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THE BANACH-SAKS THEOREM IN $C(S)$

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A Banach space X has the Banach-Saks property if every sequence (x_n) in X converging weakly to x has a subsequence (x_{nk}) with $(1/p)\sum_{k=1} x_{nk}$ converging in norm to x. Originally, Banach and Saks [**2**] proved that the spaces L^p $(p > 1)$ have this property. Kakutani [**4**] generalized their result by proving this for every uniformly convex Banach space, and in [**9**] Szlenk proved that the space L^1 also has this property.

An alternate version of the Banach-Saks property occurs in the works of other authors who replace "sequence (x_n) in X converging weakly to x" by "bounded sequence (x_n)". Using this version, Nishiura and Waterman [**5**] proved that every space with the Banach-Saks property is reflexive. Baernstein [**1**] later gave an example of a reflexive Banach space not having the Banach-Saks property.

In either version one asks that the $(C, 1)$ means of some subsequence converge in norm. The similar property with the $(C, 1)$ method replaced by an arbitrary regular summability method has been studied in [**8**] and [**10**].

Following [**7**] we denote by $S^{(\alpha)}$ the derived set of order α, where α is any ordinal number. Also, S will denote a compact metric space with metric d, $C(S)$ will be the space of all continuous complex-valued functions on S with norm

$$||f||_S = \sup_{s \in S} |f(S)|,$$

and

$$f_n \xrightarrow{w} f$$

will denote the sequence (f_n) converges weakly to f.

We will use the first version of the Banach-Saks property given above in this paper.

It is easy to show that the space $c = C(\{0, 1, 1/2, 1/3, \ldots\})$ has the Banach-Saks property. However, $C[0, 1]$ does not [**6**], so it is natural to ask for which topological spaces S does $C(S)$ have the Banach-Saks property. Our main result is contained in the following:

THEOREM. *$C(S)$ has the Banach-Saks property if and only if $S^{(\omega)} = \emptyset$.*

Proof. If $S^{(\omega)} \neq \emptyset$, Proposition 2 shows that $C(S)$ does not have the Banach-Saks property.

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Conversely, if $S^{(\omega)} = \emptyset$ we know that $\bigcap_{i=1}^{\infty} S^{(i)} = \emptyset$. Since S is compact and $S^{(i)} \supseteq S^{(i+1)}$. we conclude that there is a smallest integer i_0 for which $\bigcap_{i=1}^{i_0+1} S^{(i)} = \emptyset$. Then $S^{(i_0)}$ must be a finite set and we have

$$S^{(i_0)} \subseteq S^{(i_0-1)} \subseteq \ldots \subseteq S^{(2)} \subseteq S^{(1)} \subseteq S.$$

It is easy to see that $C(S^{(i_0)})$ has the Banach-Saks property and by using Proposition 1 i_0 times we see that $C(S)$ has the Banach-Saks property.

We must establish the propositions used in the proof of the Theorem.

PROPOSITION 1. *If $C(S^{(1)})$ has the Banach-Saks property, then so does $C(S)$.*

Proof. Suppose

$$f_n \xrightarrow{w} f \text{ in } C(S).$$

Replacing f_n with $f_n - f$ we may assume

$$f_n \xrightarrow{w} 0.$$

By [**3**, p. 265] we see that

$$f_n \xrightarrow{w} 0 \text{ implies that } |f_n| \xrightarrow{w} 0.$$

If we replace f_n by $|f_n|$ and use

$$\left\| \frac{1}{p} \sum_{k=1}^{p} f_{n_k} \right\|_S \leqq \left\| \frac{1}{p} \sum |f_{n_k}| \right\|_S,$$

we may assume that each $f_n \geqq 0$.

If $S = S^{(1)}$ there is nothing to prove so we assume $S \neq S^{(1)}$. Then there is some point $s_0 \in S \backslash S^{(1)}$ such that $d(s_0, S^{(1)}) = \delta > 0$. The set

$$F_1 = \{s \in S | d(s, S^{(1)}) \leqq \delta/2\}$$

is not empty. From the compactness of S we see that F_1 has finitely many points. Since $\lim_{n\to\infty} f_n(s) = 0$ for each $s \in S$ [**3**, p. 265], given any $\epsilon_1 > 0$ there is an integer n_1 such that $|f_{n_1}(s)| \leqq \epsilon_1$ for all $s \in F_1$. The uniform continuity of f_{n_1} on S yields a $\lambda_1 > 0$ such that $|f_{n_1}(s) - f_{n_1}(t)| \leqq \epsilon_1$ for $d(s, t) \leqq \lambda_1$. Let $\delta_1 = \min(\lambda_1, \delta/2^2)$.

We continue this process inductively. Suppose, then, that n_{k-1} and δ_{k-1} have been determined and let $F_k = \{s \in S | d(s, S^{(1)}) \geqq \delta_{k-1}\}$. As before we see that F_k is a finite set so, given any $\epsilon_k > 0$, we can find an integer $n_k > n_{k-1}$ such that $|f_{n_k}(s)| \leqq \epsilon_k$ for $s \in F_k$. The uniform continuity of f_{n_k} on S yields a $\lambda_k > 0$ such that $|f_{n_k}(s) - f_{n_k}(t)| \leqq \epsilon_k$ for $d(s, t) \leqq \lambda_k$. Let $\delta_k = \min(\lambda_k, \delta/2^{k+1})$.

Let $u \in S \backslash S^{(1)}$ and let p be any positive integer. Since $\lim_{k\to\infty} \delta_k = 0$ there exists a smallest integer k for which $d(u, S^{(1)}) \geqq \delta_k$. Denote this integer by k_0. Then, for $k \geqq k_0$, $d(u, S^{(1)}) \geqq \delta_k$ so $u \in F_k$ and hence $|f_{n_k}(u)| \leqq \epsilon_k$. By the compactness of $\{u\}$ and $S^{(1)}$ we can find a point $v \in S^{(1)}$ such that $d(u, v) =$

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$d(u, S^{(1)})$. Thus $d(u, v) \leqq \delta_k$ for $k = 1, 2, \ldots, k_0 - 1$. We now proceed to calculate a bound for $||(1/p)\sum_{k=1}^{p} f_{n_k}||_S$.

First, suppose $p \leqq k_0 - 1$. Then,

$$\left|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}(u) - \frac{1}{p}\sum_{k=1}^{p} f_{n_k}(v)\right| \leqq \frac{1}{p}\sum_{k=1}^{p} |f_{n_k}(u) - f_{n_k}(v)| \leqq \frac{1}{p}\sum_{k=1}^{p} \epsilon_k$$

and thus

$$\left|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}(u)\right| \leqq \frac{1}{p}\sum_{k=1}^{p} \epsilon_k + \left|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}(v)\right| \leqq \frac{1}{p}\sum_{k=1}^{p} \epsilon_k + \left\|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}\right\|_{S^{(1)}}.$$

Next, suppose $p \geqq k_0$. Then,

$$\left|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}(u) - \frac{1}{p}\sum_{k=1}^{p} f_{n_k}(v)\right| \leqq \frac{1}{p}\sum_{k=1}^{k_0-1} |f_{n_k}(u) - f_{n_k}(v)| + \frac{1}{p}\sum_{k=k_0}^{p} |f_{n_k}(u)|$$

$$+ \frac{1}{p}\sum_{k=k_0}^{p} |f_{n_k}(v)| \leqq \frac{1}{p}\sum_{k=1}^{k_0-1} \epsilon_k + \frac{1}{p}\sum_{k=k_0}^{p} \epsilon_k + \frac{1}{p}\sum_{k=k_0}^{p} f_{n_k}(v) \leqq \frac{1}{p}\sum_{k=1}^{p} \epsilon_k + \frac{1}{p}\sum_{k=1}^{p} f_{n_k}(v).$$

Hence

$$\left|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}(u)\right| \leqq \frac{1}{p}\sum_{k=1}^{p} \epsilon_k + 2\left|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}(v)\right| \leqq \frac{1}{p}\sum_{k=1}^{p} \epsilon_k + 2\left|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}\right|_{S^{(1)}}.$$

We have

$$\left|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}(u)\right| \leqq \frac{1}{p}\sum_{k=1}^{p} \epsilon_k + 2\left\|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}\right\|_{S^{(1)}}$$

for all $u \in S \backslash S^{(1)}$ and, obviously, for all $u \in S^{(1)}$ so we conclude that

$$(1) \qquad \left\|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}\right\|_S \leqq \frac{1}{p}\sum_{k=1}^{p} \epsilon_k + 2\left\|\frac{1}{p}\sum_{k=1}^{p} f_{n_k}\right\|_{S^{(1)}}.$$

If the restriction of f_{nk} to $S^{(1)}$ is denoted by g_{n_k} then

$$g_{n_k} \xrightarrow{w} 0 \text{ in } C(S^{(1)}).$$

By hypothesis there exists a subsequence $(g_{n_{k(i)}})$ such that

$$\left\|\frac{1}{p}\sum_{i=1}^{p} g_{n_{k(i)}}\right\|_{S^{(1)}} \to 0.$$

Arguing as above, we find

$$(2) \qquad \left\|\frac{1}{p}\sum_{k=1}^{p} f_{n_{k(i)}}\right\|_S \leqq \frac{1}{p}\sum_{i=1}^{p} \epsilon_{k_i} + 2\left\|\frac{1}{p}\sum_{i=1}^{p} f_{n_{k(i)}}\right\|_{S^{(1)}}.$$

It suffices to choose ϵ_k so that $\lim_{k\to\infty}\epsilon_k = 0$ and to notice that

$$||(1/p)\textstyle\sum_{i=1}^{p} f_{n_{k(i)}}||_{S^{(1)}} \to 0$$

in order to conclude $||(1/p)\sum_{i=1}^{p} f_{n_{k(i)}}||_S \to 0$. This establishes the proposition.

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The next proposition is essentially a generalization of the result of Schreier [**6**] and repeatedly uses some simple topological facts which we provide in the following lemmas.

LEMMA 1. *Let* (s_n), $(n = 1, 2, \ldots)$, *be a sequence of distinct points of* S *which converge to* $s_\infty \in S$. *Let* O *be any open set containing each* s_n *and* s_∞. *Let* ϵ_k *be a sequence of positive numbers such that* $\epsilon_k \to 0$. *Then, there is a subsequence* (s_{n_k}), $(k = 1, 2, \ldots)$, *of* (s_n) *and a sequence of open discs* O_n *(centered at* s_{n_k}*) satisfying the following conditions:*

(1) $n_k \neq n_j \Rightarrow O_{n_k} \cap O_{n_j} = \emptyset$;
(2) $O_{n_k} \subseteq O$ *for all* k;
(3) *the radius* r_{n_k} *of* O_{n_k} *satisfies* $r_{n_k} \leqq \epsilon_k$;
(4) $s_\infty \notin O_{n_k}$ *for all* k.

The proof is elementary.

LEMMA 2. *Let* (E_n), $(n = 1, 2, \ldots)$, *be a sequence of closed subsets of* S *such that:*

(1) *each* $E_n \subseteq O_n$, *where* O_n *is an open disc of radius* $r_n > 0$;
(2) $r_n \to 0$;
(3) *the discs* O_n *are mutually disjoint;*
(4) *the centers* x_n *of the discs* O_n *converge to a point* x.

Then $F = \bigcup_{n=1}^{\infty} E_n \cup \{x\}$ *is a closed set in* S.

Proof. Let $y \in S \backslash F$. Then there is a disc $D_\delta(y)$ and a disc $D_\lambda(x)$ such that $D_\delta(y) \cap D_\lambda(x) = \emptyset$. Take n_0 such that $r_n \leqq \lambda/4$ for $n \geqq n_0$ and take n_1 such that $d(x, x_n) \leqq \lambda/4$ for $n \geqq n_1$. Letting $n_2 = \max(n_0, n_1)$ we have $O_n \subseteq D_\lambda(x)$ for all $n \geqq n_2$. Then $\bigcup_{n=1}^{n_2-1} E_n$ is a closed set not containing y so by normality we can find mutually disjoint open sets U, V containing $\bigcup_{n=1}^{n_2-1} E_n$ and y, respectively. Let $W = D_\delta(y) \cap V$. Then W is an open set containing y which does not meet F. Since y was an arbitrary point of $S \backslash F$, we conclude that $S \backslash F$ is open and hence F is closed.

PROPOSITION 2. *If* $S^{(\omega)} \neq \emptyset$, *then* $C(S)$ *does not have the Banach-Saks property.*

Proof. Here, ω represents the first countable ordinal number so the condition $S^{(\omega)} \neq \emptyset$ is equivalent to $\bigcap_{i=1}^{\infty} S^{(i)} \neq 0$. This forces each $S^{(i)}$ to be infinite, so we can find a sequence of distinct points (s_i), $(i = 1, 2, \ldots)$, with s_i in $S^{(i)}$ for each i.

By compactness of S, the sequence (s_i) has a limit point s_∞. Passing to a subsequence, if necessary, we may assume $s_i \to s_\infty$. Notice that passing to a subsequence does not alter the fact that s_i is in $S^{(i)}$, because $S^{(i)} \supseteq S^{(i+1)}$ for all i.

Let (ϵ_k) be any sequence of positive numbers converging to 0. Using Lemma 1 with $O = S$ we obtain a subsequence of (s_n) which we denote by $s(3, \infty)$, $s(4, \infty, \infty)$, $s(5, \infty, \infty, \infty) \ldots$ and discs $D(3, \infty)$, $D(4, \infty, \infty) \ldots$ centered at these points, which satisfy the four conditions of Lemma 1.

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The point $s(3, \infty)$ is in $S^{(1)}$ so there is some sequence of points of S converging to $s(3, \infty)$. We may assume this sequence lies within $D(3, \infty)$. Using Lemma 1 with $O = D(3, \infty)$ we obtain a subsequence, which we denote by $s(3, 4)$, $s(3, 5)$, $s(3, 6)$ ... and discs $D(3, 4)$, $D(3, 5)$ ... centered at these points satisfying the four conditions of Lemma 1.

Similarly, $s(4, \infty, \infty)$ is in $S^{(2)}$ so there is some sequence of points of $S^{(1)}$ which lie within $D(4, \infty, \infty)$ and converge to $s(4, \infty, \infty)$. Applying Lemma 1 with $O = D(4, \infty, \infty)$ we obtain the points $s(4, 5, \infty)$, $s(4, 6, \infty)$, $s(4, 7, \infty)$ ... and discs $D(4, 5, \infty)$, $D(4, 6, \infty)$ ... satisfying the conditions of Lemma 1. Now, each $s(4, k, \infty)$ is in $S^{(1)}$ for $k = 5, 6, 7, \ldots$ so in each $D(4, k, \infty)$ we may repeat the argument of the preceeding paragraph, thus obtaining discs $D(4, k, k+1)$, $D(4, k, k+2)$ ... centered at points $s(4, k, k+1)$, $s(4, k, k+2)$ ... satisfying the four conditions of Lemma 1.

In general, $s(n, \infty, \infty, \ldots, \infty)$ is in $S^{(n-2)}$ so, using Lemma 1 with $O = D(n, \infty, \infty, \ldots, \infty)$ we can find discs $D(n, n+j, \infty, \infty, \ldots, \infty)$, $(j = 1, 2, \ldots)$, centered at points $s(n, n+j, \infty, \ldots, \infty)$, $(j = 1, 2, \ldots)$, such that each $s(n, n+j, \infty, \ldots, \infty)$ belongs to $S^{(n-3)}$, $s(n, n+j, \infty, \ldots, \infty)$ converges to $s(n, \infty, \ldots, \infty)$ as $j \to \infty$, and such that these discs and points satisfy the conditions of Lemma 1. Note that each parenthesis $(n, n+j, \infty, \infty, \ldots \infty,)$ has $n-1$ entries. We can apply Lemma 1 to each disc $D(n, n+j, \infty, \ldots, \infty)$ to obtain subdiscs $D(n, n+j, n+j+k, \infty, \infty, \ldots, \infty)$, $(k = 1, 2, \ldots)$, centered at points $s(n, n+j, n+j+k, \infty, \ldots, \infty)$ in $S^{(n-4)}$ such that these points converge to $s(n, n+j, \infty, \ldots, \infty)$. In other words, given a disc whose center lies in some $S^{(i)}$ we use Lemma 1 to construct a sequence of subdiscs whose centers are in $S^{(i-1)}$. Since we initially have a point in $S^{(n-2)}$, we can only repeat this process $n-2$ times. The method of indexing the discs indicates the inclusion relations between the discs in the sense that in the disc $D(n, \infty, \infty, \infty, \ldots, \infty)$

(1) $D(n, m_1, m_2, \ldots, m_k, \infty, \infty, \ldots, \infty)$ contains all thus constructed discs whose associated parentheses begin with $n, m_1, m_2, \ldots, m_k$, and only these discs. We summarize the other properties of these discs as follows:

(2) each parenthesis has length $n-1$;

(3) the entries in each parenthesis form a strictly increasing sequence;

(4) if $n, m_1, m_2, \ldots, m_k$ are fixed, then the discs $D(n, m_1, \ldots, m_k, m_k+j, \infty, \ldots, \infty)$, $(j = 1, 2, \ldots)$, with centers $s(n, m_1, \ldots, m_k, m_k+j, \infty, \ldots, \infty)$ satisfy the four conditions of Lemma 1.

Let $D(n, m_1, m_2, \ldots, m_{n-2})$ represent the general subdisc constructed as above and then let $K(n, m_1, m_2, \ldots, m_{n-2})$ be the set of centers of all thus constructed subdiscs, including $D(n, m_1, m_2, \ldots, m_{n-2})$, contained in $D(n, m_1, \ldots, m_{n-2})$. We now prove that each $K(n, m_1, m_2, \ldots, m_{n-2})$ is closed in S. To this end, suppose all the entries in $(n, m_1, m_2, \ldots, m_{n-2})$ are finite. Then $K(n, m_1, \ldots, m_{n-2})$ is just the single point $s(n, m_1, \ldots, m_{n-2})$ and is obviously closed. Now suppose some of the entries $(n, m_1, m_2, \ldots, m_{n-2})$ are ∞ and let m_k denote the last finite entry. To show $K(n, m_1, \ldots, m_k, \infty, \ldots, \infty)$ is

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closed, it suffices by (4) and Lemma 2, to show each $K(n, m_1, \ldots, m_k, m_k + j, \infty, \ldots, \infty)$ is closed. By (4) and Lemma 2 we will have each $K(n, m_1, \ldots, m_k, m_k + j, \infty, \ldots, \infty)$ closed if we know that each $K(n, m_1, \ldots, m_k, m_k + j, m_k + j + i, \infty, \ldots, \infty)$ is closed. After a finite number of such reductions we find that it is sufficient to prove that each $K(n, m_1, \ldots, m_{n-2})$ is closed if each entry in $(n, m_1, \ldots, m_{n-2})$ is finite. But this has already been established.

Now, let Z_i be the set of centers of all discs constructed as above in whose associated parentheses the integer i occurs. Notice that

$$Z_i \subseteq \bigcup_{k=3}^{i} D(k, \infty, \ldots, \infty).$$

For $3 \leqq k < i$ let L_p^k denote the finite union of all possible $K(k, m_1, m_2, \ldots, m_{p-2}, i, \infty, \infty, \ldots, \infty)$ where i occurs in the pth position. Let O_p^k be the corresponding union of all $D(k, m_1, m_2, \ldots, m_{p-2}, i, \infty, \ldots, \infty)$. The preceeding paragraph allows us to conclude that L_p^k is closed for $3 \leqq k < i$, so, since $Z_i \cap D(k, \infty, \ldots, \infty) = \bigcup_{p=2}^{k-1} L_p^k$ for $3 \leqq k < i$ and $Z_i \cap D(i, \infty, \ldots, \infty) = K(i, \infty, \ldots, \infty)$ we see that $Z_i \cap D(k, \infty, \ldots, \infty)$ is closed for $k = 3, 4, \ldots, i$ and hence $Z_i = \bigcup_{k=3}^{i} \{Z_i \cap D(k, \infty, \ldots, \infty)\}$ is closed in S for every $i \geqq 3$.

Let

$$U_i = D(i, \infty, \ldots, \infty) \cup \left\{ \bigcup_{k=3}^{i-1} \bigcup_{p=2}^{k-1} O_p^k \right\}.$$

Then U_i is an open set containing Z_i for $i \geqq 3$. Therefore, U_i^c is closed and $U_i^c \cap Z_i = \emptyset$. By Urysohn's lemma, we can find f_i in $C(S)$ such that $f_i(Z_i) = 1$, $f_i(U_i^c) = 0$, and $0 \leqq f_i \leqq 1$ for every $i \geqq 3$. We now proceed to show that

$$f_i \xrightarrow{w} 0 \text{ in } C(S).$$

Since $\|f_i\| \leqq 1$ for $i \geqq 3$, we need only show [**3**, p. 265] that $f_i(s)$ converges to 0 for every s in S. If a point s lies within no $D(i, \infty, \infty, \ldots, \infty)$ then, in particular, s is in U_i^c for $i \geqq 3$ and thus $f_i(s) = 0$ for $i \geqq 3$. So we assume s is a point lying in some disc $D(i_0, \infty, \infty, \ldots, \infty)$. Let $D(i_0, m_1, m_2, \ldots, m_{i_0-2})$ be the smallest subdisc of $D(i_0, \infty, \ldots, \infty)$ in which s lies and let m_k be the last finite entry in $(i_0, m_1, m_2, \ldots, m_{i_0-2})$. Then for every $i > m_k$ we see, by property (1), that amongst all subdiscs of $D(i_0, \infty, \ldots, \infty)$ whose associated parentheses contain i, none contain $D(i_0, m_1, m_2, \ldots, m_{i_0-2})$. Also, by the way in which $D(i_0, m_1, m_2, \ldots, m_{i_0-2})$ was chosen, s does not lie in any subdisc of it whose associated parenthesis contains an i. Thus, for $i > m_k$, s is in U_i^c and hence $f_i(s) \to 0$.

The sequence (f_i), $(i = 3, 4, \ldots)$, has been shown to converge weakly to 0. We now show that there is no subsequence (f_{n_k}) with $\|(1/p)\sum_1^p f_{n_k}\|$ converging to 0.

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Take $n_1, n_2, n_3, \ldots$ to be any strictly increasing sequence of natural numbers and let k be any natural number. Since $n_{k+1} \geqq k + 1$ all k numbers $n_{k+1}, n_{k+2}, \ldots, n_{2k}$ occur simultaneously in at least one parenthesis $(m_1, m_2, \ldots, m_{m_1-1})$ constructed above. Let $s_0 = s(m_1, m_2, \ldots, m_{m_1-1})$. Then s_0 is in Z_{n_j} for $k + 1 \leqq j \leqq 2k$ and hence $f_{n_j}(s_0) = 1$ for $k + 1 \leqq j \leqq 2k$. Thus

$$\left\|\frac{1}{2k} \sum_{j=1}^{2k} f_{n_j}\right\| \geqq \left|\frac{1}{2k} \sum_{j=1}^{2k} f_{n_j}(s_0)\right| \geqq \frac{1}{2k} \sum_{j=k+1}^{2k} f_{n_j}(s_0) = \frac{1}{2}$$

so

$$\overline{\lim_{p\to\infty}} \left\|\frac{1}{p} \sum_{j=1}^{p} f_{n_j}\right\| \geqq \frac{1}{2}.$$

This concludes the proof.

It has been brought to the author's attention by a personal communication that, using less direct methods than those employed here, the main result of this paper remains true when S is a non-metrizable compact Hausdorff space.

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