Magnetic Field Reconnection in Differentially Rotating Accretion Disks

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ABSTRACT

Differentially rotating accretion disks threaded by a uniform magnetic field have been numerically simulated. Fast reconnection followed by coalescence allows the magnetic field to drive small amplitude radial oscillations in the disk. These oscillations may be observable as the viscous stresses cause the disk to brighten and fade as the disk expands and contracts. Episodes of reconnection may also be observable as hot spots produced locally at the sites of coalescence. Cataclysmic variables, and in particular dwarf novae, provide a natural interpretation for these calculations.

1. Introduction

Accretion disks, in order to transfer mass, must diffuse angular momentum outwards and this requires a viscous couple between adjacent radial zones. The dynamics of accretion disks are presently uncertain because the key element, the viscosity, is not well understood. Ordinary molecular viscosity is too small to account for the apparent radial inflow rates and luminosities of stellar disks and it is generally considered necessary that some anomalous viscosity due to turbulence or magnetic fields be present. Accretion disks are natural environments for the amplification of field fluctuations through differential rotation and radial inflow. Studies of magnetic viscosity have thus generally concentrated on chaotic field geometries (Eardley and Lightman 1975; Ichimaru 1976; Sakimoto and Coroniti 1981; Coroniti 1981). Although the evolution of chaotic fields is central to the viscosity problem, it seems worthwhile to investigate the much simpler, but still quite formidable, problem of the action of a disk upon a field with a definite geometry. The problems that are posed in this paper concern the windup of a uniform field by a differentially rotating disk. Even in this much restricted domain, there are several interesting phenomena; magnetic field reconnection and coalescence, and large scale disk oscillations. In addition, the repeated episodes of reconnection cause a net diffusion in the plasma. Although the geometry of the uniform field is contrived, many of the observations made here carry over to more complex and chaotic magnetic field configurations.

2. Simulation of a Disk Plasma Threaded by a Uniform Magnetic Field

The initial conditions in the simulation described here were those appropriate to a "cold" disk threaded by a uniform field at $t = 0$. "Cold" here refers to purely Keplerian motions. The plasma is initially confined to a torus surrounding the

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compact star, which is not present on the grid except as a source of gravitation. In other simulations, the disk was allowed to have a larger extent, with the inner regions filled with a rigidly rotating plasma. The continuous disks were not distinguishable from the tori in the field evolution nor in the types of oscillations that were set up in the plasma. In the simulation discussed here, the initial field strength was chosen so that the Keplerian velocity, v_ϑ was about 10 times the Alfven speed ($v_A = B/\sqrt{4\pi\rho}$). This allowed the torus to rotate without being disrupted by the magnetic field. In simulations employing much weaker fields, it was found that electrostatic effects became very important and tended to obscure the role of the field. These latter simulations will be discussed in a future publication.

Two critical moments in the evolution of the plasma and field are shown in Figs. 1A and 1B. These plots show magnetic field lines along with the non-Keplerian components of the ion velocity field.

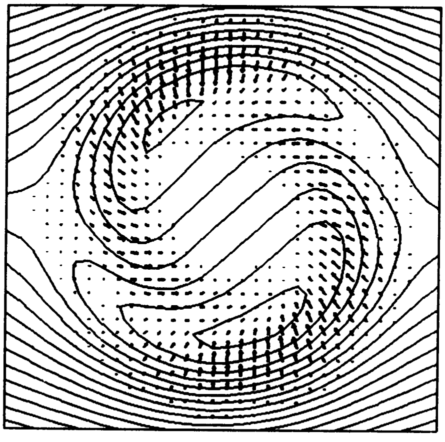

Fig. 1A

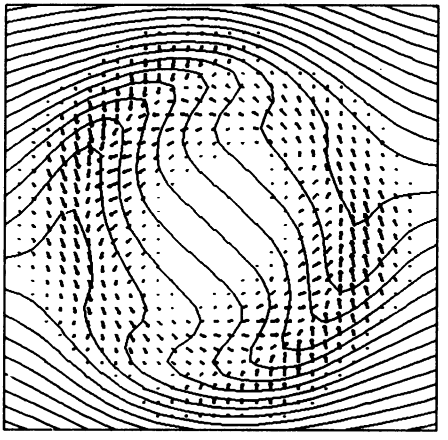

Fig. 1B

Figure 1: Simulated disk and field for the reconnection and coalescence epochs in the initial revolution of the torus. The torus in Fig. 1A has completed ½ revolution while in Fig. 1B the torus has completed ¾ revolution.

The field lines, which are frozen to the plasma, are wound up by the orbital motions in the torus. The compression of the field causes the particles to be slightly deflected from purely Keplerian orbits. During this phase, the deflection is inward since the field is retarding their motion. Once the field has rotated through π radians, X type neutral points are formed and the field undergoes fast reconnection. We emphasize here that the reconnection occurs on a timescale much shorter than the resistive timescale set by particle collisions (which have been reduced through the finite size particle effect). In Fig. 1A the formation of magnetic islands is evident. These islands rotate with the plasma and then coalesce as shown in Fig. 1B. The tendency of the magnetic field during coalescence is to snap back to a more uniform configuration. This causes the particles which were accelerated inwards during field compression to now drift outwards. This cycle is repeated in subsequent rotations of the torus. Reconnection in this geometry conserves flux; the number of field lines linking the torus is constant.

The first cycle of reconnection is special in the sense that the torus was started cold. Ion inertia causes the particles positions to lag behind the instantaneous field geometry. A steady state is eventually reached when the particles are at their maximum radial extent at peak field compression and at minimum radius when the field is returned to uniformity. This relationship is most clearly seen in Fig. 2 which depicts the energy history of the field and plasma. Time in these calculations is measured in terms of the inverse electron plasma frequency, $\omega_{pe} = \sqrt{4\pi n e^2 / m_e}$.

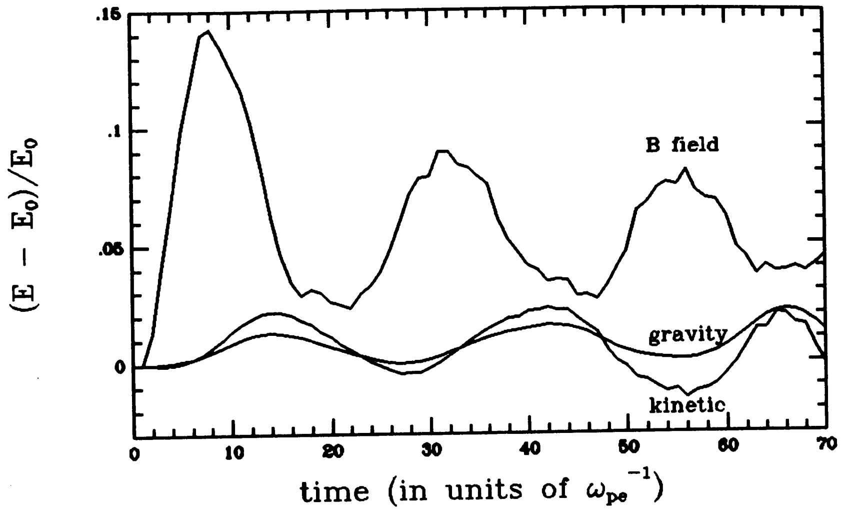

Figure 2.: Energy history of an oscillating disk plasma. Peaks in the magnetic field energy occur just prior to reconnection and at the same time as maximum disk expansion.

The particles positions can be deduced from the *kinetic* and *gravity* tracks. At maximum radius, the particles have a minimum of binding energy and consequently a minimum orbital velocity; the kinetic energy and binding energy are in phase with one another. The magnetic field energy is π out of phase, a relationship which takes several rotations to achieve. At peak compression (maximum field energy) the particles are given an inward acceleration, but it takes ½ cycle for the particles to drift inwards. Similarly, during magnetic field coalescence (return to minimum field energy) the particles are accelerated outwards with their positions lagging by ½ period.

The cycle of reconnection followed by coalescence causes the torus to radially oscillate. The maximum radial excursion is estimated by the product of the non - Keplerian drift velocity and the orbital period. The non - Keplerian ion drifts induced by coalescence are on the order of the Alfven velocity of the plasma at peak magnetic field compression. Since reconnection occurs every half cycle of revolution, the maximum field strength is roughly twice the initial field strength; there is twice as much flux per unit volume threading the torus after it has turned through π radians. The relative amplitude of the oscillation is therefore $\Delta R / R \sim 2\pi v_A / v_\vartheta$. The effect of this oscillation on the luminosity of the torus can be estimated by assuming that the torus emits the energy deposited by viscous dissipation as blackbody radiation. The viscous dissipation rate per unit area per unit time in an axisymmetric disk is (Pringle 1981) $D(R) = 2.25 M G \nu \Sigma / R^3$, where ν is the viscosity (whatever its source), M is the mass of the compact star, and Σ is the surface mass density of the torus. The disk will brighten according to $\Delta D / D \sim 6\pi v_A / v_\vartheta$. Evidently, disks which are only mildly superalfvenic, will undergo significant changes in their brightness if this type of oscillation is driven.

3. Astrophysical Disks and Quasi-Period Oscillations

A particularly interesting application of these calculations is to the oscillations observed in a subclass of the cataclysmic variables, the dwarf novae. These oscillations are observed only during eruption, lose their coherence every few cycles (and are therefore referred to as quasi-periodic oscillations), last typically about 5 days,and have periods about 50 s and amplitudes of about 0.005 mag (Robinson and Warner 1983; Robinson and Nather 1979, Patterson 1981). In some dwarf novae, quasi-period oscillations with several different periods are present simultaneously (Robinson and Warner 1983). The range of periods, the multiplicity of oscillations, and their incoherency all suggest that the oscillations are produced in the disk. The duration of the oscillations is typical of the time it takes for the disk material to drain onto the compact star in a state of high viscosity (for a discussion of the instability leading to eruption see Bath and Pringle 1982; Meyer and Meyer-Hofmeister 1981). The period lengths themselves suggest some type of orbital phenomenon although vertical oscillations (see Cox 1981) will necessarily have periods that appear to be orbital if the disk is pressure supported and not self-gravitating.

Magnetic field reconnection provides a mechanism for driving disk oscillations. The observed brightness variations require relatively modest fields; $v_A / v_\vartheta \sim 10^{-4}$. There are two difficulties that any theory must face; that the oscillations appear only during eruption (when the disk is draining onto the compact star) and that only a few discrete periods are present and these periods are relatively stable. The simplicity of our model prevents a complete accounting but we can suggest several ways in which these problems may be resolved. Since field reconnection should always be occuring (field diffusion is very slow due to the relatively high densities in these disks) it may be that it is either masked by other oscillations (the orbiting of the hot spot) or that the oscillations are too weak in a quiet disk state to be observed. Once the disk is destabilized and begins to accrete at a fast rate, flux conservation will enhance the field ($\sim R^{1-2}$) and the oscillation amplitude will increase. The discreteness of the period may be related to the observation that increased viscosity causes the disk to simultaneously expand as the bulk of the plasma drains onto the compact star. The period of the oscillation is not set by where the plasma is concentrated, but by where the field is sufficiently twisted to form X points. In the twisting of a uniform field by plasma extending over widely disparate radii, there may be several sites where reconnection occurs with a period appropriate to the characteristic radius of each site. These sites may be fixed in space although shifting with respect to the mass profile of the disk. In future calculations we hope to investigate the possibility of multiple sites of reconnection and also to include a chaotic component in the magnetic field.

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DISCUSSION

Wilson: Could you please describe the outer boundary condition on the magnetic field in your model?

Gilden: Periodic boundary conditions were used. A mesh sufficiently large to minimize the effects of this symmetry was necessitated.

Vasyliunas: In your simulation, the rotation period is about 20 inverse plasma periods. This means that the system in the simulation is relatively small in relation to the microscopic plasma length scales, so that non-MHD effects are much more important than they would be in the real, large system. This may be the reason for the rapid reconnection seen in the simulation, which may not scale up to the real case.

Gilden: Numerical limitations do not allow processes with widely disparate timescales to be simulated in a single model. Your criticism is valid, but only underscores the difficulty in interpreting numerical results, and in designing a numerical experiment. We have explored a range of ratios of the relevant time and length scales, and have not observed any sensitivity in the rate of reconnection. However, it cannot be said with certainty what the rate would be with the ratios observed in nature.

Heyvaerts: Can you guess what the difference may be between your 2-D (space) simulation, and the real flat disk behavior?

Gilden: One important difference will be the vertical motion of bouyant flux tubes out of the plane of the disk. A second important 3-D (space) effect will be the twisting of the field component normal to the disk.

Kennel: Will the exchange of gravitational energy with magnetic be so important in the real case where the Keplerian time is very much larger than the plasma period?

Gilden: Our results do not rely on the relative magnitudes of the plasma and orbital periods, as long as they are not comparable, in which case electrostatic effects become important. The exchange between magnetic and gravitational energy proceeds through windup and then reconnection of the magnetic field. Although our simulation code cannot reproduce the ratio of plasma to orbital timescales observed in nature, we have attempted to use a ratio, $\sim 10^{-2}$, which allows the energy transfer to not be corrupted by purely numerical effects.

Tajima: Because of the smaller disparity of plasma time and Keplerian time than in the accretion disk, the electrostatic energy is perhaps unrealistically large. However, there is no strong evidence that the electrostatic noise wipes out other physics. However, the two time scales are separate in the code. Once again, we should use our theoretical insight to interpret the raw data.

Krishan: Have you included radiation pressure in your simulations, since most of these accretion discs are radiation supported?

Gilden: We have not included radiation pressure. However, the accretion disks associated with the cataclysmic variables are not radiation pressure supported throughout. In their quiet state, they are cool enough so that gas pressure is dominant.

Van Hoven: What is the source of the fluctuations which the simulation produces, having started from a mixture of cylindrical and planar symmetries? Is this numerical noise?

Gilden: Fluctuations are not caused by the symmetries in the calculation, but by the use of a finite number of particles. The finite size particle technique is designed to minimize this noise.

Steinolfson: You remarked that the reconnection in your results was fast. What is it fast relative to?

Gilden: Fast relative to the collision timescale in the plasma.

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