17. COMMISSION DU MOUVEMENT ET DE LA FIGURE DE LA LUNE

PRÉSIDENT: M. T. BANACHIEWICZ, *Directeur de l'Observatoire de Cracovie, ul. Copernic 27, Cracovie, Pologne.*

MEMBRES: MM. Belkovich†, Boneff, Brouwer, Comrie†, Eckert, Guth, Kozieł, Mlle McBain, MM. Meyer, Nefediev, Sadler, Sundman†, Watts, Yakovkin.

YALE UNIVERSITY OBSERVATORY, DIRK BROUWER, *November* 11, 1951

Since the Meeting at Zürich, Dr Brouwer determined the parallactic inequality in the Moon's longitude from the occultations observed in the years 1932–42. Some details of this discussion were published as a part of an account of the Paris conference on astronomical constants in *Bulletin Astronomique*, **15**, 168–70, 1950. The solar parallax from this discussion was $8''\cdot793 \pm 0''\cdot0044$ (m.err.).

This conference also adopted the following resolution that Dr Brouwer believes might be considered by the Commission:

In order to bring the lunar ephemeris into accordance with the solar ephemeris the Conference recommends that Brown's *Tables of the Motion of the Moon* should be amended by removing the empirical term and by applying to the mean longitude the correction $-8''\cdot72 - 26''\cdot75T - 11''\cdot22T^2$, where T is measured in Julian centuries from January 0 Greenwich Mean Moon.

H.M. Nautical Almanac Office will report on the practical implications of this recommendation.

H.M. NAUTICAL ALMANAC OFFICE, R. GREENWICH OBSERVATORY, D. H. SADLER, *November* 30, 1950

The Nautical Almanac Office has continued its routine programme of prediction of occultations for sixty-four stations. The Occultation Machine, on which preliminary times with a probable error of about one minute are obtained for some eighty stations, is at present being overhauled and remodelled to facilitate operation. Some consideration has been given to the publication of machine times for a large number of regularly spaced stations, instead of the present programme; it is hoped that the matter can be discussed in more detail at the meeting.

The discussions of reduced observations have been published for the years 1943–46 in the *Astronomical Journal* (1943, Vol. **53**, 163, 1948; 1944–45, Vol. **55**, 7, 1949; 1946, Vol. **55**, 47. 1950) and that for 1947 is in course of publication. These occultations are being reduced with $k = 0\cdot2724953$ and without limb corrections; it is planned to reduce afresh the whole series from 1943 onwards with limb corrections when Mr Watt's results are available.

A combined list of all observations received for 1943 to 1947, showing details of observations and reduction together with residuals from the adopted solution for each lunation, is being prepared for publication in *Greenwich Observations*. This list will be circulated to all observers.

An investigation has been started into the best value of the ratio k of the Moon's diameter to that of the Earth to be used in the reduction of occultations. In view of the recent advances in the technique of observation of eclipses and occultations, using photo-electric methods and cinematography, and the incomplete work of Mr Watts on the marginal zones of the Moon, it is felt that it would be better to defer any change in the currently adopted value $k = 0\cdot2724953$, at least until this new material has been discussed.

U.S. NAVAL OBSERVATORY, C. B. WATTS, *December* 8, 1950

The work on the survey of the marginal zone of the Moon has progressed as follows. The photo-electric measuring engine (*Trans. I.A.U* **7**, 431, 1950) was completed and placed in service in December 1949 and has been found quite satisfactory During the past year 175 photographs were measured out of 650 on hand, which were made at Washington (50%), the Yale-Columbia Southern Station, Johannesburg (40%), and the Lowell Observatory, Flagstaff, Arizona (10%). The preparation of libration, orientation, and refraction corrections for most of these photographs has been completed; the application of the corrections to the profiles constructed by the measuring engine has been deferred (except experimentally) pending the completion of an apparatus to facilitate this operation.

Much thought has been given to the special problems arising in such a survey. The most difficult of these appears to be the determination of the spherical datum surface to which the measured elevations should be referred. This portion of the task is complicated by the existence of apparent gradients, transverse to the marginal zone, created by mountainous features sufficiently high to dominate a portion of the limb, even when located well in the background or foreground. These fictitious slopes appear to extend in some cases across nearly half the width of the zone and lie well above the real surface over much of this distance. The danger of introducing systematic level errors in such measures was pointed out by Banachiewicz (loc. cit. p. 173). A tentative plan for treating this problem has been developed which will be tested as the work progresses.

CENTRAL INSTITUTE OF ASTRONOMY, ONDŘEJOV U PRAHY, DOC., DR VLADIMIR GUTH

Occultations

(*a*) *Observations.* The occultations of stars by the Moon were in the years 1947–50 systematically observed at the Ondřejov Observatory (*B.A.C. Prague*, **1**, 107, 1949, and **2**, 47, 1950, and MS.) at the Technical University, Prague (*B.A.C.* MS.), at the Novák's private observatory Prague-Smichov (*B.A.C.* **1**, 106, 1949) and at the K. H. Otavský's private observatory Dolní Mokropsy (MS.). In the year 1950 the observation of occultations was further on the regular programmes of the following observatories: Skalnaté Pleso, Technical University, Brno, Štefanik Observatory, Prague. The observations made in Czechoslovakia are organized by V Guth in collaboration with the International Centre for Occultations in England (H.M. Nautical Almanac Office).

(*b*) *Predictions.* The predictions of NZC stars for Prague according to H.M. Nautical Almanac Office are given in the international supplement of *Hvězdářská Ročenka* (*Czech Astronomical Yearbook*). The elements of occultations during the total eclipses of the Moon are also included. The predictions of occultations of faint stars at Prague for the year 1950 were prepared at the Technical University, Prague (Plavec, Syrový) and distributed by letters.

(*c*) *Reductions.* Reductions by Comrie's method were done by Guth and Plavec (*B.A.C. Prague*, **1**, 21, 1948, and **2**, 46, 1950). The irregularities of the Moon's limb according to Hayn's charts were allowed for in the first paper. Guth elaborated a differential method of reduction of occultations using Comrie's method as a base, which is useful for the reduction of multiple observations at the same place or in the neighbourhood.

Observations of Partial Eclipses of the Sun. (Method of chords and contacts.) Observations of the eclipse 1945, July 9, were reduced by J. Bouška (*B.A.C Prague*, **2**, 13, 1950), of 1949, April 28, by J. Bouška, University Observatory, Prague (*B.A.C. Prague*, **2**, 13, 1950) and the same eclipse observed at the Fišer's private observatory, reduced by Bouška and K. Fisher (*B.A.C.* **2**, 13, 1950), and at the Technical University, Prague (*B.A.C.* **1**, 152, 1949) by J. Prochazka.

Suggestions. Guth suggests issuing ecliptical charts ($\pm 6°$) designed in Mercator's projection, containing stars up to the 9th magnitude, for the equinox 1950.0; these charts should serve for the graphical-numerical method of prediction of occultations for faint stars by Döllen's method. The star charts should be supplemented by an approximate ephemeris of the Moon calculated for the equinox of the charts. A. Bečvař is willing to prepare these charts with the collaboration of the proposer.

Work Carried out in the U.S.S.R. in 1948–51

Prof. A. A. Yakovkin, *Kiev, University Observatory*

At the Engelhardt Astronomical Observatory the late I. V Belkovich completed his heliometric observations of the Moon begun in 1932. He obtained a total of 247 observations. Belkovich worked up in the usual manner, the first part (1932–42) consisting of 151 observations, and obtained the following constants of physical libration and the co-ordinates of the crater Moesting A:

$$\lambda = -5^\circ\ 9'\ 20'' \pm 9'' \text{ mean error.}$$
$$\beta = -3^\circ\ 10'\ 41'' \pm 7.$$
$$h = 15'\ 32''.80 \pm 0.32.$$
$$I = 1^\circ\ 32'\ 0'' \pm 14''$$
$$f = 0.67 \pm 0.03.$$

These results constitute a part of Belkovich's fundamental work, 'Physical Libration of the Moon' (*Engelh. Observ. Kazan Univ. Publ.* No. 24, 1949), in which the Kazan series of observations of the Moon with heliometer are worked over and generalized. In the first place, Belkovich resolved the conditional equations for Banachiewicz's series with a corrected coefficient sin 2ω in the expression for physical libration in longitude. For the function of the moments of inertia $f = \frac{C-B}{C-A} \cdot \frac{B}{A}$ he obtained $f = 0.70 \pm 0.03$, instead of $f = 0.74 \pm 0.03$. In order to get a better value for f from Yakovkin's series (1916–31) and Belkovich's series (1931–42) f was given the values of 0·67, 0·68, 0·70, 0·75, and 0·80 and the equations with the remaining unknowns solved in succession. An f was determined by interpolation in which the sum of the squares of the remaining deviations would be at a minimum. For Yakovkin's series $f = 0.71$, for Belkovich's series 0·71. Thus, from the three Kazan series (loc. cit. p. 190) $f = 0.71$.

In order to determine the free libration in longitude, Belkovich joined Hartwig's series to the Kazan series and reduced the phases to a common initial epoch. The amplitudes are trustworthy but the phases show a progressive shift of about 100 degrees from series to series. The mean epochs of these four series differ for about ten years, while the phases are equal to 95°, 219°, 310°, 419° (loc. cit. p. 204). Belkovich admits that in the given case there may be a secondary fluctuation in the deviations of v_e with amplitude exceeding the amplitude of free libration. This makes doubtful the existence of a free libration.

An analysis of the remaining errors in the equations for longitude reveals a fluctuation with a period of 2ω, that is 1095 days; it is difficult to attribute such a fluctuation to the effect of the limb. From the four large series, the amplitude of this fluctuation is found to be $29'' \pm 5''$

A fluctuation with a period of 4ω, that is 2190 days, with an amplitude of $14'' \pm 5''$ is also quite evident. But upon simultaneous solution of all the four series, a free libration with a period of 1205 days (for $f = 0.72$) is not found (the amplitude is found equal to $4'' \pm 5''$).

We believe that the progressive shift of the initial phases of free libration in the four series has to be explained by the error of the value of the period, accepted as 1184 days. If we assume a period equal to 1085 days for the free libration, the initial phases will be brought into agreement. This period permits a single determination of the value of $f = 0.65$, whereas by the usual method of determination of f, from the coefficients of forced libration, two solutions are found for f, as has been indicated by Belkovich in 1939. The final solution can be obtained by the method of approximation. Such a method of determination of f from the observed period of free libration was pointed out in 1948.

An investigation of the libration effect in the radius of the Moon in the series of Hartwig, Banachiewicz, Yakovkin, and Belkovich showed that, in the first place, the radius of the eastern limb in all the series is greater than that of the western on the average for $0''{\cdot}14 \pm 0''{\cdot}03$ (mean error) and, in the second place, that the libration effect is one-half as great on the eastern limb, as on the western (*Engelh. Obs. Publ.* **24**, 242, 1949).

Nefedjev obtained at the Engelhardt Observatory in the years 1938–45, 143 observations of Moesting A co-ordinates from which he deduced the following constants (*Astr Circ. U.S.S.R.* Nos. 98–9, 1950):

$$\lambda = -5^\circ\ 10'\ 13'' \pm 14'' \text{ mean error.}$$
$$\beta = -3^\circ\ 11'\ 46'' \pm 9.$$
$$h = 15'\ 33''{\cdot}90 \pm 0{\cdot}45.$$
$$I = 1^\circ\ 32'\ 04'' \pm 15.$$
$$f = 0{\cdot}65 \pm 0{\cdot}045.$$

In second approximation two values were obtained for f· 0·60 and 0·71. Belkovich obtained 0·62 and 0·71 from Yakovkin's series, and 0·60 and 0·71 from his own observations.

In this series, the libration effect of the lunar radius is $+0''{\cdot}041 \pm 0''{\cdot}013$ per 1° of libration in agreement with other determinations.

The mean radius of curvature of the eastern limb was found to be greater than that of the western by $0''{\cdot}14$ in agreement with other observations. Nefedjev continued his work along this line and has obtained 252 observations.

Yakovkin recalculated his observations of the Moon for 1916–31, taking into consideration the change of lunar profile (*Kiev Obs. Publ.* No. 3, 1950). Values obtained by him for the constant of libration differ considerably from the initial ones; the new values were as follows:

$$\lambda = -5^\circ\ 11'\ 13'' \pm 14'' \text{ mean error.}$$
$$\beta = -3^\circ\ 13'\ 11'' \pm 11.$$
$$h = 15'\ 34''{\cdot}52 \pm 0''{\cdot}6.$$
$$I = 1^\circ\ 33'\ 48'' \pm 17''$$
$$f = 0{\cdot}82 \pm 0{\cdot}03.$$

In his paper 'Inclination of the Lunar Orbit and the Libration Effect' (*Kiev Obs. Publ.* No. 4, 1951) Yakovkin showed that as a result of the dependence of the visual lunar radius upon libration in latitude, different values of the inclination of the lunar orbit should be obtained from observations of the declination of the northern or southern limb. Corrections of the tabular values of the inclination of the lunar orbit were deduced from Greenwich and Washington meridian observations. From a treatment of a total of 2721 observations obtained for different periods of time it was found that the inclination of the southern limb is greater, on the average by $0''{\cdot}52$, than the inclination of the northern limb. This should be regarded as a new proof of the existence of a libration effect in the radius of the Moon, of the same order as that in heliometric observations.

At the Kiev Astronomical Observatory 112 plates with 336 photographs of the Moon have been obtained in 1950 with an astrograph ($f = 4{\cdot}3$ m.), with photographs of stars used for accuracy. Similar work is being done at the Engelhardt Observatory with a horizontal telescope ($f = 8$ m.).

Recommendations

1. Having considered as established, the existence of a considerable effect of libration in the radius of the Moon and of an asymmetry of the lunar profile in relation to the equator, the Commission thinks the study of the figure of the margin of the Moon a problem of prime importance.

2. The Commission thinks that, after the completion of the first problem, all the heliometric observations of the past and the current century have to be worked up in a uniform manner to determine the constants of the physical libration of the Moon.

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The memoir of K. Kozieł on the Moon's libration as derived from Hartwig's Dorpat heliometric observations, appeared in the *Acta Astronomica*, **a**, **4** 61–193, 1948–49. Its numerical results, as obtained 'with Hayn', i.e. taking into account the irregularities of the lunar disk according to Hayn, are summarized as follows: λ, β, h are three selenographical co-ordinates of Mösting A, I inclination of the Moon's mean equator to the ecliptic, f the quantity $(C-B)\,B:(C-A)\,A$ (A, B, C being the principal moments of inertia of the Moon).

Observer Hartwig, June 1884–December 1885

	I root for *f*	*II* root for *f*
λ	$-5°\,11'\,50'' \pm 12''$	$-5°\,11'\,16'' \pm 13''$
β	$-3°\,10'\,27'' \pm 17''$	$-3°\,9'\,56'' \pm 18''$
h	$15'\,32''{\cdot}88 \pm 0''{\cdot}56$	$15'\,33''{\cdot}81 \pm 0''{\cdot}58$
I	$1°\,31'\,36'' \pm 23''$	$1°\,31'\,10'' \pm 22''$
f	$0{\cdot}71 \pm 0{\cdot}051$	$0{\cdot}60 \pm 0{\cdot}05_5$
Mean error of the weight unity	$\pm 0''{\cdot}614$	$\pm 0''{\cdot}575$

Thirty-six observations, each consisting on the average of 10 distances.
Heliometer 108 mm. aperture, 1·6 m. focal length.

The solution being a double one, the problem raised by this investigation (and set forth in the Draft Report for the Zürich meeting, 1948, August; *Trans. I.A.U.* **7**, 172) is: which of the two values of f is the true one—that in the vicinity of 0·71, or that near 0·60. The choice between them influences other unknowns.

Some other results and features of this paper are:

1. The removal of the uncertainty relative to the coefficient of the term sin 2ω of the physical libration in longitude.

2. The allowing for the convergency of hour circles in the reduction of the measurements.

3. An exact application of the method of least squares giving the unknowns directly, instead of using the apparent co-ordinates of Mösting A as intermediary independent unknown quantities. The departure from the method used in this problem since Bessel increases the accuracy of the solution and gives the *a posteriori* mean error of one measured distance. This method, developed by Banachiewicz, also permits the utilization of incomplete evenings.

4. The employment throughout of the Cracovian calculus.

5. The confirmation (loc. cit. p. 192) of the librational term $b\beta_0°$ in the Moon's radius, if the measurements are treated by the old method giving the apparent radii from semicircles and apart from other unknowns. On the other side, the term is disproved, as far as it is possible because of limited observational material, by the new method in which the observations of both limbs are treated together.

The last result being very important for the actual problems of the figure of the Moon, it would of course be advisable to prove it by other observations. Meanwhile, Banachiewicz has investigated whether the observations of Hartwig are not affected by a systematic error of phase, similar to that found recently by Belkovich. (The Kazan observer remarked that his observations give the longitude of Mösting A 122″ smaller, if the altitude of the Sun for an observer on the crater is below 55° and the crater is connected with limb II, *Eng. Obs. Publ.* **24**, 161.) With this end in view, the writer utilized 157 residuals of Hartwig's observations 1890–1915, as collected by Belkovich (loc. cit. pp. 194–6); the result proved negative. The residuals of the observations of Hartwig do not show any simple correlation with the phase. The mean residual of cos $\delta\Delta\alpha$ for the observations made more than three days after the full moon $-0''{\cdot}03$, especially, leads to the unwarranted correction of selenographic longitude of $+7''$ only. This is too small a quantity to influence the conclusion of Kozieł, it being supposed that the earlier observations of Hartwig are similarly free from the phase-error.

Dr Kozieł reports he is now at work on Hartwig's 1877–79 Strassburg observations, to which he applies the Cracow method, to obtain a broader basis for his results.

It follows from simple geometrical considerations, as well as from numerical formulas (cf. *Acta Astr.* p. 192, footnote), that an apparent increment of the radius of the Moon, as obtained from one limb only, may result either from the increment of the polar-region radius, or from the decrement of the equatorial-region radius, or from the combination of both. Therefore, an increment of the lunar radius, as obtained from limb I or II, both limbs being taken separately, does not involve the increase of the radius, computed from I and II taken jointly. For this reason the librational term of the radius, as obtained by the old method of separate limbs, does not necessarily call for the existence of a similar term in the method of joint limbs, although very particular slopes of the equatorial regions of the Moon would then be required. The whole matter is somewhat puzzling and appears to demand further consideration.

We have identically

$$\text{(The Figure of the Moon)} = \text{(Sphere)} + \text{(Deviations from the Sphere)}, \qquad (1)$$

and according to Hayn, after the reduction to the mean distance

$$\text{(The Profile of the Moon)} = \text{(Circle of constant radius)} + \text{(Irregularities)}, \qquad (2)$$

the terms of (2) depending on the topocentric libration λ_0, β_0, of the Moon. We denote by (irreg. H.) the irregularities, as given by the charts of Hayn. These irregularities may demand certain corrections, ΔH. Then

$$\text{(The Profile of the Moon)} = \text{(Constant circle)} + \text{(Irreg. } H.) + \Delta H. \qquad (3)$$

Let us now take a set of observations on the profile of the Moon reduced 'with Hayn' It means that at the basis of the reduction are the first two terms (circle) + (irreg. H.) of the second member of (3), or what might be called 'the model of the Moon after Hayn' If any deviations from this basis are shown by the observations, they have no direct bearing on the figure of the Moon, but only on Hayn's model of this figure. To deduce from them anything referring to the true figure of the Moon, the underlying heights according to Hayn must therefore be taken into account. This conclusion applies both to the inequality of the halves of the Moon, and to the apparent libration of the radius. (N.B. we do not call Yakovkin's term $b\beta_0$ into question, because it comes out also from the reductions 'without Hayn', and does explain different inclinations of the lunar orbit as deduced by Yakovkin from the upper and lower limbs of the Moon.)

The ΔH, which appear as a librational term of the radius of the Moon, measured with the heliometer and reduced 'with Hayn', are large, of the order of $0''\cdot058\beta_0{}^\circ$; according to (3) they indicate, first of all, the systematic errors of the heights after Hayn. What is the origin of these systematic errors? This question is of great importance, because the method used by Hayn is *mutatis mutandis* universally employed in the determination of the lunar radius and centre from photographs. The primary cause of its failure seems to be the systematic nature of the lunar polar-region elevations. This systematic character of the polar slopes is to some extent plausible also *a priori*. Let the prime meridian of the Moon be an ellipse with $a-b=10$ km., with the great axis of its visible part directed, in accordance with Prof. Hopmann's views, somewhere towards Alphonsus. The mean increase of the polar radius of the Moon for one degree of the libration in latitude would be, in the prime meridian, 111 m., or, geocentrically, $0''\cdot06$, increasing the limb-radius by $0''\cdot06 \times 0\cdot848 = 0''\cdot05$, i.e. by the required order of greatness (0·848 being the double coefficient of our formula (*Acta Astr.* loc. cit.)).

To be sure it is often supposed, not excepting our Zürich report, that much harm originated because the photographs lacked a scale. To obtain the heights two other unknowns must, however, be determined, besides the radius of the Moon, viz. the co-ordinates of its centre, and this is also a source of errors.

The above-cited result of Kozieł that the lunar disk appears not to change (with the exception of polar radius) according to the librational term $b\beta_0$, is confirmed by the occultations of the Pleiades, which occur for the libration in latitude of about $-4°$ in the mean, but lead to practically the same, or even greater radius $15' 32''\cdot77$, than other occultations (cf. H. Illigner, *A.N.* **235**, p. 306, October 1934; the correction $+0''\cdot11$ made there for the change of parallax is to be cancelled). If the knowledge of the scale would be sufficient then one should expect, on the ground of the above result, that the heliometer measurements, with their best known scale, would lead to the true, constant dimensions of the Moon. As a matter of fact, they give variable and therefore false dimensions (also if reduced 'without Hayn'). The same must happen also with the photographs. And if the radius were not trustworthy, neither could the heights be right.

Another question arising here is why the librational coefficient b is approximately the same, 'without' and 'with' Hayn. Thus the observations of Schlüter gave to Yakovkin $b = 0''\cdot09$ 'without', his own 'with' $0''\cdot05$; the great series of Hartwig-Naumann gave to Banachiewicz 'without' $0''\cdot054$, 'with' $0''\cdot029$. To explain this, let us suppose that the limb of the Moon was observed by heliometer as well as photographed at the same time and in the same place. The resulting semicircle is obtained from the heliometer and the astrograph by nearly the same method, and must be the same, abstracting from the errors of observations and the limited number of points. If then the heliometer measurements are corrected according to the heights determined photographically, the resulting semicircle must be again the same and must be affected by the same systematic error. In reality the photographic and the heliometer measurements do not of course refer to identical, but only to more or less distant limbs, and the errors will be somewhat different. Moreover, the profile according to Hayn is a little smoothed, and this is reflected in the smaller 'with' values of b.

In the heliometer measurements the systematic errors can be considerably diminished by the joint treatment of the two limbs, as in the Cracow method, thanks to the existence of the common crater as a point of reference, and the knowledge of the position-angles and the scale. But the photographic determinations of the scale are often less favourably situated in this respect.

The prediction of occultations by the Moon has been made by the Cracow Observatory since 1920 for four to five places in Poland. Until 1934 the semi-graphical method of Kowalski (employed by the writer since 1899, with some refinement, and about 1926 described by Comrie) was used to this end, but then it was replaced by a purely numerical method (*Acta Astr* **a**, **3**, 67–76). In 1950 a new procedure was given by Banachiewicz, it economizes more than 50% of time and mental work. The innovation consists simply in constructing the tables of the local co-ordinates u and v in function of the *round* value of the Greenwich (not local) hour angles of the occulted star, and in the determination of the co-ordinates x and y of the Moon for the moments of these hour angles. The tables of u and v are now twice as great, but the values for all places can be juxtaposed, and instead of the painstaking double interpolation hitherto needed, one relative to the declination and another to the hour angle, there remains only the first. The auxiliary tables were computed by S. Milbert.

Various Investigations

The systematic observations and reductions of the motion of the Moon are progressing regularly and will not be discussed here.

Belkovich finds (*Astr. Circ. Acad. U.S.S.R.* No. 81, December, 1948) that the Kazan heliometer measurements admit a double solution for f, one near 0·6, another near 0·7. The first solution fits the measurements of Yakovkin somewhat better and, moreover, according to the estimate of the author, it causes the disappearance of the fluctuation of the longitudes of Mösting A, having the period 2ω, remarked by Belkovich in the observations of Hartwig (1890–1915), Banachiewicz, Yakovkin, and himself. To determine the value of f near 0·6, Belkovich extends from $f=0{\cdot}55$ to $f=0{\cdot}65$ the calculation of $[vv]$, made previously in his fundamental paper (1947) from $f=0{\cdot}67$ to $f=0{\cdot}80$. In his paper of 1947, Belkovich made, *inter alia*, an extensive search for the free libration.

Banachiewicz considers (*Ac. Pol.* ser. A, 113–14, 1950), from the mathematical point of view, the method employed by Kozieł in the determination of the best approximation for f In *Acta Astr.* **c**, **4**, 144, the same author finds that Saros is the smallest period of approximate repetition of the geocentric libration.

T. Weimer gives in *C.R.* **229**, 105, 1949, the values $f=0{\cdot}57\pm0{\cdot}06$ and $I=1^\circ 31' 25''\pm 15''$, as resulting provisionally from his fresh investigation of the forty plates of Puiseux (*Ann. Obs. Paris*, **32**, 1925). He took as his basis the values of the physical libration as obtained from the apparent positions of seventy-two craters. The purpose of the author was to clear up the way. A second root for f was not especially sought for and did not appear.

Dans une autre note, *C.R.* **230**, p. 1834 (1950), M. Weimer décrit un procédé d'enregistrement de profils lunaires, d'après des clichés, à l'aide d'une chambre claire.

Prof. J. Hopmann, Hannover, beschäftigte sich mit der Neureduktion der Breslauer Messungen von Franz für 150 Fixpunkte.

'In Verbindung mit der Haynschen Profilkarte des Mondrandes hat es sich ergeben: (*a*) Der Mondrand ist—von den Bergformationen abgesehen—mit der Unsicherheit 1 : 5000 genau ein Kreis. (*b*) Die uns zugekehrte Mondseite wölbt sich über einer entsprechenden Kugel auf. Wird sie als die Hälfte eines 3-achsigen Ellipsoids aufgefasst, so liegt der Pol der grossen Achse in SW Quadranten der Mondscheibe, nahe dem Westrand von Alphonsus, sie ist 6·8 km. länger als der mittlere Radius. Die kleine Achse ist 5·0 km. kürzer als die mittlere und liegt nahe dem Nordpol des Mondes bei Pythagoras.

'Die Rittersche Höhenschichtenkarte ergibt ein ähnliches Bild wie die obigen 150 Fixpunkte, sowohl was die Längen der Achsen wie die Lage der Pole betrifft. Sie befinden sich in SW Quadranten bei Stöffler bzw. im NO bei Harpalus. Es hat den Anschein, als ob dieser Aufwölbung der uns zugekehrten Mondhälfte kein Gegenstück auf der Rückseite entspricht.'

Two Finnish expeditions for the solar eclipse of May 20, aiming to make use of the known motion of the Moon for geodetic purposes (*Trans. I.A.U* **7**, 144) report chronocinematographic observations of two central contacts on the Gold Coast (T. J. Kukkamäki) and one contact in Brazil (R. A. Hirvonen), *Proc. Finn. Acad.* 1950, pp. 103–13 and 115–25.

Photo-electronic physical registration was applied by the U.S. Army Map Service to the observation of the moments of occultations (*A.J* **55**, 177; cf. also *A.J.* **55**, 75). Be the error $\pm 0''{\cdot}0052$ (mean or probable?) in $\Delta\sigma$ as indicated for an individual observation of one phenomenon more or less unreal—it cannot be estimated for the time being with any degree of precision—it appears that the new method opens a new era of observations in this domain, because the progress in accuracy it affords is of one digit or rather more. To cancel out, as far as possible, the effect of limb irregularities, a prominent feature of the limb is selected, and the field stations are so placed that at each the star disappears behind the chosen feature. The U.S.A. observations were used to determine the distance of the Moon in kilometres, as deduced from the relative speed of the Moon. It may be remarked that the uncertainty of the distance of the summit of the mountain, supposed to be near the lunar equator, from the edge of $0^\circ{\cdot}2$ only, brings uncertainty of 16 mm. in the relative displacement of the lunar shadow during 1 second, amounting to 29 m. during half an hour. In the reduction of these observations the influence of the bending of lunar rays should be at times taken into account; this displaces the place of occultation by about 3 m. in the vertical of the Moon, in the direction perpendicular to its rays, for the zenith distance $z=45°$ The displacement increases proportionally to $\sec z . \mathrm{tg}\, z$, approximately, for z under 76°

By inverting the problem such observations may serve to establish a connection between the geodetic nets of different continents. For particulars of projected operations sponsored by the International Geodetic Association, cf. *Bull. Geod.* No. 19, March 1951, p. 88:

Le matériel d'observation comprend un télescope du type Cassegrain et un dispositif d'enregistrement à cellule photo-électrique. Ces instruments ont été étudiés et mis au point par l'Army Map Service et l'Université de Wisconsin au Washburn Observatory, Madison, Wisconsin, sous la direction du Dr Whitford.

Le miroir primaire du télescope à une ouverture de 30·5 cm. à $f/4$. Le miroir secondaire est situé à 91·5 cm. du précédent et il a une distance focale, négative de 40·6 cm. La distance focale de l'ensemble est de 487·7 cm. pour un encombrement total de 122 cm. Après réflexion par le second miroir, la lumière est renvoyée à travers une série de diaphragmes destinés à supprimer les lumières parasites. Elle parvient enfin, après réflexion, sur un prisme à réflexion totale sur la cellule photo-électrique où vient se former l'image de l'étoile.

For the time being, the observations cannot afford the positions of the centre of the Moon with an accuracy even approximately corresponding to the new precision of observed moments, the irregularities of the limb being known only with a mean error surpassing $\pm 0''\cdot 25$. To exploit the new method in this direction it should be applied first to the determination of the irregularities of the Moon from occultations. This determination, similar to that made by Przybyllok (*Mitt. Heidbg.* **11**, 1908), would be, moreover, a by-product of the observations of the motion of the Moon. If further experience justifies the bright prospects and proves the high value of the method, the observations in question must be regarded as being of the greatest importance for this branch of celestial mechanics. Their full utilization would demand, however, a great increase of the accuracy of the positions of the occulted stars.

Meanwhile the question is open, of how to utilize the numerous existing photographs of the Moon for the present purposes in order to obtain the profile of the limb from them; or, in other words, how to avoid the systematic errors of the usual method? One solution would be the employment of the heliometer measurements to obtain the basic heights of the different points of the limb, to be further adjusted by photographs. The measurements should be calculated by the direct method. It would, therefore be advisable to modernize (instead of dismantle) the heliometer, the instrument which has proved itself in these problems. For this purpose the scales should be observed photographically and their shifting could be performed by motor. This would greatly accelerate measurements. The visual observations have here the advantage over the photographic of the choice of the moments, when the images of two objects viewed together are the best.

T. Banachiewicz
President of the Commission

Supplementary report of C. B. Watts, *U.S. Naval Observatory, Washington*

The survey of the marginal zone of the Moon is now (February 1952) in the following state. About 400 plates have been measured, of which 102, exposed on 52 nights, have been selected as temporary standards. The selection was based on photographic quality and distribution with respect to the librations and the position angle of the mid-point of the bright limb. The necessary systematic corrections having been applied to the measures, preliminary datum lines were fitted to the profiles. These lines, representing circular arcs, were derived in each case from an analysis of ordinates spaced at intervals of two degrees. Differences among the standard profiles at the points that they have in common are now being analysed with the object of deriving a second datum surface. This surface, it is expected, will better represent those portions of the marginal zone that actually contribute to the outline of the Moon's limb. A third and final datum surface will result from an analysis of all the profiles. In deriving it consideration will be given to the distinction between the apparent transverse gradients (created by prominent physical features) and the real gradients that lie beneath them.

Supplementary report of H.M Nautical Almanac Office

The overhaul of the occultation machine mentioned in the previous report has been completed. The list of stations for which predictions are published has been revised and, beginning with 1954, occultation machine times will be recorded and accurate predictions provided for 70 stations; longitude and latitude coefficients enable these predictions to cover many other stations.

As predictions are provided for all known observers, the elements of occultations were little used and their publication in the *Nautical Almanac* has been discontinued from 1952. The apparent places of occulted stars are still given.

The discussion of reduced occultation observations in 1947 was published in the *Astronomical Journal*, **55**, 247, 1951, and that for 1948 is in the Press. The combined list of observations for the years 1943–47 has been prepared and is being published as an Appendix to *Greenwich Observations*, 1939; it should be issued shortly.

Both predictions and reductions are now being computed by punched card machines. Although the time taken for the computation is thereby much reduced, it is essential that all the material should be handled at once.

Report from Prof. Y Hagihara, *Chairman, Nat. Comm. of Astronomy, Science Council of Japan*

Visual observations of occultations have been carried out by professional and amateur astronomers of Japan. The results were sent to the Royal Greenwich Observatory before publication. Research on the photo-electric observation of occultations is satisfactorily in progress. The impersonal timing of an occultation within 0·01 sec. or so proved to be a fairly easy matter for a single photo-electric observation. Reappearances of stars from the Moon's dark limb could also be observed with the same accuracy. Partial results are reported by K. Osawa in the *Proc. Jap. Acad.* (in the Press). Some test observations of occultations for geodetic application have also been carried out. J. Ueta of the Kyoto University criticized use of the occultation observations for astronomical-geodetical purposes (*Publ. Astr. Soc. Japan*, **2**, 87, 1950).

Work carried out in the U.S.S.R. (according to a supplementary notice by Prof. A. A. Yakovkin)

1. The libration paper of A. A. Nefediev appeared in full in the *Publ. of the Engelhardt Observatory of the Kazan State University*, no. 26, 1951.

2. A. A. Nefediev continued his observations of Mösting A and had obtained 296 observations up to 1 January 1952.

3. In 1950–51, 232 plates of the Moon were taken at the Kiev Astronomical Observatory and 108 plates at the Engelhardt Observatory.

4. The first recommendation is to be changed as follows: Taking for granted the existence of the libration effect in the radius of the Moon and the asymmetry of its profile relative to the equator, the Committee considers as an immediate task the study of the relief of the marginal zone of the Moon and the construction of corresponding charts.

In reply to the inquiry of the undersigned Prof. Yakovkin communicated that he understood by the radius of the Moon the radius of the circle representing most exactly single observations (taken separately) of the Moon's half-limb. Such a radius is therefore to be regarded as to some extent a random variable, and it does not need to be constant.

Compte-rendu supplémentaire du Prof. T. Banachiewicz

M. José Antonio Madeira, de l'Observatoire de Lisbonne, nous écrit qu'il s'est servi des occultations, pour chaque lunaison comprise entre 1923 et 1948, ainsi que des observations méridiennes de la Lune de 1933 à 1934 et de 1944 à 1948, pour l'étude des irrégularités à courte période de la rotation de la Terre. Il a obtenu des courbes montrant une ressemblance avec les variations saisonnières de la marche des pendules. M. Madeira est d'avis que, pour le moment, les occultations sont trop entachées d'erreurs diverses—entre-autres celles qui proviennent des irrégularités du bord lunaire—pour pouvoir être concluantes sous ce rapport.

L'Observatoire de Paris a publié en 1952 l'atlas de profils lunaires de M. Th. Weimer, première partie, bord Est de la Lune. L'atlas contient 71 profils lunaires sur 6 planches dimensiones de $0{\cdot}74 \times 1{\cdot}02$ m. Comme documents de base ont été utilisés les clichés de la Lune obtenus entre 1894 et 1909 par Loewy et Puiseux au grand équatorial coudé de l'Observatoire de Paris. Malgré les deux ou trois mille clichés qui étaient à la disposition de l'auteur, il ne lui a pas été possible de donner un profil suffisamment voisin de toutes les valeurs de 2 en 2 degrés des librations en longitude aussi bien que les librations en latitude. Ces librations idéales fournissent cependant une nomenclature facilitant le choix des clichés correspondant le mieux à une valeur donnée de la libration.

La différence essentielle entre les cartes de Hayn et les profils de M. Weimer consiste en ce que les altitudes de Hayn se trouvent sur ses cartes en fonction des coordonnées sélénocentriques P et D, tandis que les hauteurs de l'Atlas de Paris se rapportent aux points donnés par leurs angles de position topocentriques sur le bord de la Lune pour la libration apparente λ, β. La distance de la Lune mise à part, les altitudes doivent être déterminées suivant l'Atlas, dans le cas général, par une double interpolation. On peut se demander s'il ne serait pas plus commode que les graphiques soient mis en Tables.

Un autre travail à signaler sur la figure de la Lune est 'Selenodätische Untersuchungen' de M. J. Hopmann, directeur de l'Observatoire de Vienne (à paraître dans les *Publ. de l'Acad. de Vienne*). M. Hopmann a fait une nouvelle réduction de 5 clichés de l'Observatoire Lick, antérieurement étudiés par Franz et il a obtenu pour chacun des 150 points de Franz 3 coordonnées sélénocentriques. En outre, il a déterminé avec M. Behrmann les diamètres et les profondeurs des cratères, de même que les hauteurs de leurs remparts. En moyenne, il a obtenu pour les diamètres 15 km., pour les profondeurs 2·23 km., pour les hauteurs des remparts 0·45 km. En supposant que le profil lunaire—abstraction faite des montagnes—soit un cercle, l'auteur en obtient pour le pôle du grand axe de l'ellipsoïde lunaire $\lambda = 0°$, $\beta = -8°{\cdot}1$, et pour le pôle du petit axe $\lambda = 0°$, $\beta = +81°{\cdot}9$, le demi-grand axe surpassant de 7·0 km. le demi-petit axe. Avec $\beta = -32°$ du grand axe, que l'auteur obtient d'après la carte des profils de Ritter, on est conduit à l'accroissement apparent observé du rayon polaire de la Lune accompagnant l'augmentation de β. Pour les autres paramètres de l'ellipsoide lunaire, ainsi que pour les conséquences relatives à la cosmogonie qu'en tire M. Hopmann, nous renvoyons le lecteur à son ouvrage.

An appendix to *Greenwich Observations*, 1939, published by the Greenwich Observatory in 1952, contains 3588 observations of lunar occultations 1943–47, together with their reduction, compiled and discussed by Flora M. McBain. The deviations of the Moon from her tabular latitude and mean longitude are shown in figures. There is no obvious periodic term in the longitude but there appears a nearly annual term in latitude with an amplitude of about $0''{\cdot}7$. The results are considered as preliminary pending the applications of corrections for limb error.

Discussion générale

La détermination du profil de la Lune paraît être maintenant le problème le plus actuel concernant la Figure de la Lune. Il nous paraît utile de formuler clairement ce problème difficile et d'indiquer les principes d'une solution possible.

Qu'est-ce que le profil de la Lune (étant données la libration et la distance de la Lune)? Au point de vue de l'astronomie dynamique ce n'est nullement la figure seule du bord lunaire. C'est l'ensemble des coordonnées des points du bord par rapport à la projection sur la sphère du centre de gravité *G* de la Lune. Pour trouver la solution, il convient de diviser la question en deux problèmes séparés plus simples:

(1) Détermination (toujours pour chaque distance et libration données) de la figure du bord par rapport à un point donné, appelons-le Mösting *A*, de la surface de la Lune.

(2) Détermination de la position de Mösting *A* par rapport au centre de gravité *G* de la Lune.

On cherche souvent le profil de la Lune par rapport à un cercle passant le mieux par les points observés, du bord. Que peut-on en obtenir? La variabilité systématique (Yakovkin) du rayon du cercle obtenu montre que l'hypothèse de la forme sphérique de la Lune correspond mal à la réalité. D'autre part, suivant l'expérience numérique (*Acta Astron.* **c**, **5**, 29–32) l'hypothèse de la circularité peut fort bien conduire à des altitudes erronées et à un centre géométrique entièrement faux. Ce qu'on devrait chercher, en se basant sur les observations, ce sont les coordonnées des points du bord par rapport aux points connus rattachés au point central de la surface lunaire. On obtiendrait, en partant d'elles, les coordonnées polaires, par rapport au point central du disque, rattaché à Mösting *A*. Les observations, directes (méridiennes) ou indirectes (p. ex. occultations) de ce point central, donneraient sa position par rapport à *G*. Les opérations inverses conduiraient au profil dynamique de la Lune.

Le programme ci-dessus concerne les mesures des points repères du bord. Les profils, qu'on détermine actuellement à l'aide de photographies, par des procédés ingénieux, fourniraient les détails du bord par rapport à la surface de référence choisie de manière à convenir le mieux aux points fondamentaux.

Pour les différents points relatifs à l'Ephéméride de la Lune et de son mouvement de translation, nous renvoyons le lecteur au 'Joint Supplementary Report of the (British) Nautical Almanac Office and the Office of the American Ephemeris', publié dans le même volume des *Transactions*.

Prof. T. BANACHIEWICZ
Président de la Commission

PAPERS OF YAKOVKIN

General Characteristic of the contour of the Moon

All lunar observations are connected with observations of its limb and therefore require a knowledge of the position of the limb centre. This is obvious in the case of the determination of the corrections to the lunar ephemerides from meridian observations and from occultations of stars. For the reduction of such observations the first approximation is considered sufficient, in which the profile of the Moon is supposed to be circular, with the visual radius depending only on the parallax, and the centre of mass considered as coinciding with the centre of the circumference. Under close examination the first admission appeared to be wrong. As was proved in the paper by Yakovkin, Belkovitch and Nefedjev, the lunar radius depends upon the libration in the latitude. The same is confirmed with some reservation by Banachiewicz and Kozieł. The incorrectness of the second assumption was established by Hansen and expresses itself in the fact that according to observations, the Moon's latitude is always less than the theoretical, the discordance not exceeding one second of arc.

For greater clearness of the ensuing reasoning it is necessary to come to an agreement of what we mean by the word 'radius' of the Moon. In measuring the photographs of the Moon (works by Hayn) and in reducing the heliometric observations we always have to do with the western or the eastern limb. The lunar radius, deduced from such observations, is the radius of the circumference which gives the best fit for the one or the other limb.

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It was proved by us that the lunar radius, as used in that sense, depends upon the libration in latitude and does not depend upon the libration in longitude. We obtained an average for both limbs:

$$R = R_0 + 0''.04\beta_0.$$

Such a result is obtained in the case when the observations are corrected for the irregularities of the limb according to the maps of Hayn in which the heights of the relief are related to the most probable circumference in the above sense. With no correction for relief a somewhat greater libration effect is obtained. One of the probable explanations offered by us in 1934 (1), supposes that there is a vast plateau in the southern Polar region of the Moon (or of a surface, surrounding an accumulation of ring-shaped mountains or rocks). The total height of such a 'Large relief' for various polar distances might be calculated from the value of the libration effect in the radius of the Moon.

The existence of the 'Large relief' explains also the constant correction of the latitude of the moon $-0''.5$. Another independent proof of the libration effect of the radius was obtained by us in studying the inclination of the lunar orbit (2), (3): the inclination determined from observations of the southern limb is $0''.51$ greater than that given by those of the northern. This method of observation of the libration effect gives a value of the same order as the heliometric measurements, viz. $+0''.04$ for each degree of libration.

In accordance with the changing thickness of the 'Large relief', the radius of the total disk of the Moon must also have the effect of libration if by the radius of the lunar disk is understood the radius of the most probable circumference, showing the best coincidence with the visual contour of the Moon.

For the idealized picture of the western and the eastern limb of the Moon a curve is supposed, having two parameters and consisting of a quarter of the circumference having a radius r_0 for the northern quadrant and of a quarter of the ellipse with semi-axes r_0 and $r_0 + a$ for the southern quadrant. r_0 does not depend upon libration, a shows a linear dependence on libration.

A more detailed analysis shows that the libration effect of the radius of the eastern limb sets in only when a large positive libration in latitude exists, and generally is from two to three times less than at the western limb.

Belkovitch found that the average radius of the eastern limb is $0''.14$ greater than that of the western; this result is obtained for all the series investigated without exception. If the radii vectores of the points of the limb measured on the limb of the Moon be reduced for the large relief the libration effect of the radius will disappear. The systematic correction to the lunar latitude will disappear as well.

The correct statement of the problem might be as follows.

To determine the barycentric lunar co-ordinates of the points of the limb, which might enter into the revised profile of the Moon, it is expedient to replace the barycentric radii vectores of the points of the physical surface of the Moon by the heights above the standard sphere whose centre coincides with the centre of the lunar mass.

Measurements of the limb of the Moon are of course insufficient for the solution of this problem—it is necessary to determine the orbital motion of the Moon. The centre of the lunar mass will be determined as the point moving according to the theory of gravity.

To be certain that the model so constructed is correct two requirements must be satisfied.

1. After the reduction of the radius vector according to the derived maps, the radius of the Moon must be constant for any libration (in the above-mentioned sense). The maps of Hayn do not satisfy that condition.

2. The latitudes of the Moon, reduced from observations of both limbs, must not show any systematic disagreements with the theory, and the errors remaining in the latitude must be of a random character. The maps of Hayn also do not satisfy that condition.

When these two conditions are fulfilled it will follow that the inclinations of the lunar orbit determined separately for the northern and the southern limbs will be identical.

The model of the Moon offered by us is one of the possible models. It depends upon the two parameters, the radius of the equatorial belt and the optical libration in latitude, and might therefore be better suited to the actual limb of the Moon. This model satisfies the two requirements in respect to the radius and latitude of the Moon. It establishes a general surface level for the record of heights. This is the sphere of constant radius limiting the northern hemisphere of the Moon. It is supposed that the centre of the mass of the Moon coincides with the centre of this sphere.

A. A. YAKOVKIN

The free Libration of the Moon

The usual method of a reduction of the heliometric observations of the Moon takes into account only the terms which represent the forced libration. The periodic terms representing the free libration are isolated from the errors of the crater longitude. The period of the free libration is calculated from the results obtained for the forced libration. The initial phases, amplitudes and periods of the free libration according to longitude, calculated in this manner, have produced, in some series of observations, contradictory results. The existence of the free libration was therefore considered doubtful (4).

The author recommends the reverse procedure: to determine first the periods, for which the initial phases of the free libration are approximately coincident for all series of observations. If such a period is known, the moments of inertia of the Moon and the terms of the forced libration can be determined.

This method was applied in the reduction of the four large Kazan series of observations, embracing the period from 1910 to 1945. The constants of the free libration of the Moon, calculated for each of these series with a period of 1085 days, show satisfactory coincidence, as is seen from the following comparison:

Years	Observer	No. of observations	Free libration	
1910–15	Banachiewicz	130	51 sin	[248° + 0·3318 $(t-t_0)$]
			±11	±15
1916–31	Yakovkin	250	46	206° + 0·3318
			±10	±12
1931–42	Belkovich	147	60	187° + 0·3318
			±13	±13
1938–45	Nefedjev	143	78	184° + 0·3318
			±16	±14

Total 670 obs. $t = 2{,}412{,}000$ Julian days

The period of free libration of 1085 days which is obviously capable of some improvement, corresponds to a value:

$$f = \frac{C-B}{C-A} \cdot \frac{B}{A} = 0{\cdot}65.$$

For free libration the following average expression might be taken:

$$\underset{\pm 11}{55''} \sin [\underset{\pm 14}{204°} + 0°{\cdot}3318(t - 2{,}412{,}000)].$$

The libration effect in lunar radius had not been taken into account in such calculations, but in longitudinal equations it is of small significance.

It is of interest to note that the period of free libration so found differs by only 10 days from the period of a term of the forced libration, the argument of which equals the doubled distance of the perigee of the lunar orbit from its ascending node. The value $f = 0{\cdot}65$ is extremely close to the critical value of the latter, equalling 0·662, for which the coefficient of the term mentioned is infinitely large. It is difficult to say whether this is an accident or the result of dynamical effects.

A. A. YAKOVKIN

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Compte rendu des séances

PRÉSIDENT: Prof. T BANACHIEWICZ.

SECRÉTAIRE: Mrs GOSSNER.

1ère séance. 10 septembre 1952

Le Président ouvre la séance en faisant les communications suivantes:

1. Environ 3600 observations d'occultations lunaires avec leur réduction ont été publiées récemment à Greenwich. Les résultats sont considérés comme provisoires vu le manque de données sur les irrégularités du bord de la Lune.

2. L'Observatoire de Paris a publié un atlas de profils lunaires de M. Weimer. Il y a une différence essentielle entre les cartes de Hayn et les profils de M. Weimer. Chez Hayn on trouve les altitudes en fonction des deux coordonnées sélénocentriques, tandis que les hauteurs de l'atlas de M. Weimer sont rapportées à la libration apparente α, β, et à l'angle de position topocentrique. Il semble que ce soit un progrès, mais l'interpolation double est un peu incommode. L'atlas de M. Weimer repose sur l'hypothèse que, si l'on fait abstraction de ses irrégularités, le bord de la lune est un cercle.

3. Dans un article publié récemment dans les *Acta Astronomica*, sér. C, vol. v, M. Banachiewicz appelle l'attention sur le fait que la méthode usuelle employée pour déterminer les profils de la lune est défectueuse. Dans cet article l'auteur donne des résultats qu'obtiendrait un astronome résidant sur la Lune en appliquant pour la détermination de la figure de la Terre la méthode qu'on applique pour déterminer la figure de la Lune. Pour le rayon de la Terre il résulterait une valeur inférieure de 3 km. à celle du ½ axe polaire, et le centre du cercle serait distant de 22 km. du centre vrai. Pour éviter les erreurs systématiques dans les altitudes on devrait employer des points de référence sur la surface de la Lune, servant entre autres à rattacher les coordonnées des points d'un bord aux points de l'autre bord. Les irrégularités du bord de la lune peuvent pareillement, comme l'ellipticité, fausser le rayon et le centre du cercle de référence.

Mr Clemence remarked that the indetermination can be removed by photographs of the N. and S. limb at full moon, as is being done at the moment by Mr Watts at the U.S. Naval Observatory.

M. Martinov indique que des astronomes de l'Observatoire d'Engelhardt ont essayé de faire des observations du bord de la Lune pendant des éclipses totales de la Lune, mais que de telles observations sont très difficiles à réaliser.

Mr Atkinson remarked that it would be useful to observe the lunar disk during annular eclipses of the Sun. Le Président dit qu'il a fait la même remarque dans son article recent dans les *Acta Astronomica*.

Ensuite le Président résume la théorie de M. Yakovkin sur la figure de la Lune et il ajoute qu'il est bien probable qu'on ne puisse pas admettre que le bord de la Lune soit un cercle ou une courbe simple; alors se pose la question: comment peut-on déterminer le profil de la Lune? Il s'agit d'un problème dynamique, et pour le résoudre il convient de le diviser en deux parties:

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1. Détermination de l'ensemble des points du bord par rapport à un certain point—par exemple, Mösting *A*—de la surface de la Lune.

2. Détermination de la position de ce point par rapport au centre de gravité *G* de la Lune.

Monsieur Clemence remarque que le problème est très difficile.

2me séance. 12 septembre 1952

Après une brève discussion entre MM. Brouwer, Atkinson, et Banachiewicz, la Commission adopte la proposition qu'il serait désirable d'observer les éclipses annulaires du Soleil.

M. Sadler lit la communication de M. Yakovkin sur la figure de la Lune.

Le Président fait un bref commentaire dans lequel il discute la différence observée pour la valeur de la libration du rayon, selon que l'on emploie la méthode de Bessel, ou la méthode Cracovienne.

M. Martynov mentionne que Yakovkin a basé ses résultats sur plusieurs centaines d'observations, ainsi que sur les études du mouvement de la Lune.

M. Sadler lit ensuite la seconde communication de M. Yakovkin, donnant les résultats de son travail sur la libration libre de la Lune.

'Mr Brouwer wonders about the change in phase which appears through the years in Yakovkin's results and suggests that maybe an improved value of the period should be adopted.' Le Président explique que les calculs ne sont qu'approximatifs et que l'ajustement devrait se faire en déterminant ensemble toutes les inconnues y compris celles de la libration physique.

'Dr Atkinson described his method of observing total or annular eclipses, for the purpose of determining the position of the Moon. If the observer is located outside the path of centrality he will see a small crescent which rotates rapidly during the phenomenon. By observing how the presence of mountains near the cusps of the crescent affects the length of the cusps and by comparing these observations with accurate data on the profile of the Moon, such as provided by Mr Watts, an accurate position of the Moon and a value of the lunar radius can be derived. Mr Atkinson presented some results obtained by this method at the eclipse of 25 February 1952.'

'Mr Watts reported on the progress of the survey of the marginal zone of the Moon now being done at Washington, and discussed the methods by which the reductions are carried out. It is intended to exhibit the results of the survey by contours in a series of libration frames corresponding to position angles spaced at intervals of one-fifth of a degree. The horizontal and vertical arguments will be the selenographic co-ordinates of the observer on the Earth. The contour interval will be one-tenth of a second of arc. From these 1800 charts it will be possible to extract the corrections required for the various measures that involve the limb of the Moon.'

'Mr Markowitz reported on the dual-rate lunar position camera. With the completion of the corrected lunar ephemerides it will be possible to use the Moon to determine ephemeris (uniform) time. The time-lapse when transits or occultations are used is far too great to meet the needs of standard frequency broadcasts. A photographic method of determining the Moon's position was therefore developed. The orbital motion of the Moon during the 10 sec. exposure is removed by means of a tilting glass plate, 1 mm. thick. By a suitable contact a record is made on a chronograph of the time when this plate is parallel to the photographic plate. The observational programme is now in progress at the U.S. Naval Observatory. The first plates reduced indicate a probable error for one night of about $\pm 0^{s}\cdot 3$ in T It appears possible in the future to determine T to better than $\pm 0^{s}\cdot 1$.'

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