NUMERICAL SIMULATION OF MASS TRANSFER AND ACCRETION DISC FLOW IN BINARY SYSTEMS

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Abstract. A numerical simulation of equilibrium gas flow in a binary system is presented for the case in which an accretion disc forms around the accreting star (primary). Preliminary results are shown for the mass ratios 0.4, 1.0 and 2.5. Two main conclusions are reached. First, the accretion disc is quite well defined and is comparable in size to the Roche lobe of the primary. Second, all but a few per cent of the mass transferred is accreted by the primary.

1. Introduction

Accretion discs have been receiving considerable attention from astronomers in recent years. In this paper we consider the accretion disc that arises when Roche-lobe mass transfer takes place in a semi-detached binary system in which the accreting star is too small for the stream of transferred material to strike it directly. Such discs are of particular relevance to binary X-ray sources (Rees, 1976) and to dwarf novae (Warner, 1976).

Close to the accreting star (henceforth the primary), where most of the accretion energy is liberated, the gas flow can be treated as a standard, relatively well understood accretion disc in which the main flow is circular and Keplerian. For accretion to take place at all there must be some form of viscosity which acts to transfer angular momentum (relative to the primary) outwards and to allow material in the disc to spiral slowly inwards, releasing energy as it does so.

When the binary orbit is circular and the mass-transferring (secondary) star is rotating synchronously with the orbit and is sufficiently centrally condensed (that is, when the Roche approximation may be applied) the flow near the inner Lagrangian point (L_1) is also relatively well understood. Since the flow velocity at L_1 is comparable to the sound speed in the atmosphere of the secondary and since this velocity is in general much smaller than the orbital velocities, it has long been assumed (Kuiper, 1941; Kopal, 1956; Gould, 1957, 1959; Kruszewski, 1964) that the initial trajectory of the stream is the same as that delineated by a test particle falling from the L_1 point in the restricted three-body problem. Additional support for this assumption has been provided by Lubow and Shu (1975).

The understanding of the interaction between the accretion disc around the primary and the mass stream from the secondary is of crucial importance to a number of problems. Together with a better understanding of the radiation processes involved it is central to the interpretation of the optical light curves of dwarf novae and of the optical and X-ray light curves of some binary X-ray sources. Perhaps most important of all it can tell us how much of the mass transferred through the L_1 point actually ends up on the primary and how much mass and/or angular momentum (if any) is lost from the system as a whole. A definitive answer to this problem would greatly simplify calculations of the evolution of binary stars.

Suppose for the moment that no viscous processes are occurring in the gas. In this case the general aspects of the gas flow are relatively easy to understand. The region

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where the disc/stream interaction takes place is sufficiently close to the primary that angular momentum about the primary is approximately conserved during the interaction. Thus the dissipation of energy that occurs during the interaction allows the material in the stream to form a ring about the primary and the radius of the ring is roughly determined by the condition that the angular momentum with respect to the primary for particles flowing round the ring is equal to that for particles in the incoming stream. Calculations based on these assumptions have been made by Warner and Peters (1972), Flannery (1974) and Lubow and Shu (1975). Note, however, that the assumption that no viscous processes are acting means that no accretion can take place onto the primary and that the transferred material continues to build up in a ring around it.

The effect of viscosity on a ring of material circling a central gravitating object has been calculated by Lynden-Bell and Pringle (1974). Although the details of their results depend on the assumptions made about the nature of the viscosity, the general aspects of the flow do not. Viscosity acts to spread the ring out. The inner radius of the ring decreases, taking most of the mass with it and so releasing gravitational potential energy. The outer radius increases, in order to absorb the angular momentum which is lost by mass at the inner radius and which is transferred outwards by the viscous mechanism. If the initial radius of the ring is R_h and if the material (and angular momentum) flowing outwards is absorbed at a radius R_L then the fraction of mass that is absorbed at R_L in order to allow the rest to fall from R_h to R_1, the radius of the primary (assumed $\ll R_h$), is approximately $(R_h/R_L)^{\frac{1}{2}}$. For this reason Prendergast and Burbidge (1968) predicted that about half the mass transferred in such a binary system would not be accreted by the primary.

Thus we see that if viscous processes are important in the gas flow (or, in other words, if accretion takes place) the outer edge of the accretion disc must be larger than that calculated under the assumption of zero viscosity. In fact, if the flow settles down to a steady state we do not necessarily expect all the transferred mass to be accreted and may expect some of it to be lost from the Roche lobe taking with it the excess 'angular momentum about the primary'. Thus we may expect the outer edge of the disc to be comparable to the size of the Roche lobe.

Because of the enormity of the problem a full-scale hydrodynamical calculation of the equilibrium flow is not really feasible at this stage. Prendergast and Taam (1974) have considered the simpler situation in which the primary is large enough that a disc does not form. Flannery (1975) has computed the flow resulting from a constant mass transfer rate for only one orbital period. He, however, excludes consideration of viscous processes and therefore finds that the transferred material forms a ring around the primary. Sorensen *et al.* (1975) have considered the accretion flow resulting from a stellar wind driven from the surface of the secondary. Their results are complementary to those presented here.

We have adopted a simpler approach. In the gas flow we wish to simulate, pressure is relatively unimportant since radiation timescales are for the most part much shorter than dynamical ones. On the other hand some kind of viscous or dissipative process must be present in order to allow accretion to proceed. We suspect that once an equilibrium flow has been established, the overall properties of the flow may be relatively independent of the properties of the dissipative process. Rather than attempting to use a computer to provide an approximate solution of the fluid equations (which are, after all, just analytic

expressions of conservation laws) we have adopted a many-body approach to the problem. We approximate the fluid as being composed of a few thousand individual particles. For most of the time these particles move under Newton's laws as isolated test particles in the potential of the binary system (restricted three-body problem). Every so often each particle is forced to interact instantaneously with its neighbours in a viscous manner. In Section 2 we discuss the details of the viscous interaction and our numerical technique. We present the results of the computations in Section 3 and comment on them in Section 4.

2. Methods

2.1. PARTICLE DYNAMICS; THE RESTRICTED THREE-BODY PROBLEM

The relevant equations have been discussed by several authors and we recapitulate briefly to establish our notation. We choose units so that the binary separation, the total mass of the system and the orbital angular velocity are unity. We work in a frame rotating with the binary system. We take the centre of mass of the primary (mass m_1) to be at the origin $(0,0,0)$ and the centre of mass of the secondary (mass m_2) to be at $(-1,0,0)$. We take the x-y plane to be the orbital plane and for convenience ignore any motion in the z-direction. The equations of motion for a test particle are then

$$\frac{d^2x}{dt^2} = 2\frac{dy}{dt} + x + 1 - \mu + \frac{(1+x)(\mu-1)}{r_2^3} - \frac{\mu x}{r_1^3} \tag{2.1}$$

$$\frac{d^2y}{dt^2} = -2\frac{dx}{dt} + y\left\{1 + \frac{\mu-1}{r_2^3} - \frac{\mu}{r_1^3}\right\} \tag{2.2}$$

where $\mu = q/(1+q) = m_1/(m_1+m_2)$, $r_2^2 = (x+1)^2 + y^2$ and $r_1^2 = x^2 + y^2$.

The conserved quantity along the orbit of an isolated particle is the Jacobi integral

$$C = 2U - (\dot{x}^2 + \dot{y}^2)$$

where

$$U = \frac{1}{2}\left[\mu r_1^2 + (1-\mu) r_2^2\right] + \frac{\mu}{r_1} + \frac{1-\mu}{r_2}. \tag{2.3}$$

2.2. THE VISCOUS INTERACTION

Initially the particles are allowed to follow the orbits prescribed by the Equations (2.1) and (2.2). After a small time interval Δt the particles have moved a finite distance without any interaction between themselves. At this instant any particle with a sufficiently close neighbour (or neighbours) is allowed to interact instantaneously with it (or them). Suppose that at one instant there are n particles which we are allowing to interact, with positions r_i and velocities v_i $(i = 1, n)$ (with respect to an inertial frame) before the interaction. Since the interaction is instantaneous, during it we must conserve (Newton's third law) the particle's total momentum and total angular momentum with respect to an inertial frame. The action of viscosity is to reduce *locally* the shear and (bulk viscosity) the dilatation of the fluid. Therefore the simplest form of viscous interaction to consider is to assume that after the interaction the particles end up rotating rigidly

about their local centre of mass and remain in their original positions. The position of their centre of mass is given by

$$r_c = \frac{1}{n}\sum_{i=1}^{n} r_i. \tag{2.4}$$

The velocity of the centre of mass is given by

$$v_c = \frac{1}{n}\sum_{i=1}^{n} v_i. \tag{2.5}$$

We define $R_i = r_i - r_c$ and $V_i = v_i - v_c$.
After the interaction the velocities of the particles are given by

$$u_i = v_c + R_i \wedge \Omega$$

where

$$\Omega = -\frac{\sum_{i=1}^{n} R_i \wedge V_i}{\sum_{i=1}^{n} R_i \cdot R_i}. \tag{2.6}$$

More generally, we may assume that shear and bulk viscosities are not efficient enough to achieve local rigid body rotation within an interacting region completely in a time Δt. For example if a fraction α of the *local* shear and of the *local* dilatation is dissipated in the interaction, the velocities of the particles after the interaction may be written

$$u_i = v_c - \alpha R_i \wedge \frac{\sum_{j=1}^{n} R_j \wedge V_j}{\sum_{j=1}^{n} R_j \cdot R_j} + (1-\alpha) V_i . \tag{2.7}$$

Of course, for the particles taking part in a given interaction, their state of lowest energy (given momentum and angular momentum) is rigid body rotation. Thus the process we describe is of necessity a dissipative one. In this sense the interactive process used in our numerical simulation of the fluid flow contains the necessary properties of a dissipative process.

We assume that the energy liberated is radiated away instantaneously and we record the amount and position liberated during each interaction. This should be a reasonable approximation since the radiation timescale for gas in the disc is short compared to the flow timescale.

2.3. NUMERICAL TECHNIQUE

To follow the particle orbits we use fourth and seventh order Runge-Kutta integration subroutines. In the region near the compact object primary ($r_1 < r_{cr}$, where r_{cr} is a prescribed parameter taken to be 0.05) we use the seventh order routine since the accelera-

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tions due to the gravitational field is larger there. We use the fourth order routine elsewhere.

After each timestep the position of each particle is checked to determine whether it has other particles close enough to it to interact with. This position checking process can be time-consuming and for maximum efficiency we use a mixture of two methods.

(i) Close to the primary within an area A (see Figure 1) we divide the space up into $k \times m$ mesh boxes each of size $l \times l$ (typically k and m lie in the range 50–80). If a particle is within A at the end of a timestep it is allocated to a mesh box. All the particles in each mesh box are then allowed to interact in the manner described in Section 2.2.

(ii) Outside A, where the particle density is smaller each pair of particles is checked to see if they lie within a distance l of each other. If they do, they are allowed to interact in the same manner setting $n = 2$ in the equations of Section 2.2. We find that our results do not depend on the size of A.

In addition, at each timestep those particles that have returned to the secondary, those that have been accreted by the primary and those that have been lost from the system

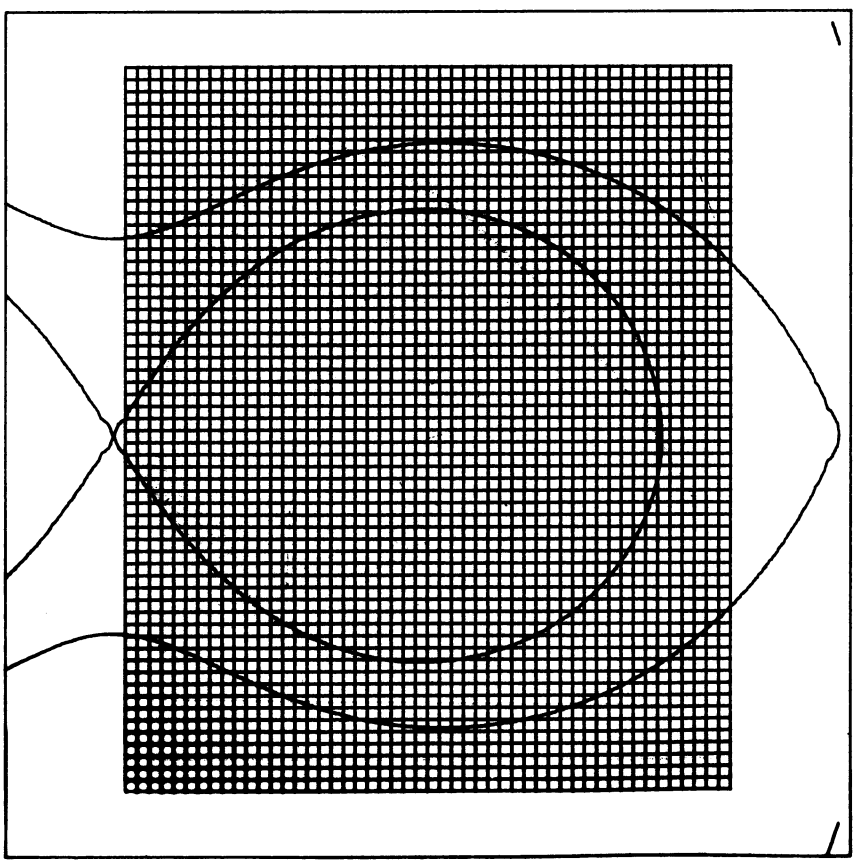

Fig. 1. The Roche lobe with $q = 1.0$ showing the area A defined in Section 2.3. Also shown is the potential which passes through the outer Lagrangian points.

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are removed and recorded. A particle is deemed to have been lost from the system if it has a velocity greater than twice the escape velocity from the system and is at a distance of greater than 10 binary separations from the primary.

3. Results

We start our computations at the onset of mass transfer and continue until the flow reaches some kind of equilibrium – in particular until the number of particles within two binary radii of the primary tends to a constant value (Figure 2). We then allow the flow to proceed for a few more binary orbits and take a time average of the results.

In all the computations presented here we have used a timestep $\Delta t = 0.01$, a mass transfer rate of one particle per timestep, and the viscous interaction (2.7) in the form with $\alpha = 1$. We find that particles conserve the Jacobi integral along their orbit to within a few parts in 10^6 per timestep Δt. This is similar to the rounding error incurred during each interaction. Thus the cumulative error incurred during five orbits of equilibrium

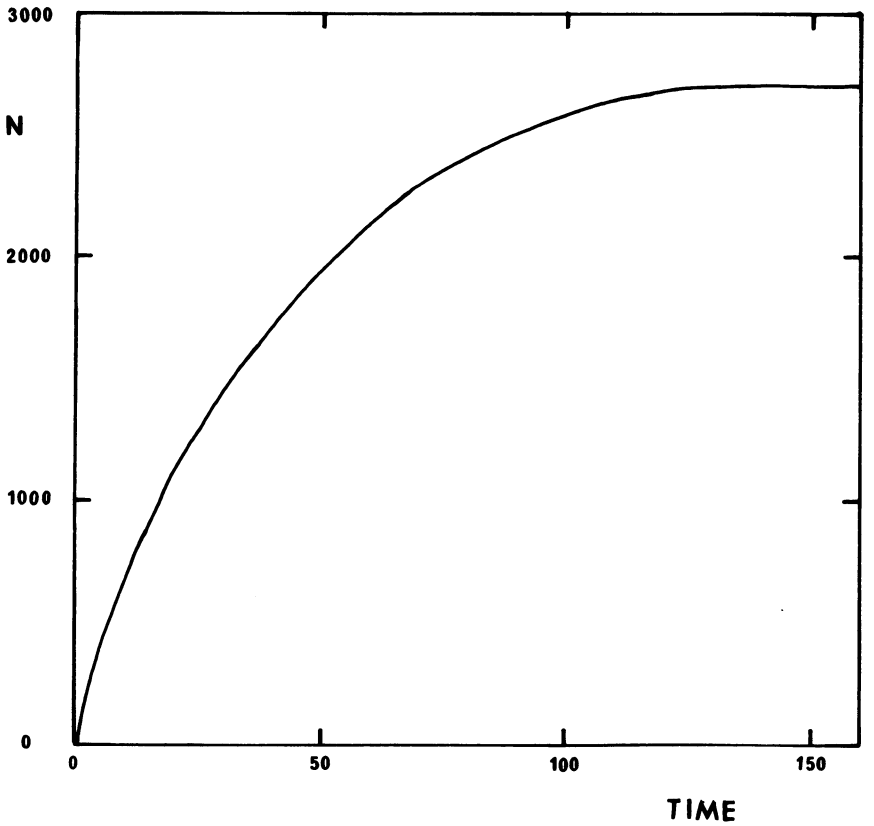

Fig. 2. A graph of the number N of particles within two binary radii of the centre of mass of the primary as a function of dimensionless time for $q = 1.0$. Equilibrium flow is established after a time of 130 which corresponds to 20 orbital periods.

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flow is less than a few parts in 10^4. We take the interaction length $l = 0.02$ and the radius of the primary to be $R_1 = 0.01$.

The results are presented in Figures 3–5 and some of the details are summarized in Table I. In all cases the orbital motion is anti-clockwise. The equipotential surfaces corresponding to the L_1 and L_2 points are shown. The density contours are shown in the figures marked (a). Since the density of the particles in the disc depends on the space-dependence of the viscosity, the levels of the individual contours probably reflect directly our assumptions about the viscous process and as such are not necessarily very meaningful. We do find, however, that the outer edge of the disc is quite well defined and in each case the density drops quite sharply outside the outermost contour shown. Near the primary the contours are circular and the density decreases, as expected from standard accretion disc theory (Shakura and Sunyaev, 1973).

The velocity fields are shown in the figures marked (b). The flow in the disc is anti-clockwise about the primary. The length of the line shown corresponds to the magnitude

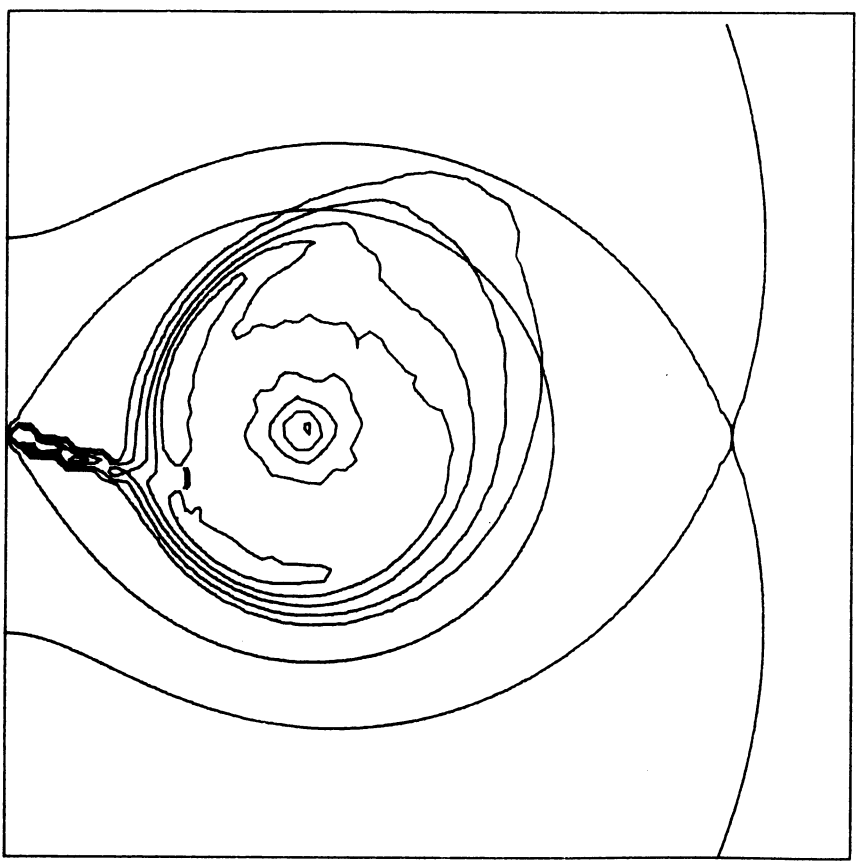

Fig. 3a. $q = 1.0$, $l = 0.02$. Density contours at relative levels 1, 2.5, 5, 7.5 and 10. The density is low at the outside and at the centre of the disc and rises to a 'volcanic' rim in between. This variation is just a result of the assumptions made about viscosity.

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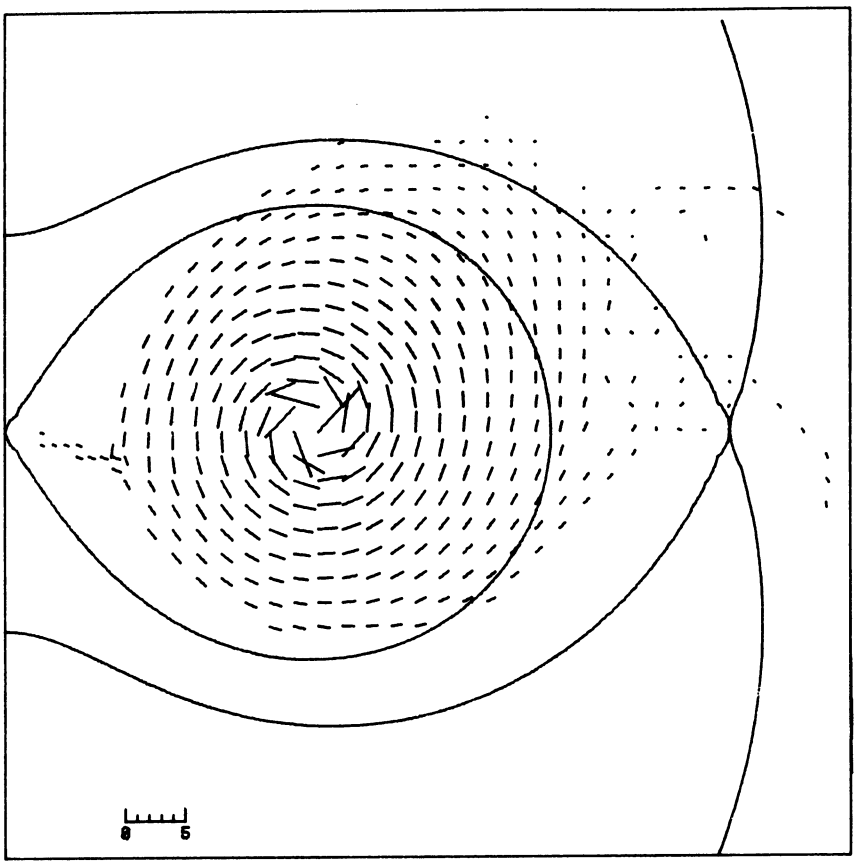

Fig. 3b. $q = 1.0$, $l = 0.02$. The velocity field. Flow in the disc is anti-clockwise and the initial points of the vectors lie on a square mesh at the points corresponding to the velocity indicated by the length of the vector. The scale is shown.

TABLE I

Model parameters

$q = M_1/M_2$	n_0	N	f	R_S	R_h
0.4	20	1500	0.06 ± 0.01	0.19	0.07
1.0	20	2700	0.03 ± 0.01	0.27	0.09
2.5	40	5000	0.03 ± 0.01	0.40	0.13

of the velocity. Each line starts on a square grid corresponding to the point at which the velocity is measured. The scale shown corresponds to unit velocity in the units used. The presence of a shock where the stream strikes the disc is quite marked.

The contours of radiation intensity (that is, power per unit area) are shown in the figures marked (c). In an equilibrium flow, these contours should be reasonably independent of the assumptions made about the viscosity. We note a region of enhanced emission

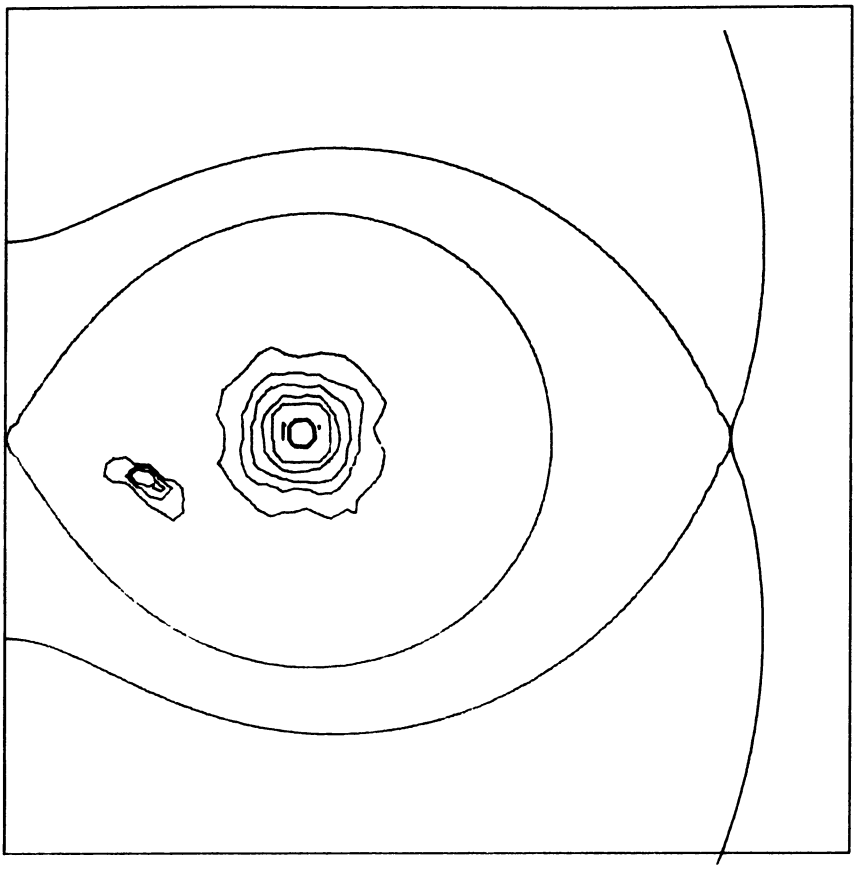

Fig. 3c. $q = 1.0$, $l = 0.02$. Contours of radiation intensity at relative levels of 1.0, 2.5, 5, 10, 20, and 80.

where the stream strikes the disc. Close to the primary the contours are roughly circular and follow the expected r_1^{-3} dependence.

In Table I we show for each case the number of orbits required to reach equilibrium (n_0), the number of particles in the equilibrium disc (N), and the fraction of particles transferred that are not accreted (f). We also show the radius R_s at which the disc stream energy dissipation takes place, and the radius R_h at which we would expect it to occur were no viscosity present (Flannery, 1975).

4. Discussion

Those wishing to apply the results presented here directly to observational data should proceed with considerable caution. We have shown a numerical simulation of a time-average of the equilibrium flow of a pressure-free, pseudo-viscous fluid which is being transferred from one star to another in a binary system. We feel that the overall properties of the flow are probably well represented, despite the simplifications we have made, but that any detailed conclusions drawn should be treated with care. We have not presented

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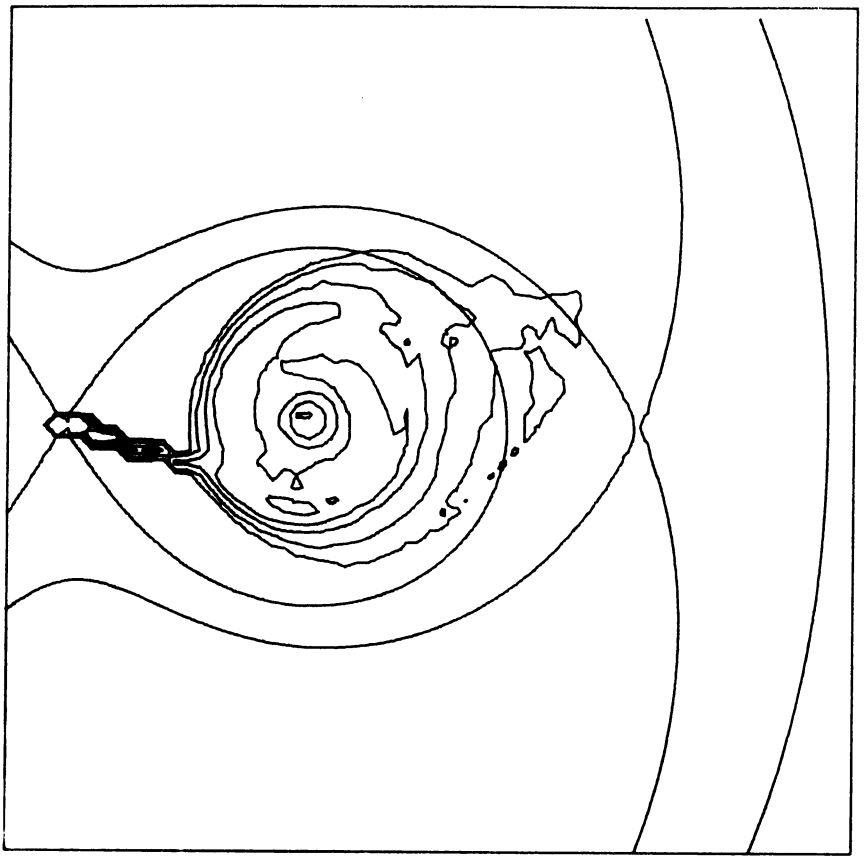

Fig. 4a. $q = 0.04$, $l = 0.02$. Density contours at relative levels of 1, 2.5, 5, 7.5 and 10. The density is low at the outside and at the centre of the disc and rises to a 'volcanic' rim in between. This variation is just a result of the assumptions made about viscosity.

any of the time-dependent features of the flow, although we are preparing a 16-mm cine film of some of it. We hope that this will give us further insight into the mass-transfer process. For this reason we cannot, as yet, comment upon time-dependent accretion phenomena, for example the outbursts of dwarf novae. We have also, for simplicity and to save time omitted any consideration of fluid motions perpendicular to the plane of the binary system. For this reason we are unable to shed much light on phenomena that are probably associated with variations in the thickness of the disc (that is, that are essentially three-dimensional), for example the pre-eclipse dips in Her X-1 and the hump in the orbital light curve of dwarf novae.

Nevertheless we feel that the following conclusions can be tentatively drawn from our results.

(i) A high percentage of the matter transferred from the secondary is actually accreted by the primary (see Table I). This is in contrast to the simple estimates (Section 1) of the amount of matter that is required to be lost in order to remove the necessary angular

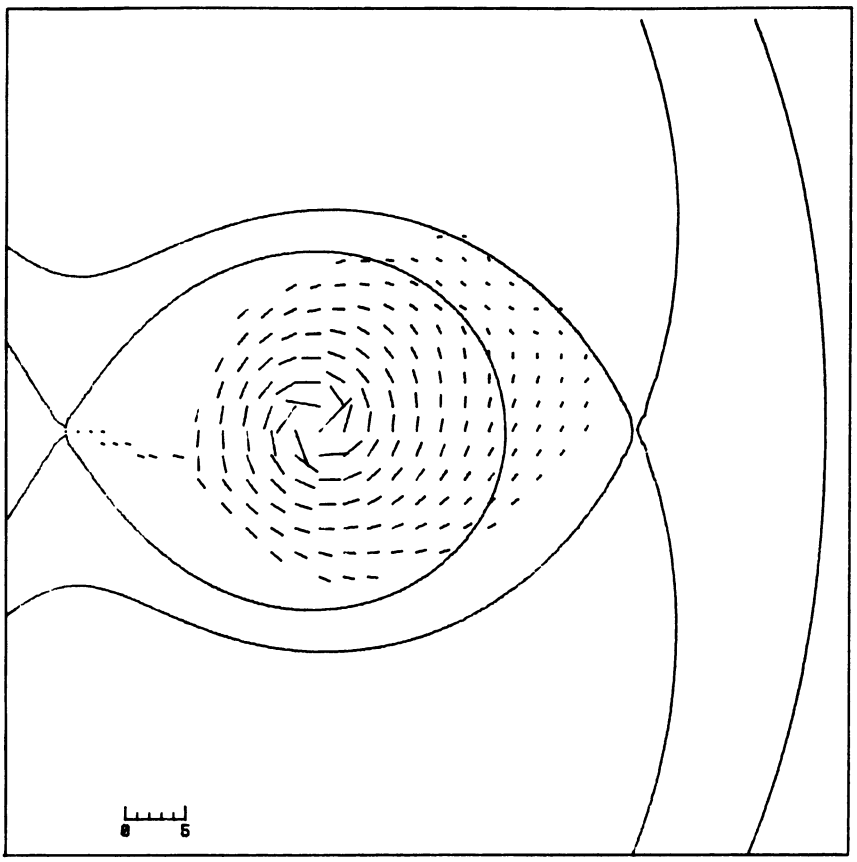

Fig. 4b. $q = 0.4, l = 0.02$. The velocity field.

momentum with respect to the primary. There are probably a number of reasons for this. Firstly the radius of the primary R_1 is finite, but for the cases we have considered it is not large enough to explain the discrepancy entirely. Secondly, the estimate was made for particles orbiting a single gravitating mass for which the concept of angular momentum is good. For orbits close to the primary, angular momentum with respect to the primary is similarly a good concept. However, near the edge of the accretion disc, particles are influenced by the gravitational fields of both stars, and angular momentum about the primary is no longer even approximately conserved along a particle's trajectory. Thus we expect in any case our somewhat naive estimate to be in error. Thirdly the outer regions of the disc may be thought of as a viscous field which is rotating faster than the orbital angular velocity. Tidal interactions would then tend to slow down the rotation, and to transfer the necessary amount of angular momentum to the orbital motion. If this process were efficient enough there would be no need for any of the transferred matter to be lost from the Roche lobe of the primary. However, since this process depends on viscosity it seems probable that its relative efficiency compared to mass loss from

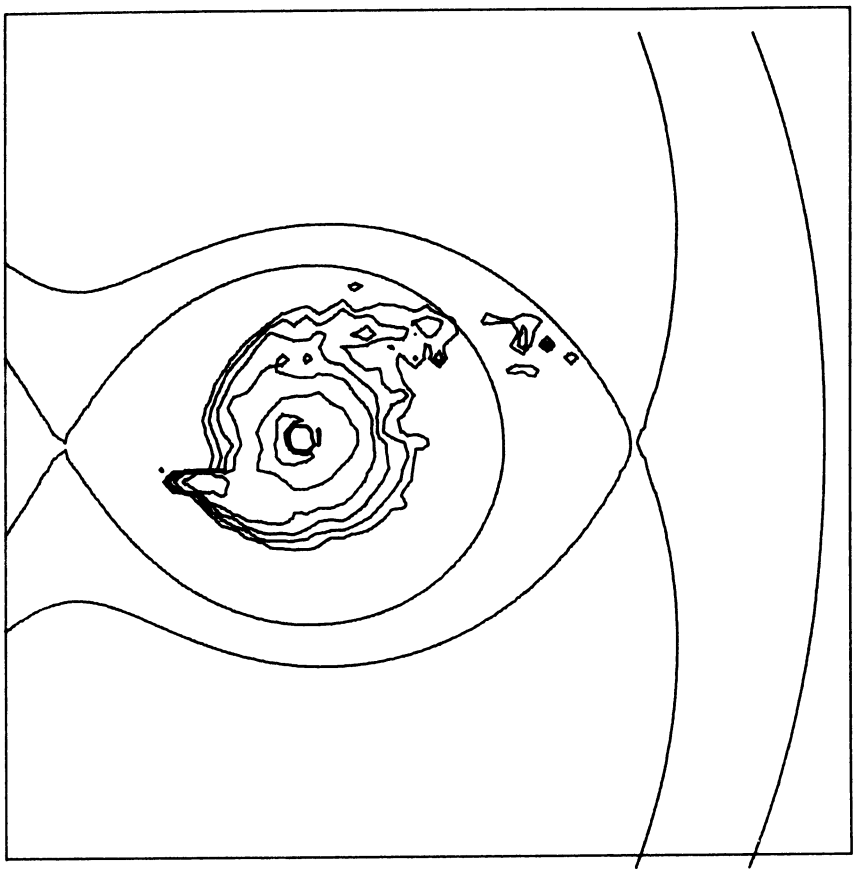

Fig. 4c. $q = 0.4$, $l = 0.02$. Contours of radiation intensity at relative levels of 0.5, 1, 2, 4, 16 and 128.

the Roche lobe may be independent of the value of the viscosity in an equilibrium situation.

(ii) In equilibrium the accretion disc is a fairly well defined entity (i.e. the particle density falls off rapidly at the edge of the disc), and more or less fills the Roche lobe of the primary. This is simply a consequence of the apparent requirement that matter must be lost from the Roche lobe to carry any of the excess angular momentum relative to the primary. The material leaves the Roche lobe roughly in the neighbourhood of the relevant outer Lagrangian point (L_2 or L_3) with relatively small velocities. Our calculations cannot be relied upon to tell us what happens to the matter that escapes. There are essentially two possibilities. Either the matter escapes from the system, in which case it is no trivial problem to calculate how much angular momentum it removes with it; or the matter returns to the secondary, in which case it is an additional factor to mass exchange itself in tending to desynchronize the rotation of the secondary, *particularly in its outer layers.* Slight non-corotation of the outer layers of the secondary can severely

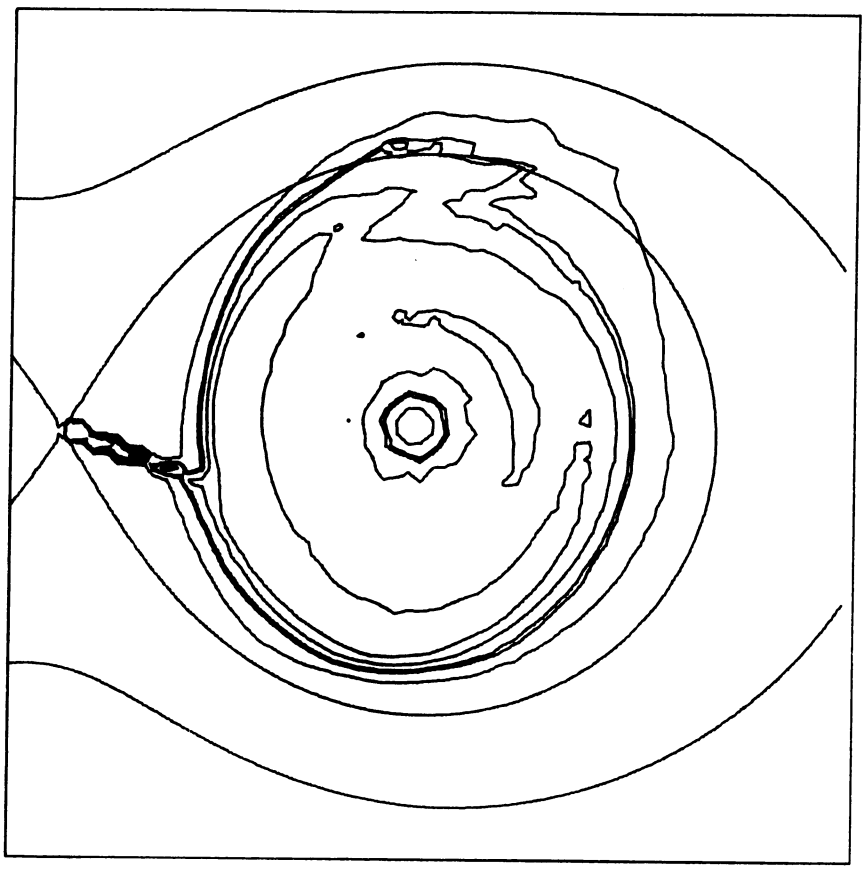

Fig. 5a. $q = 2.5, l = 0.02$. Density contours at relative levels 1, 2, 3 and 4.

alter the trajectory of the stream of transferred material. Since the velocity of the material as it leaves the Roche lobe is quite low, we expect the latter possibility to be strongly preferred for large values of M_1/M_2.

In summary, we have presented a numerical simulation of equilibrium viscous gas flow in a binary system. Although the approach we have adopted is somewhat naive, it makes computation relatively easy, and does, we believe, embody the essential properties of such a flow, at least in a time-averaged sense. Study of the flow and of the approximations involved is still in progress and the full results will be published elsewhere.

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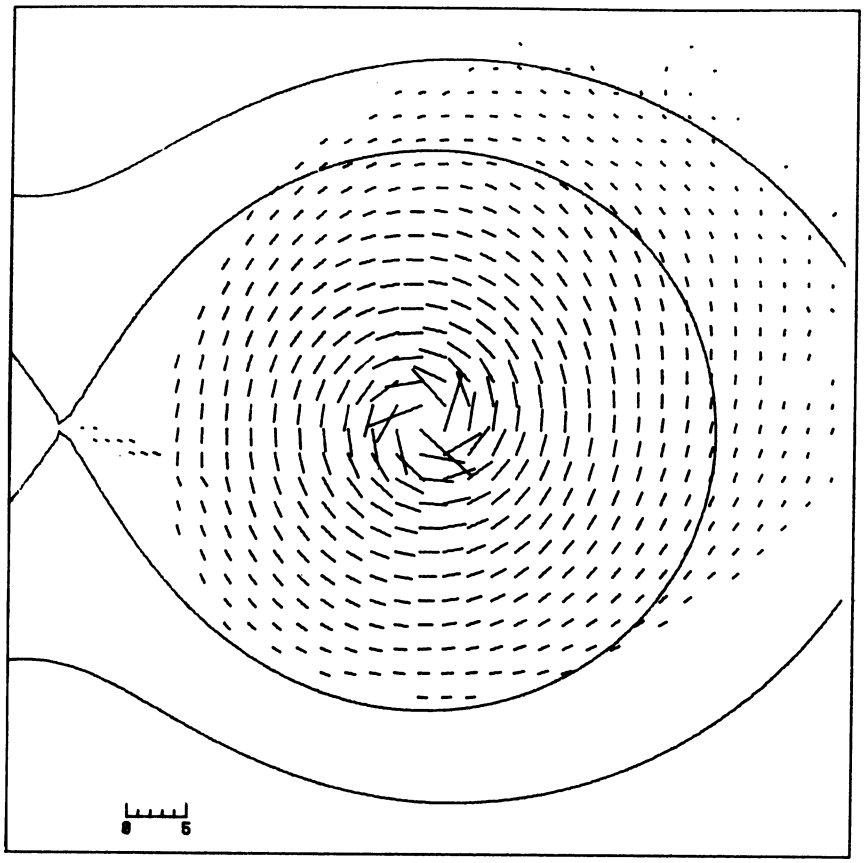

Fig. 5b. $q = 2.5, l = 0.02$. The velocity field.

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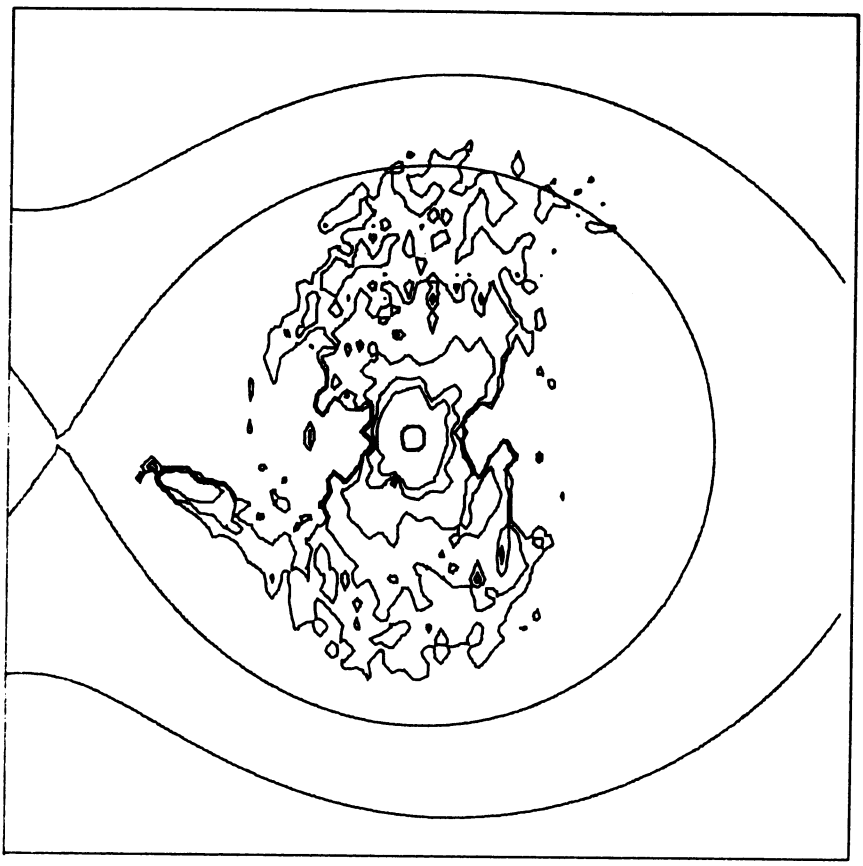

Fig. 5c. $q = 2.5$, $l = 0.02$. Contours of radiation intensity at relative levels 1, 4, 16, 64 and 128. We note that the dissipation in the outer regions of the Roche lobe is distributed in a non-circular manner with respect to the primary. This may be indicative of tidal dissipation.

DISCUSSION

Paczynski: How much matter is lost from the Roche lobe in which the disc is located? Does this matter carry away the excess of angular momentum from the accretion disc?

Pringle: Only a small fraction of the mass transferred is lost from the Roche lobe of the primary (see Table I). Loosely this matter must take away the excess of angular momentum from the accretion disc, but I think a more honest answer would be that it depends what you mean by angular momentum.

Shu: I think you would be the first to agree that what you do is not fluid mechanics. It does give some aspects of the role of viscosity, but not all. Furthermore, I would suspect that the results of the calculation are quite sensitive to the value chosen for the parameter l.

Pringle: I think it is a bit strong to say that what we are doing is not fluid mechanics. We treat the mechanics correctly and I would contend that we are dealing with a fluid. The real question is whether or not the equation of state and properties we have bestowed upon our fluid are sufficiently realistic for our present purposes. We feel that in some respects they probably are. Since the size of the parameter l is directly related to the effective viscosity in the fluid, I agree that some of the results of the calculation must depend upon it. We are at present in the process of examining the dependence in more detail.

Jones: A comment *a propos* Dr Shu's remark. While one would entertain serious doubts about the validity of such a numerical model in a non-steady situation, it may not be too bad in the steady state. The reason is that in a rapidly time-varying situation only a few particles are used to evaluate the effects of the pseudo-viscosity on the particle motions, whereas in the steady state viscous effects are evaluated in a given cell many times and averaged. The most serious problem concerns the nature of the equation of state of this pseudo-fluid. It will be important to check this 'hydrodynamic' model against known flows, and in particular to verify whether across a shock the Rankine–Hugoniot relations are satisfied.

Pringle: I agree that it is important to check this model against known flows and we have already started to do this with some success. I would have thought that since the Rankine–Hugoniot relations are just conservation equations they must be satisfied by our 'pseudo-fluid'.

Icke: One may try to mimic 'true' hydrodynamics by changing the amount of shear and the amount of dilatation which is removed by particle interaction. There is even a hope of thus distinguishing the effects of bulk viscosity and kinematic viscosity, without unduly increasing the number of particles.

Flannery: The position of the hot spot seems much too far from the central star. Has the method been checked against known flow patterns?

Pringle: Our calculations merely indicate where the energy is *generated* in the interaction between the disc and the incoming stream. I am afraid that it will take some lengthy calculations before I would want to make any comparison with observations of dwarf novae light curves. To be able to predict what the resultant hot spot would look like, one must know in addition not only where this energy is radiated, and in what form it is radiated (bolometric corrections can be substantial) but also in which directions it is radiated.