N-BODY SIMULATIONS AND THE VALUE OF Ω

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We have recently completed a series of N-body simulations of galaxy clustering in an expanding universe (Aarseth, Gott and Turner 1977). The initial conditions and our results concerning galaxy clustering will be summarized by Sverre Aarseth at this meeting. In this paper I would like to tell about the implications of these models for the value of $\Omega = 8\pi G\rho_o/3H_o^2$ (where ρ_o is the present mean density of the universe and $H_o = 50$ km s^{-1} Mpc^{-1} is Hubble's constant). In the standard Friedmann models with $\Lambda = 0$, $\Omega > 1$ implies that the universe will eventually recollapse while $\Omega < 1$ implies the expansion will continue forever. As discussed in Gott, Gunn, Schramm, and Tinsley (1974), there are a number of theoretical arguments to suggest that even the unseen matter in the universe is clustered the way the galaxies are so that virial mass determinations from groups and clusters and statistical virial theorem methods can provide good estimates of the mean mass density in the universe. We can utilize our N-body simulations to check the accuracy of these techniques.

Our simulations contain 1000 equal point masses representing galaxies contained in a spherical volume of present radius ~ 50 Mpc (see Gott 1977 for more details). Two primary models have been analyzed, an $\Omega = 1$ Einstein - de Sitter model and an $\Omega = 0.095$ open model with a mean density compatible with a variety of arguments including the cosmological abundance of deuterium (Gott, Gunn, Schramm, and Tinsley 1974). At the points in the simulations corresponding to the present epoch both models have virtually identical power law covariance functions stretching over five decades in radius. Over the observed range ($10^4 > \xi(r) > 1$) both models have covariance functions which are best fit by

$$\xi(r) \propto r^{-1.9}$$

(Gott, Turner, Aarseth 1977). This is in remarkable agreement with the relation

$$\xi(r) \propto r^{-1.8}$$

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observed by Peebles (1974) over the same range. As Sverre Aarseth has mentioned in his talk we believe the slope of the covariance function in our models is influenced by relaxation effects. We find that the slope of the covariance function is rather insensitive to initial conditions. In particular for the models we have done it is impossible to separate the $\Omega = 1$ and the $\Omega = 0.1$ models by looking at their covariance functions. Thus it appears that there is little hope of determining the value of Ω from studies of the covariance function over the range $10^4 - 1$.

We have recently completed a preliminary analysis of the three point correlation functions (Stark, Gott, and Aarseth 1977). The results for both $\Omega = 1$ and $\Omega = 0.1$ models appear to be in good agreement with the observational data of Peebles and Groth 1975.

While the $\Omega = 1$ and $\Omega = 0.1$ models have similar clustering properties they have rather different velocity distributions. The velocity dispersions of galaxies relative to the Hubble flow and within clusters in the $\Omega = 1$ model are ~ 3 times as large as in the $\Omega = 0.1$ model. This is simply because the galaxies in the $\Omega = 1$ model, weigh 10 times as much as those in the $\Omega = 0.1$ model. This large difference in velocity dispersions makes it possible to easily distinguish between the two models.

As Ed Turner described in his talk, we have used these N-body simulations to check the group catalogue techniques of Gott and Turner 1977. We can see how well virial mass estimates from groups reflect the true masses of the galaxies in the models. The N-body simulations show that these techniques are accurate to about a factor of 2. The simulations can be used to correct these methods for any systematic errors. For the observational data this leads to corrected values of Ω in the range

$$0.06 \leq \Omega \leq 0.14$$

(Turner et al. 1977). This includes estimates using median M/L values from all groups, and mean values from uncontaminated binaries, and uncontaminated groups with 3 or more redshifts. It is interesting that binaries give similar mass to light ratios ($\Omega = 0.09$) as do groups ($\Omega = 0.06$) and clusters ($\Omega = 0.13$).

Recently there has been renewed interest in statistical virial theorem methods. Fall (1975) has pointed out that the excess potential energy δW (per galaxy) due to the clustering can be calculated by integrating $(\xi(r)/r)\, d^3r$. Since the amplitude of the covariance function is fixed by observation, the potential energy per unit mass δW is proportional to Ω. Fall gives theoretical arguments suggesting that $\delta T \approx (2/3)\delta W$ where $\delta T = \frac{1}{2}V_p^2 = \frac{1}{2} < (\vec{V}-\vec{V}_H)^2 >$. V_p is the root mean square peculiar velocity of all galaxies in the sample relative to the uniform Hubble flow. The N-body simulations show that $\delta T \approx (2/3)\delta W$ for all models at the present epoch to an accuracy of 50% (Gott, Martin, Aarseth 1977).

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Fall adopted $V_p \sim 300$ km s^{-1} and using the amplitude of the covariance function found by Peebles deduced:

$$\Omega = 0.05$$

Davis, Geller and Huchra (1977) have reanalyzed this problem using a complete redshift sample of galaxies brighter than 13th magnitude. If all galaxies had peculiar velocities V_p relative to the Hubble flow then random pairs of galaxies should have line of sight velocity differences of $\Delta V_r = \sqrt{2}\, V_p/\sqrt{3}$. Such velocity differences between galaxies can be measured for galaxies with separations of ~ 1 Mpc. The amplitude of the covariance function is high enough that most such pairs seen in the sky are real pairs and not background foreground projection effects. The r.m.s. value of ΔV_r is computed using the method of Geller and Peebles (1973): they find $\Delta V_r \sim 300$ km s^{-1} as compared with $\Delta V_r \sim 270$ km s^{-1} found previously by Geller and Peebles with a smaller sample of galaxies. This result is supported by Gott, Martin, and Aarseth (1977) who find $\Delta V_r \sim 300$ km s^{-1} for an incomplete redshift sample in the northern sky. (In all these studies the Virgo cluster is excluded from the samples because with it removed the covariance functions of these samples are equivalent to those obtained in deeper surveys and have the appropriate power law shape. If Virgo is included it dominates the covariance function and the extra potential energy due to it would have to be included. Also Virgo may contain background foreground contamination problems.)

Using $V_p = \sqrt{3}\,\Delta V_r/\sqrt{2}$, Davis et al. deduce $\Omega = 0.46$ for the northern galactic cap and $\Omega = 0.23$ for the southern galactic cap. They also estimate the mean luminosity density in each region and find $1.0 \times 10^8\ L_\odot\ \text{Mpc}^{-3}$ and $5.5 \times 10^7\ L_\odot\ \text{Mpc}^{-3}$ respectively. From deeper surveys they deduce that the mean luminosity density for a fair sample of the universe is $6 \times 10^7\ L_\odot\ \text{Mpc}^{-3}$. Thus it is no mystery why the northern galactic cap yields a value of Ω that is higher by a factor of two; that region simply contains twice as many galaxies as the average for the universe. If the values are normalized to the average luminosity density, both the north and south give similar estimates of Ω. The average is $\Omega = 0.26$.

A study of the velocity distributions in the N-body simulations by Gott and Aarseth (1977) indicates the pair velocity differences are quite isotropic at all scales: thus if we pick any pair of galaxies their peculiar velocity difference vector is uncorrelated with their separation vector. This means when we sample close pairs in the sky $\Delta V_r \approx \Delta V_{tot}/\sqrt{3}$. Both $\Omega = 1$ and $\Omega = 0.1$ models show ΔV_r to be independent of radius for scales near 1 Mpc. This is in agreement with the observations. Gott, Martin and Aarseth (1977) have shown that the statistical method of Geller and Peebles does yield approximately correct estimates of ΔV_r at 1 Mpc. An interesting result found by Gott and Aarseth (1977) is that the true value of V_p is given by approximately $V_p \sim \Delta V_r$ where ΔV_r is the radial velocity difference of pairs at 1 Mpc, (this relation holds for both the $\Omega = 1$ and the $\Omega = 0.1$ models) rather than the naive

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estimate $V_p \sim \sqrt{3}\,\Delta V_r/\sqrt{2}$. This is perhaps even more surprising when one considers that motions on scales larger than 1 Mpc could in principle boost V_p above the naive estimate. However, it is easy to see how this comes about. V_p is the r.m.s. average value for all galaxies, while ΔV_r is the average value for pairs. Consider the following example: one cluster of 100 members and a velocity dispersion of 1000 km s^{-1}, 10 small groups with 10 members each and velocity dispersions of 300 km s^{-1} and 100 field galaxies with velocities of 100 km s^{-1} relative to the Hubble flow. Say further that the clusters and groups have sizes ~ 1 Mpc so that all have the same M/L ratio; assume the field galaxies have no neighbors within 1 Mpc. For this sample, V_p = 606 km s^{-1}. Now the 100 galaxies in the cluster produce 4950 pairs, while the 100 galaxies in the groups produce only 450 pairs and the 100 field galaxies produce no pairs at all, giving ΔV_r = 785 km s^{-1}. So $V_p = 0.8\,\Delta V_r$ for this case. In principle one must know the multiplicity function of galaxies (i.e. the distribution of group sizes (cf. Gott and Turner 1977) to correct ΔV_r for these statistical effects and determine V_p. The multiplicity function may be determined by making a group catalogue. With a proper treatment even the statistical virial theorem methods require some knowledge of the groups present. This brings us surprisingly close to the group catalogue methods with which we started. Those methods do not throw away the additional information available as to which pairs actually go together to form a group. By utilizing more of the available information group catalogue methods may be even more accurate than the statistical virial theorem methods. The N-body simulations show that only ~ $\frac{1}{2}$ of the galaxies have neighbors within 1 Mpc and that there are a large range of cluster sizes. The N-body simulations have multiplicity functions quite similar to those observed so we can regard the estimate $V_p \sim \Delta V_r$(1 Mpc) as reasonably reliable. This lowers the estimate of Davis _et al_. by a factor of 3/2 to give:

$$\Omega = 0.18$$

with an uncertainty of a factor of 2 due mainly to the uncertainty in the amplitude of the covariance function. If we used this value of V_p ~ 300 km s^{-1} with the amplitude given by Peebles we would obtain Fall's result $\Omega = 0.05$;the difference in Ω values is due to the fact that Davis _et al_. find an amplitude of the covariance function that is considerably lower than that found by Peebles.

Davis _et al_. also use solutions of the truncated BBGKY hierarchy equations obtained by Davis and Peebles (1977) which give $V_p \sim \sqrt{3}\,\Delta V_r$(1 Mpc) and yield values of $\Omega \sim 0.6$. The N-body simulations indicate that this BBGKY technique overestimates V_p by a factor of $\sqrt{3}$ and Ω by a factor of 3. In solving the truncated BBGKY equations a number of ad hoc approximations are made concerning both the evolution of the two and three point correlation functions and the form of the velocity distributions and the problem may lie in one or more of these approximations.

Peebles (1976) has formulated a statistical virial theorem method based on the observed amplitude of the three point correlation function (Peebles and Groth 1975). This predicts ΔV_r(3 Mpc) ~ 830 $\Omega^{1/2}$ km s^{-1}. Since the observations show ΔV_r(3 Mpc) ~ 300 km s^{-1} this gives

$$\Omega = 0.13$$

This figure is based on Peebles original estimates of the amplitude of the covariance function and should be compared with Fall's value of $\Omega = 0.05$ for the same assumptions.

The Geller and Peebles (1973) statistical virial theorem gives M/L ~ 140 and with the luminosity density found by Davis et al. yields $\Omega = 0.12$.

In conclusion, the different suitably corrected statistical virial theorem methods yield values of Ω in the range

$$0.05 \leqslant \Omega \leqslant 0.18$$

These results are consistent with those found by the group catalogue methods and are inconsistent with $\Omega > 1$ due to matter associated with galaxies. The results are consistent with the value of $\Omega = 0.1$ implied from cosmological production of deuterium with H_o = 50 km s^{-1} Mpc^{-1} (Gott, Gunn, Schramm, and Tinsley 1974).

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DISCUSSION

Jones: If the peculiar velocities on large scales were as large as indicated by the 24 hr microwave background anisotropy (say 650 km/sec), how would this affect your estimate of Ω?

Gott: Sandage and Tammann's studies indicate that perturbations of the Hubble flow within the local supercluster are less than $\sim$ 250 km s^{-1}. In any case if a peculiar velocity of the Earth were produced by galaxies within $\sim$ 20 Mpc, then the direction of the predicted motion should be roughly in the direction of the Virgo cluster and should be $\lesssim$ 250 km s^{-1}. The recent microwave background studies, if correct, suggest a velocity of $\sim$ 600 km s^{-1} in a different direction. This we would have to ascribe to a bulk motion of the whole local supercluster due to clustering on scales $\gtrsim$ 50 Mpc. The values of Ω, deduced above from comparing the peculiar velocities of galaxies relative to the local supercluster with the clustering within the supercluster would be unaffected. A separate estimate of Ω can be obtained from the bulk motion of the supercluster, if one knew the shape of the covariance function from 50 Mpc out to the current Hubble radius. Unfortunately, no observational data on this exists. Using the theory of Gott and Rees for the covariance function at large scales, I have recently calculated that a motion of 600 km s^{-1} for the local supercluster would imply a value of $\Omega \sim 0.2$.

van der Laan: If in your simulations you were to introduce a mass spectrum and a schematic form of tidal friction with its resulting mass segregation, have you any idea of the effect on your results?

Gott: We have new simulations using 4000 bodies in which the masses of galaxies are distributed according to a realistic Schechter type luminosity function, but we have not analysed these yet. We have done simulations where 2/3 of the galaxies have mass 1.0 and 1/3 of the galaxies have mass 2.0. At the end the heavy galaxies have a covariance function with approximately the same slope, but twice the amplitude of the low mass galaxies, in accordance with theoretical expectations. There is some evidence from studies of binaries and groups that E and SO galaxies have M/L values $\sim$ 2 that of spirals. This might explain why Davis and Geller find that in a magnitude limited survey the covariance function of E and SO galaxies is just twice the amplitude of that for spirals.

Audouze: With the values for Ω which come out from your talk ($\Omega \sim 0.1$) it seems to me that according to Gott, Gunn, Schramm and Tinsley deuterium may not be synthetized in sufficient quantities in a canonical model of Big Bang nucleosynthesis.

Gott: For H_o = 50 km s^{-1} Mpc^{-1} the value of Ω predicted by deuterium synthesis is $\Omega = 0.1$. I think that given the uncertainties, the estimates of Ω from deuterium production and dynamical measurements are in

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nice agreement. Of course, we think that it is quite suggestive that these two completely different methods give similar values.

Ozernoy: Why did you not obtain by numerical stimulations a cutoff in the covariance function due to the fact that gravitational instability does not work at redshifts smaller than about Ω^{-1}?

Gott: The cutoffs at $\xi(r) \sim \Omega^{-3}$, predicted by some theories for the low Ω models due to exactly the effect you mention, have not shown up in the N-body simulations. We have several lines of evidence to suggest that non-linear relaxation effects are important in establishing the slope of the covariance function over the observed range. Aarseth will talk about this tomorrow.

Peebles: I hope it is accepted that the fact that richer groups contribute more pairs than poorer ones causes no systematic error in the esimate of ΔV_r, if one does it right. In the form of the virial theorem I like best at the moment, one uses ΔV_r directly, with no attempt to deduce V_r, and one relates this to an integral of the three-point correlation function. This gives rather a higher Ω than Dr Gott mentioned.

Gott: The statistical virial theorem methods mentioned by Peebles do calculate ΔV_r in the proper way but they have implicit assumptions that may bias the results in large virialized clusters. In these clusters, which contribute a significant fraction of the total pairs, the velocities of the individual pairs of galaxies are not due to their motion about each other but to their random motion in the whole cluster. This may well make the cosmic virial theorem estimates somewhat too high. If one uses $\xi(r) \approx 68\ r^{-1.77}$ as found by you and substitutes $\Delta V_r \approx 300$ km s^{-1} as found by Davis, Geller and Huchra into your cosmic virial theorem using the 3-point correlation function it gives $\Omega = 0.13$.

Davis: I would like to disagree slightly with the conclusions you reached concerning my work with Geller and Huchra. We derive a lower limit of Ω in the South of 0.26, which if translated to a fair sample density would suggest $\Omega \gtrsim 0.3$. This estimate of Ω is a lower limit because it does not include any peculiar motion on large scales and it is not quite fair to conclude that large scale motion does not exist in the Universe because it is not found in the N-Body simulations of the Universe.

Gott: The lower limit you found used the naive estimate $V_p = (\sqrt{3}/\sqrt{2})\ \Delta V_r$. The N-body simulations indicate that rather than a lower limit this is in fact an overestimate. Large scale motions do boost V_p as you suggest, but the statistical effects I mentioned have an even stronger effect in decreasing V_p. The N-body simulations certainly do have large scale peculiar motions as can be seen by inspection of redshift space pictures. The simulations include both effects and give $V_p \approx \Delta V_r$. Thus we would correct the $\Omega = 0.3$ estimate you mention downward by a factor of (3/2) to give $\Omega = 0.17$.

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Fall: In attempts to estimate the rms velocity of galaxies with respect to the Hubble flow (V_p) by comparison with the relative velocities of pairs (ΔV_r) it is important to recognize that in principle V_p and ΔV_r can have different scale dependences. Would you comment on the results of your numerical experiments within this context?

Gott: The N-body simulations indicate that $\Delta V_r \sim$ const for pair separations 100 kpc < r < 3 Mpc as is found in the observations. The V_p we are interested in measuring is the rms peculiar velocity of galaxies with respect to the mean Hubble flow defined for a large homogeneous sample (r ∿ 50 Mpc). Motions on scales 1 Mpc < r < 50 Mpc can boost V_p relative to ΔV_r measured at 1 Mpc. However the statistical effects I mentioned in my talk make V_p lower with respect to ΔV_r than one would otherwise expect. The N-body simulations which produce reasonable covariance functions include both these statistical effects and the effects of large scale motions. They give the empirical result $V_p \sim \Delta V_r$.

Tully: Implicit in your discussion is the assumption that most of the mass in the Universe is distributed like the galaxies. This assumption may well not be correct.

Gott: These simulations do assume that the majority of the mass in the Universe is clustered like galaxies. This includes any unseen matter which falls into groups and clusters. While it is conceivable that most of the mass is in some homogeneous component which does not participate in the clustering, there are theoretical difficulties with this as outlined by Gott, Gunn, Schramm and Tinsley.

Fridman: Did you consider plane systems?

Gott: No.