. Taftāzānī had earlier given two conditions for modal syllogisms in this figure:

1. Either the minor premise is a perpetuity proposition [for example "Every J is always B"] or the major premise is one of the six modality propositions that convert when negative.

2. That a possibility proposition is not used in this figure except alongside a necessity proposition or a major premise that is a descriptional necessity proposition.

In what follows, the two conditions set forth in Taftāzānī's "general rule" will be correlated with the productive moods they are meant to cover.

Condition (1):

(a) The middle term has subject generality.

(b) Either (i) the middle is connected to the minor in actuality or (ii) the middle is predicated of the major.

This condition is met by all productive moods of the first and third figures:

(BARBARA) Every J is B & Every B is A
(CELARENT) Every J is B & No B is A
(DARII) Some J is B & Every B is A
(FERIO) Some J is B & No B is A
(DATISI) Some B is J & Every B is A
(DISAMIS) Every B is J & Some B is A
(FERISON) Some B is J & No B is A
(BOCARDO) Every B is J & Some B is not A
(FELAPTON) Every B is J & No B is A
(DARAPTI) Every B is J & Every B is A

In each of these moods, the middle term B is the subject of at least one universally quantified premise (1a), and – given that modal syllogisms in the first and third figures must have a minor premise that is modally stronger than mere possibility – the middle term B is subject to or predicated of the minor term J actually (1b[i]).

Condition (1) is also met by the following moods of the fourth figure:

(BAMALIP) Every B is J & Every A is B

(DIMATIS) Every B is J & Some A is B

(CALEMES) No B is J & Every A is B

(FESAPO) Every B is J & No A is B

(IV, 7) Every B is J & Some A is not B

(IV, 8) No B is J & Some A is B

In these moods, the middle term B is the subject of a universally quantified premise (1a) and is predicated of the major term (1b[ii]).

Condition (2):

(a) The major term has subject generality.

(b) The premises are of different quality.

(c) There is opposition between (i) the relation of the description of the middle to the description of the major and (ii) the relation of the description of the middle to the substance of the minor.

The productive moods of the second figure meet conditions (2a) and (2b):

(CESARE) Every J is B & No A is B

(FESTINO) Some J is B & No A is B

(CAMESTRES) No J is B & Every A is B

(BAROCO) Some J is not B & Every A is B

In these moods, the major term A has subject generality in the premises (2a), and the premises are of different quality (2b).

As for showing that condition (2c) is also met, this requires a more involved discussion. In the remaining part of this section, I will explicate condition (2c) in more detail and discuss some objections that were raised in the commentary tradition.

To reiterate, the conditions of modal syllogisms in the second figure are:

1. Either the minor premise is a perpetuity proposition or the major premise is one of the six types that convert when negative: (i) absolute necessity, (ii) absolute perpetuity, (iii) descriptional necessity, (iv) specific descriptional necessity, (v) descriptional perpetuity, and (vi) specific descriptional perpetuity.

2. That a possibility proposition is not used in this figure except

alongside a necessity proposition or a major premise that is a descriptional necessity proposition.

The perhaps most influential commentary on this section of *Tahḏīb al-manṭiq* is by the Persian scholar Mullā ʿAbdullāh al-Yazdī (d. 981/1573–4), still studied in traditional madrasas in Iran and South Asia. Yazdī explained that the first of these conditions means that at least one of the following is true:[10]

(a) The minor premise is a perpetuity proposition or stronger (i.e., a necessity proposition). The other premise is modally at least actual (i.e. stronger than mere possibility) – for otherwise we are dealing with condition (2) in which one of the premises is a possibility proposition. But if one premise is a perpetuity proposition and the other premise is modally at least actual but differing in quality, then there is opposition between the modality of the middle-minor relation and the modality of the middle-major relation. To avoid misunderstanding, Yazdī made it clear that the incompatibility is not between the modal relation between J and B and the modal relation between A and B. The incompatibility is rather between the two modal relations if applied to the same two extremes. If X is perpetually Y, then it cannot be the case that X is necessarily or actually not Y.

(b) The major premise is convertible when negative, i.e., it must be one of the following six propositions: (i) absolute necessity, (ii) absolute perpetuity, (iii) descriptional necessity, (iv) specific descriptional necessity, (v) descriptional perpetuity, and (vi) specific descriptional perpetuity. The other premise must be affirmative (for the two premises must have different qualities) and modally stronger than mere possibility (for possibility premises are treated under condition (2)). Again, this means that there is opposition between the modality of the middle-minor relation and the modality of the middle-major relation.

The second of the conditions for the productivity of second-figure modal syllogisms is that a possibility proposition is only used along with a necessity proposition or a major premise that is a descriptional necessity proposition. This, according to Yazdī's commentary, has the same implication. If one premise is a possibility proposition and the other a necessity proposition, and if the premises differ in quality, then there is opposition between the modality of the middle-minor relation and the modality of the middle-major relation.

Another commentator on Taftāzānī's *Tahḏīb al-manṭiq*, the Persian

---

[10] Yazdī, *Šarḥ Tahḏīb al-manṭiq*, p. 318–322 [p. 319 and 321 are devoted to glosses by later scholars].

scholar Mīr Abū l-Fatḥ al-ᶜArabšāhī (d. 976/1567–8), raised an objection concerning the supposed incompatibility between the modal relations in the two premises.[11] He gave the following counter-examples:

> Every J is possibly B
> Every A is necessarily not B as long as it is A
> ───

> Every J is at some time B
> Every A is necessarily not B as long as it is A
> ───

Both syllogisms meet the stated conditions for productivity in the second figure. Yet, according to Mīr Abū l-Fatḥ, there is no incompatibility between the modal relations in the premises. There is no incompatibility between "Every X is possibly Y" and "Every X is necessarily not Y as long as it is X." (Consider the case in which X = bachelor and Y = married.) Nor is there incompatibility between "Every X is at some time Y" and "Every X is necessarily not Y as long as it is X." (Consider the case in which X = sleeper and Y = awake.)

Mīr Abū l-Fatḥ's objection arguably fails. To see this, it is helpful to reiterate the exact phrasing of condition (2c). Taftāzānī had not merely stipulated that there is opposition between the modalities of the middle-minor and middle-major relations. He had, to be exact, stipulated that there is opposition between (i) the relation of the description (*waṣf*) of the middle to the *description* of the major and (ii) the relation of the description of the middle to the *substance* (*ḏāt*) of the minor. Consider one of Mīr Abū l-Fatḥ's counter-examples:

> Every J is possibly B
> Every A is necessarily not B as long as it is A
> ───

In the minor premise, the modal relation of *affirmative possibility* obtains between the description "B" and each and every entity (*ḏāt*) falling

under "J:" each and every J is said to be possibly B. In the major premise, the relation of *negative necessity* obtains between the description "B" and the description "A:" Individual entities might be A at one time and B at another time, but they cannot possibly be B while also described as "A" (for example, if A = bachelor and B = married). And it is clear that there is opposition between affirmative possibility (the relation that obtains between the middle term B and individual J's) and negative necessity (that obtains between the middle term B and the description "A").

Mīr Abū l-Fatḥ seems to have anticipated some such response to his objection. He countered that if the response is allowed then the criterion becomes too loose and would admit non-valid modal syllogisms, such as the mentioned counterexample with the order of premises switched:

> Every J is necessarily not B as long as it is J
> Every A is possibly B

This should not be valid. But conditions (2a) and (2b) are met: The major term A has subject generality, and the premises are of different quality. Is condition (2c) also met? At first sight, Mīr Abū l-Fatḥ's objection seems strong: If there is opposition between the modal relations of the two premises in the counterexample, then this opposition should remain when the order of premises is switched. But again, the exact wording of Taftāzānī's rule circumvents the worry. The relation of the description of the middle ("B") to the substance of the minor (the thing falling under the subject term "J") is *negative descriptional necessity*. The relation of the description of the middle ("B") to the description of the major ("A") is indeterminate. It might be *affirmative possibility* (for example if A = human and B = writer) but it could be *negative descriptional necessity* (if A = dry human and B = wet). In any case, it is clear that there is no formal incompatibility between the two modal relations in the premises.

Again, Mullā ʿAbdullāh Yazdī's commentary provides a sure-footed commentary.[12] He explained carefully that the standard conditions for modal productivity in the second figure are co-extensive with the presence of the opposition between the modal relations. As mentioned above, he explained that when the standard conditions for productivity are met, then the opposition obtains. So, by contraposition, when the standard

---

[12] Yazdī, *Šarḥ Tahḏīb al-manṭiq*, 322–324 [p. 323 is devoted to glosses by later scholars].

conditions are not met, then the opposition does not obtain. If the minor premise is weaker than a perpetuity proposition, and if the major premise is not one of the six that convert when negative, then the strongest premises would be of the following form:

> Every J is necessarily not B as long as it is J, not always
> Every A is necessarily B at some time, not always

This modal argument does not meet condition (c). The relation of the description of the middle ("B") to the description of the major ("A") is *affirmative necessity at some time*, and the relation of the description of the middle ("B") to the substance of the minor (the things that are J) is *negative descriptional necessity*. There is no opposition between the two modalities. And, Yazdī added, if there is no opposition between these two premises, then there is no opposition between the rest of the modality propositions that are weaker.

To take stock, the moods of the first and third figures all satisfy the first condition of Taftāzānī's general rule, as do six out of eight moods of the fourth figure. The productive moods of the second figure satisfy the second condition. This leaves two moods of the fourth figure unaccounted for. These are:

(FRESISON) Some B is J & No A is B

(IV, 6) Some B is not J & Every A is B

These do not satisfy condition 1, for the middle term B does not have subject generality in one premise. It is striking that later commentators on *Tahḏīb al-manṭiq* do not consider *potential* subject-generality, which would allow FRESISON to pass muster (for the major premise converts simply to "No B is A"). Instead, commentators tended to say that these two moods satisfy the second condition. Now, it is clear that they satisfy (2a) and (2b): the major term A has subject generality and the premises differ in quality. But do the moods satisfy (2c)? Taftāzānī's phrasing of condition (2c) suggests that he had second figure syllogisms in mind, not fourth-figure syllogisms. In the minor of a fourth-figure syllogism, the middle term B is not predicated of individual J's – rather the minor term J is predicated of individual B's.

The commentary literature on *Tahḏīb al-manṭiq* supports the supposition that Taftāzānī did not consider syllogisms in the fourth figure to meet requirement (2c). According to Yazdī's commentary, the two moods of the fourth figure satisfy (2a) and (2b) and are not meant to satisfy (2c) given that Taftāzānī is not attempting to capture the conditions for

modal syllogisms in the fourth figure.[13] The same point is made in an earlier commentary on *Tahḏīb al-manṭiq* written by Taftāzānī's great-grandson Aḥmad b. Yaḥyá al-Ḥafīd al-Harawī (d. 916/1510).[14]

4. WHAT IS THE POINT OF THE "GENERAL RULE?"

One of the numerous commentators on Taftāzānī's *Tahḏīb al-manṭiq*, ᶜUbaydullāh al-Ḥabīṣī, who was active in Central Asia in the 1540s, simply left out the passage on the general rule.[15] His commentary came to be standard in the Azhar college in Cairo and elicited numerous glosses by Egyptian scholars from the seventeenth century to the nineteenth. One of the few Egyptian glossators who showed awareness of the missing lemma was Ḥasan al-ᶜAṭṭār (d. 1250/1835), who had spent some years in Istanbul and was more abreast of logical works used in the Turco-Persianate world than most of his compatriots. ᶜAṭṭār speculated that the passage was missing from some copies of Taftāzānī's *Tahḏīb* (as opposed to it being purposefully left out by Ḥabīṣī) and opined that there was simply no need for it because the specific conditions of productivity for each figure had already been mentioned in Taftāzānī's handbook.[16] The comment brings out a larger question: What exactly is the point of trying to formulate a general rule for syllogistic productivity, especially one so intricate as Taftāzānī's? The commentators who did discuss the passage devoted little if any attention to this question. They had obviously mastered the modal syllogistic of the post-Avicennan logical tradition and brought this to bear on the relevant passage of the handbook. But their focus was entirely on a textual problem, namely whether Taftāzānī's formulation adequately captured all and only the productive categorical syllogisms in the four figures and the modal syllogisms of the first three figures. None of them questioned the received, figure-specific conditions for syllogistic productivity.

[13] See Yazdī's treatise on Taftāzānī's general rule, written before his commentary on the entirety of Taftāzānī's handbook, and printed as an appendix to Yazdī, *Šarḥ Tahḏīb al-manṭiq*, p. 404 (Taftāzānī's general rule does not encompass the modal conditions of the fourth figure); 410 (the fifth and sixth moods of the fourth figure satisfy conditions 2a and 2b).

[14] Ḥafīd al-Taftāzānī, *Šarḥ Tahḏīb al-manṭiq*, ed. ᶜAbd al-Ḥamīd al-Turkmānī (Amman: Dār al-Nūr al-Mubīn, 2019), p. 192–193.

[15] Ḥabīṣī's commentary was dedicated to the Uzbek ruler ᶜAbd al-Laṭīf Ḫān (r. 1540–1552).

[16] Ḥasan al-ᶜAṭṭār, *Ḥāšiya ᶜalá Šarḥ al-Ḫabīṣī* (Cairo: al-Maṭbaᶜa al-Azhariyya, 1318/1900), p. 245.

In the medieval and early-modern Latin tradition, there was an attempt to use the concept of distribution to ground the figure-specific conditions. For example, the fundamental rule – the so-called *dictum de omni et nullo* – that what is predicated of a distributed term is predicated of any item falling under that term, and that what is negated of a distributed term is negated of any item falling under that term, was used to support the conditions for productivity of first-figure syllogisms and thus indirectly of all syllogistic arguments.[17] Consider, for example, a BARBARA syllogism:

<div align="center">

Every J is B

Every B is A

———————

Every J is A

</div>

The fact that the subject of the major premise (B) is distributed means that A is predicated of all that falls under the subject term "B," and the first premise tells us that every J falls under B. The doctrine of distribution and the *dictum de omni et nullo* could thus be presented as grounding the productivity of BARBARA. To be sure, some modern observers have doubted whether it is possible to ground syllogistic productivity in this way.[18] My point is not that medieval and early-modern Latin logicians successfully managed to ground syllogistic productivity in the *dictum de omni et nullo*, but rather that they *tried* to do so. It is fair to say that the concept of "subject generality" was not employed for that purpose in the Arabic tradition. For good or for bad, the general rules involving "subject generality" remained an alternative or summary way of setting forth the conditions of productivity, as opposed to being used to ground the figure-specific conditions.

One possible objection here would be that Ibn Wāṣil, in his commentary on Ḫūnaǧī's *Ǧumal*, did use the general rule to derive the more fa-

---

[17] Ashworth, *Language and Logic in the Post-Medieval Period*, p. 230–239; Broadie, *Introduction to Medieval Logic*, p. 175. The view that the *dictum de omni et nullo* grounds the productivity of first-figure syllogisms – and indirectly all arguments – is also clearly stated in Richard Whately's well-known *Elements of Logic* (London: B. Fellowes, 1829), p. 74–75 and, a century later, in Susan Stebbing's influential *A Modern Introduction to Logic* (London: Methuen, 1961), p. 86–92.

[18] For skeptical discussions, see H.W.B. Joseph, *An Introduction to Logic*, 2nd revised ed. (Oxford: Clarendon Press, 1967), p. 294–334; J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* (Oxford: Clarendon Press, 1957), p. 46–47, 74. For recent discussions that are more optimistic about the logical relevance of the *dictum*, see J. Barnes, *Truth, etc.: Six Lectures on Ancient Logic* (Oxford: Clarendon Press, 2007), p. 386–419 and M. Malink, *Aristotle's Modal Syllogistic* (Cambridge, Mass.: Harvard University Press, 2013), p. 34–44.

miliar, figure-specific conditions. To dispel this objection, it will be help-
ful to quote Ibn Wāṣil's manner of proceeding. Here is how he "derives"
the condition that a first-figure syllogism must have an affirmative mi-
nor premise:

> As for the first figure, if its minor premise is one of the
> two negatives, then according to this general rule [set forth
> by Ḫūnaǧī] it must be sterile, for the first sentence is absent,
> for one of its qualifications does not obtain, and this is that
> the middle term is subject to the minor term affirmatively,
> and the second sentence is also absent, for one of its two qual-
> ifications is not present, and this is that the middle term is
> affirmed of all of the major term, for it is a subject [and not
> a predicate] in the major premise, and if converted it is only
> affirmed of some. So of the sixteen [possible] moods, eight
> can be discounted – by multiplying the two negative minors
> with the four majors – and one must set as a condition the
> affirmativeness of the minor premise.[19]

Yes, Ibn Wāṣil thus "derives" one of the standard conditions of produc-
tivity for a syllogistic figure from Ḫūnaǧī's general rule. But the "deriva-
tion" is only a matter of presentation. We are not told – in the quoted
lemma or elsewhere in Ibn Wāṣil's commentary – why the general rule
is more fundamental than the figure-specific conditions. We are not told,
in other words, why it is proper to start from the general rule and show
that the figure-specific conditions accord with it, rather than – as I have
done above – starting with the figure-specific conditions and showing
that the general rule accords with them. Indeed, we are not given any
reason to believe that Ḫūnaǧī's general rule is adequate except that it
captures all and only those moods that are independently known to be
productive. So, on pain of circularity, it cannot be that the general rule
grounds or explains syllogistic productivity.

There were, it should be pointed out, attempts in the medieval Arabic
tradition to ground the figure-specific conditions, for example the follow-
ing passage from Ḫūnaǧī's longer summa of logic *Kašf al-asrār*:

> The first figure: The condition of its productivity is the af-
> firmativeness of the minor, for otherwise the minor would not
> be subsumed under the middle … and the judgment about
> the middle with the major would not transmit, affirmatively

---

[19] Ibn Wāṣil, *Commentary on the Jumal on Logic*, p. 118.

or negatively, to it [i.e. to the minor] … and the universality of its major premise, for otherwise it may be that the part that is judged in the major premise is not the same as the minor term, and the judgment [in the major premise] is not transmitted to it [i.e., to the minor term].[20]

In other words, a productive first-figure syllogism must have an affirmative minor premise and a universal major premise because otherwise the minor term would not be "subsumed" under the middle term, and the judgment about the middle term in the major premise would not "transmit" to the minor term. In contrast to the cited passage from Ibn Wāṣil, this is a genuine attempt at grounding the standard conditions of productivity. But it is noteworthy that the concept of "subject generality" was not invoked for this purpose. Conversely, the concepts of "subsumption" (*indirāǧ*) and "transmission" (*taʿaddī*) were not mentioned in the "general rule" (*ḍābiṭa*) of syllogistic productivity in *Al-ǧumal*. The concepts of "subsumption," "transmission," and "subject generality" are clearly related, and it would arguably not have been difficult to make the relation explicit. But Ḫūnaǧī did not do so. For him, justifying the figure-specific conditions and giving a "general rule" for syllogistic productivity were two separate endeavors. The fact that he mentioned the general rule in *Al-ǧumal*, his shortest work on logic, and did not discuss it in *Kašf al-asrār*, his magnum opus that was almost certainly written after *Al-ǧumal*, strongly suggests that he thought of the general rule as simply synthesizing the figure-specific rules and therefore appropriate to a short, condensed presentation. Had he thought of the general rule as more basic than the figure-specific rules, then he would presumably have revisited the topic in his longest work.

Taftāzānī, too, seems to have proposed his general rule as a summary or synthesis. Two factors are telling in this respect: First, he presented the general rule after having gone through the figure-specific conditions, instead of starting with the general rule and trying to derive the figure-specific conditions from it. Second, commentators agreed that Taftāzānī's general rule was meant to capture the figure-specific conditions for the productivity of categorical syllogisms across the four figures

---

[20] Afḍal al-Dīn al-Ḫūnaǧī, *Kašf al-asrār ʿan ġawāmiḍ al-afkār*, ed. Khaled El-Rouayheb (Berlin & Tehran: Institute for Islamic Studies & Iranian Institute of Philosophy, 2010), p. 249. Taftāzānī offers a very similar account in his commentary on *Al-risāla al-šamsiyya* by Najm al-Dīn al-Kātibī (d. 1276); see Saʿd al-Dīn al-Taftāzānī, *Šarḥ Al-risāla al-šamsiyya* (Lucknow: al-Matbaʿ al-Yūsufī, 1317/1899), p. 58.

and the conditions for modal syllogisms in the first three figures, all of which Taftāzānī had outlined in the immediately preceding section. The fact that the general rule was not meant to capture the modal conditions for the fourth figure indicates that Taftāzānī was not proposing a fundamental rule that would ground syllogistic productivity as such, but was instead proposing a summary formulation of those figure-specific conditions that he had previously mentioned.

5. CONCLUSION

Ḫūnaǧī's and Taftāzānī's formulations of general rules of syllogistic productivity across the four figures are a noteworthy chapter in the history of Arabic logic. Both made use of the concept of "subject generality," by which they meant being the subject of a universally quantified proposition. This applies to the subjects of universal-affirmative propositions (Every J is B) and universal-negative propositions (No J is B). A term was said to have *potential* subject generality if it is the predicate of a universal-negative proposition (No J is B) and can, by conversion, become the subject of a universally quantified proposition (No B is J). The concept of "subject generality" is reminiscent of, but not equivalent to, the medieval and early-modern Latin concept of "distribution" – one important difference being that the predicates of particular-negative propositions are distributed but do not have subject-generality even potentially.

Another important difference between the two traditions is that the doctrine of distribution was linked by Latin logicians to the so-called *dictum de omni et nullo* which was often thought to ground syllogistic productivity. By contrast, the general rules proposed by Ḫūnaǧī and Taftāzānī were not used to derive the figure-specific conditions of productivity from more basic logical principles. They merely gave an alternative presentation of the rules of syllogistic productivity – a presentation that did not lay claim to being more fundamental than the familiar, figure-specific rules.