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EPHEMERIS TIME

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Abstract. In this article is described the measurement of time from a more general point of view than has been taken before. It is shown that mean solar time and ephemeris time are two particular applications of the general principle of measuring time by observing angular motions, a principle that can be applied to any angular motion whatever, provided that an adequate theory of the motion is available.

Space and time may be measured in a fashion that makes the errors of measurement independent. That is, a measure of distance and a measure of time may be defined in such a way that an error in a measurement of distance will not produce any error in the measurement of time, and vice versa. It is true that the measurements must be restricted to the local frame of reference. Special relativity teaches us that the errors are no longer strictly independent if two observers in relative motion exchange and combine the results of their observations. But this restriction does not destroy the principle of independence, it means only that one observer must have a care in interpreting the observations of another.

It is easily possible to define the fundamental units of measurement in such a way that the errors in measurements of time and distance are not independent. For example, the fundamental unit of length might be taken as the meter and the fundamental unit of velocity as the velocity of light. The unit of time would then be a derived one, obtained as the quotient of distance by velocity, and then any error in measurements of length would be carried over into practical determinations of time. This procedure would not, however, destroy the duality of space and time, which is intrinsic and, for any one observer, absolute.

An alternative choice of fundamental units is to take the meter as unit of length, and the time required for light to travel a distance of one meter as the unit of time. In this case also the errors would not be independent; practical measurements of time-intervals would be affected by the error in the length of the meter-bar used, as well as by the accidental errors of measurement of the time-intervals themselves.

The non-independence of the errors in the two cases mentioned is a result of defining one thing in terms of another. Let us consider a third case of a different sort. Suppose that the wave length of some monochromatic electromagnetic radiation is taken as the unit of length, and the inverse of the corresponding frequency as the unit of time. In this case the errors may be independent or not, according to the techniques of measurement employed. If no reference is made to frequency when measuring lengths, and if no reference is made to wave length when measuring time-intervals, then the errors are independent. On the other hand, the product of a wave length by the corresponding frequency is an absolute constant, the velocity of light. If the value of this constant is known, it then becomes possible to infer lengths from measurements of frequency, and vice versa; and if this technique is employed, the errors will not be independent. With this choice of units it would become necessary to examine the techniques used in every experiment, in order to ascertain whether the errors are independent or not.

Until now, the fundamental units of length and time have always been defined in such a way as to preserve the intrinsic independence of the errors of measurement. In particular, the recent redefinition of the second preserves the independence. It is impossible to infer anything about time from measurements of distance, and vice versa. I remark, however, that it has been a matter of choice rather than necessity. It would have been possible to define the unit of distance as the distance through which a body would fall in a unit of time, under the action of gravity; and if this had been done, the errors of measurement would not have been independent.

Any angle that is a known continuous function of the time, and that can be measured independently of measurements of distance, is suitable for the measurement of time. It is not necessary that the angle increase uniformly with the time, nor even that it always increase with the time; it is only necessary that an adequate theory of the motion is available. As an example of an angle that does not always increase with the time, we have the motion of a simple pendulum; but in this case we have not a suitable fundamental measure of time because we have not an adequate theory of the motion, which is unpredictably disturbed by variations in gravity, variations in the length of the pendulum, and imperfections in the suspension.

The cases where theories of angular motion have been brought to the highest state of perfection are the cases of motions of the bodies in the solar system. They are of two sorts, motions of rotation and motions of revolution.

Imagine a point fixed on the surface of a rotating body, at some distance from the poles of rotation. As the body rotates once on its axis, this point moves nearly in a circle, describing an angle of nearly 2π radians. The equations of angular motion of this point are known and may be solved, giving

$$\theta = a + bt + \text{periodic terms}, \qquad (1)$$

where θ is the amount of angle described since some fundamental epoch, t is the time reckoned from the fundamental epoch, and a and b are constants of integration. The periodic terms are functions of the time and of a and b, and may involve additional constants of integration; their amplitudes are very much smaller than 2π, which makes the solution of the equation of motion possible by successive approximations. The periodic terms express the amounts of precession and nutation since the fundamental epoch. The number of equations of the type (1) is the same as the number of bodies in the system.

Next imagine a line drawn from the center of mass of a body to the center of mass of the body about which it revolves. During one revolution of the body this line changes its direction in space, describing an angle of nearly 2π radians. The equations of orbital motion may be solved for this angle, giving

$$v = A + nt + \text{periodic terms}, \qquad (2)$$

where v is the amount of angle described since the fundamental epoch, t has the same meaning as before, A and n are constants of integration, and the periodic terms now result from the gravitational action of other bodies in the system. The number of equations of the type (2) is one less than the number of bodies in the system, the sun being excepted.

The constants, a, b, A, and n for each body of the system must be determined by observation. But it is impossible to determine their values for any body until a measure of time has been adopted. In order to define a measure of time it is necessary and sufficient to choose numerical values for a and b, or for A and n, for one of the bodies. Once that has been done, then the remaining two constants for that body and the four constants for every other body may be determined by observation. So the number of arbitrary constants of integration for the whole system is two less than four times the number of bodies in the system (if we neglect the sun and the additional constants of integration that may be present in the periodic terms, and in the expressions for the other components of the complete motions).

In choosing the two constants that define the measure of time we are at liberty to take any numerical values whatever. The constant b or n defines the unit of time and the constant a or A defines the fundamental epoch from which the units are counted.

Consider first the rotation of the earth, and the value of b. If we take b equal to 2π, then the unit of time is approximately one rotation of the earth, but not precisely, owing to the presence of the periodic terms in Eq. (1). The precise unit of time is one *mean* rotation of the earth, the mean value of θ being defined as its value when the periodic terms in Eq. (1) are suppressed. Again, if we take b equal to 4π, then the unit of time will be two mean rotations of the earth. Or, if we take b equal to 2π divided by 86400 and multiplied by 1.002737811 8868, then the unit of time would be the second of mean solar time. The value of a was determined by the condition that the mean sun should be over the meridian of Greenwich at noon. Having fixed the values of a and b for the earth it then becomes possible to determine time by observation of the rotation of the earth, and to determine by observation the values of a and b, and of A and n, for other celestial bodies. But having adopted mean solar time as the fundamental measure of time it is no longer possible to regard a and b for the earth as being constants of integration in the usual sense; their values cannot by any means be determined by observations.

From what has been said it is now easy to see how Eq. (2) for the earth may be used to define a measure of time. The values of A and n adopted were those of Newcomb, because there was no reason to prefer any others, and the new definition of the second depends on his value of n. The values of A and n are now not subject to corrections determined by observations. It is now meaningless to speak of an error in the tabular mean longitude of the sun, or of an error in its tabular mean motion. On the other hand it is now possible to determine a and b for the earth by observations. The measure of time defined by Eq. (2) for the earth, with Newcomb's constants, is ephemeris time.

Whether Eq. (1) or Eq. (2) is used to define the measure of time, the practical determination of time is the same in principle. A table is constructed, showing in adjacent columns values of t and of v (or θ). A determination of time consists in measuring the value of v (or θ), and consulting the table to ascertain the corresponding value of t.

It is particularly to be noticed that no assumption is made about the variability or invariability of the measure of time. The only assumption is that the equations of motion are known. In the case of mean solar time, Eq. (1) for the earth has been shown to be inconsistent with Eq. (2) for the earth, the moon, Mercury, Venus, and the great satellites of Jupiter, whereas the Eqs. (2) mentioned are consistent with one another. It is this result that has persuaded us that the theory of rotation of the earth is incomplete, and to discard mean solar time in favor of ephemeris time.

DISCUSSION

Dr. W. FRICKE,* referring to the paper read by Dr. Nemiro, said that the revision of the fundamental system of star positions FK3 which will result in the formation of FK4 is expected to be completed early in the year 1960. At this moment a statement is already possible about the accuracy of the individual corrections to FK3 which were published in 1957. According to the results obtained from the observations at Danjon's astrolabe at the Observatory of Paris during the last few years, these corrections are an effective improvement of FK3. The systematic corrections to FK3, which are being deduced at present by the use of absolute star catalogues, will be partly based on catalogues from the Pulkovo Observatory, which are excellent. Nevertheless, Dr. Nemiro's remark is true, that also in FK4 the influence of the errors of older star catalogues will not be entirely eliminated. Therefore, his proposals have to be welcomed that the time determinations with modern transit instruments be organized so that they give a possibility of deriving the accidental and systematic errors of the right ascensions of the stars of the fundamental catalogue with highest precision. Moreover, the use of the impersonal astrolabe has to be recommended for high precision observations.

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Dr. CLEMENCE thought that in the future the Danjon astrolabes should play an important role in the construction of fundamental catalogues of bright stars. By reason of the numerous observations made in the time services these instruments could greatly reduce systematic errors of various sorts in existing catalogues. It would still be necessary to rely on meridian circles for the equinox and equator, and for the positions of reference stars needed in the reduction of photographic catalogues.

Prof. PAVLOV mentioned that in recent years the time service is experiencing a great change. The function of the International Time Service is turning more and more into a service for checking the rotation of the earth; at the same time, the determination of uniform time is becoming in greater degree dependent on the atomic standards of frequency.

In connection with the first problem, the requirements of the precision of astronomical determinations of time, and particularly of catalogues of star positions, have significantly increased. Prof. Danjon has made a bold attempt to deduce seasonal variations of the rotation of the earth from observations conducted at a single observatory. His work is of great interest. The possibility of a similar kind of deduction from observations with transit instruments requires, first of all, a careful elimination of systematic errors in star catalogues.