

Sequential algorithms in nonlinear programming

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In this thesis, a class of methods for solving the following non-linear programming problem

$$\begin{aligned} & \text{minimize } f(x) \\ & \text{subject to } x \in S = \{h_i(x) = 0, i = 1, \dots, l; g_j(x) \geq 0, j = 1, \dots, m\} \end{aligned}$$

is investigated. The class of methods under investigation consists of the ones which reduce the computational process to a sequence of unconstrained minimization subproblems. These methods are generally noted for their simplicity and robustness. The aim is to analyse and extend existing results on the structure and convergence of these methods, and, in certain cases, to introduce modifications for improving the numerical performance of existing algorithms.

Because of the possible effects degeneracy and near-degeneracy may have on the numerical performance of algorithms, this thesis is initially concerned with the structural differences between degenerate and non-degenerate problems. This leads naturally to a new interpretation of degeneracy, some structural differences between degenerate and non-degenerate problems, necessary and sufficient conditions for 'isolated' solutions, the concept and properties of an 'extended' problem, and a sensitivity analysis for degenerate problems.

Barrier function methods, specifically first order (logarithmic function) methods, are then investigated. It is shown that, if certain regularity conditions are satisfied, the successive estimates $x(r)$ of the

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solution $x(0)$ lie on a smooth trajectory. If the non-degeneracy assumption is not satisfied, then the convergence to $x(0)$ is order $r^{\frac{1}{2}}$ compared with order r for non-degeneracy. A measure of sensitivity is introduced which becomes large when the non-degeneracy assumption is close to violation. It is shown that this sensitivity measure is related to the growth of $\frac{d^i x}{dr^i}$ with respect to i for fixed r . Using this information the extrapolation procedure can be analysed and the number of useful extrapolation steps can be predicted. A source for a number of shortcomings in the classical algorithm is identified, and a modification is proposed. Properties and convergence of the modified algorithm are analysed. The numerical performance of the classical and the modified algorithms is compared for a range of test problems.

Penalty function and multiplier methods, specifically the quadratic penalty function method and the corresponding augmented Lagrangian methods, are treated next. Order of convergence results for the quadratic loss penalty function and the augmented Lagrangian methods are extended to include problems with degenerate inequality constraints. For an equality constrained problem, it is known that an augmented Lagrangian method is just an unconstrained maximization of the augmented dual function, $D_{\sigma}(v) = \min_x \left\{ f(x) - v^T \cdot h + \frac{\sigma}{2} h^T \cdot h \right\}$, with respect to v for σ large enough. It is now shown that the Newton iteration for v based on maximizing $D_{\sigma}(v)$ can be decomposed into a Powell/Hestenes iteration followed by a Newton-like correction to v . Utilizing the decomposition result, two acceleration techniques are proposed for improving the convergence of the Powell/Hestenes multiplier method. The first of these two techniques is a simple device superimposed on the original Powell/Hestenes iteration to make use of information from the previous iteration. For problems with only one constraint, this acceleration technique is equivalent to replacing the second (Newton-like) part of the decomposition by a finite difference approximation. The other acceleration technique is just a quasi-Newton technique. Numerical results using various updating schemes for v are reported and discussed.

Finally, possible uses for the methods developed in this thesis are

discussed including constrained optimization without derivatives and optimal control problems with terminal constraints.