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ROBUST STABILITY OF IMPULSIVE SWITCHED SYSTEMS WITH DISTURBANCE

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Abstract

This paper studies a class of impulsive switched systems with persistent bounded disturbance using robust attractor analysis and multiple Lyapunov functions. Some sufficient conditions for internal stability of the systems are obtained in terms of linear matrix inequalities (LMI). Based on the results, a simple approach for the design of a feedback controller is presented to achieve a desired level of disturbance attenuation. Numerical examples are also worked out to illustrate the obtained results.

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1. Introduction

Switched systems have been studied extensively in recent years, see [2, 3, 5–7, 9, 10] and references therein. Such systems have many applications such as hybrid control of mechanical systems, the automotive industry, flight and air traffic control, switching power converters, intelligent vehicle highway systems, *etc.* The stability of switched systems is studied in [9, 10] by the technique of the common Lyapunov function, and in [2, 5, 7] by the multiple Lyapunov functions method. In real life, many fault-prone dynamic systems may experience abrupt changes in their structures, states and parameters, caused by phenomena such as component failure or repairs, changing subsystem interconnections, and abrupt environmental disturbances. Such systems may be modelled by impulsive switched systems, where structure and parameter

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changes are considered as operations in different forms and are thus described by system switches, and state changes are described by impulses. Switched systems may be considered as special cases of impulsive switched systems. It is shown in [5] that stability in each mode of a switched system does not guarantee the stability of the switched system. On the other hand, a pair of unstable linear systems may form a stable switched system under an appropriate switching rule [10]. Similarly, it is shown in [4] that impulses may destabilise or stabilise a dynamical system. Numerous results on switched systems and impulsive systems have appeared during the past ten years. However, there has been very little investigation into impulsive switched systems.

The objective of this paper is to study a class of impulsive switched systems with persistent bounded disturbance. In practice, it is of interest to consider the effect of persistent bounded disturbances on a dynamical system. We shall utilise the multiple Lyapunov functions method together with robust attractor analysis and establish some sufficient conditions for internal stability of the linear impulsive switched systems with persistent bounded disturbance. We also show, under some suitable conditions, that the system has ρ-performance, which will be defined in the next section. The rest of this paper is organised as follows. In Section 2, we shall present some preliminary results and definitions and notation. Then we state and prove, in Section 3, our main results. We firstly present a linear state-feedback controller for every subsystem and establish some sufficient conditions for the stability of the whole system. Then we discuss ρ-performance of the system under stable conditions. Finally, we give, in Section 4, some numerical examples to illustrate our main results.

2. Preliminaries

Let $\Omega_P = \{X : X^T P X \leq 1\}$, where P is a positive definite matrix. We use the norm $\|X\| = \sqrt{\sum_{i=1}^{n} x_i^2}$ for $X \in \mathbb{R}^n$. Let $\Phi = \{W : \|W\| \leq 1, W \in \mathbb{R}^p\}$. Denote by $\mathrm{Re}(\lambda)$ the real part of a complex number λ, by A^T the transpose of a matrix A, and by A^{-1} the inverse matrix of A.

Consider the following impulsive switched system with disturbance:

$$\begin{cases} \dot{X}(t) = A_\alpha X(t) + B_\alpha U_\alpha + C_\alpha W(t), & t \neq t_k; \\ \Delta X(t_k) = X(t_k^+) - X(t_k^-) = E_k X(t_k), & t = t_k; \\ Y(t) = D_\alpha X(t), \end{cases} \tag{2.1}$$

where $X : \mathbb{R} \to \mathbb{R}^n$, $U_\alpha : \mathbb{R} \to \mathbb{R}^m$ and $W : \mathbb{R} \to \mathbb{R}^p$ are the state, the input and the external disturbance vectors, respectively. Here $Y : \mathbb{R} \to \mathbb{R}^p$ is the controlled output, $\alpha : \mathbb{R} \times \mathbb{R}^n \to P$ is a switching signal, where $P = \{1, 2, \ldots, N\}$, that is, $\alpha(t, x)$ is a piecewise constant function. We denote by S the family of all the switching signals of system (2.1). We note that A_α, B_α, C_α, D_α and E_k are known real constant matrices of appropriate dimensions, where $k = 1, 2, \ldots$, $X(t_k) = X(t_k^-) = \lim_{h\to 0^+} X(t_k - h)$, $X(t_k^+) = \lim_{h\to 0^+} X(t_k + h)$. Here t_k is the k-th switching point, $0 < t_1 < \cdots < t_k < \cdots$, and $t_k \to \infty$ as $k \to \infty$.

DEFINITION 1. System (2.1) is called internally stable, if $X = 0$ is asymptotically stable when $W(t) = 0$.

DEFINITION 2. A set Ω is called a robust attractor of system (2.1), if all the state trajectories $X(t)$ of the system (2.1) initiating from the exterior of Ω eventually enter and remain in Ω for all $W(t)$, and those from the interior of Ω always remain in Ω.

DEFINITION 3. The system (2.1) is said to have ρ-performance, if for all $X(t)$ ($t > 0$) from the origin (that is, $X(0) = 0$), $\|Y(t)\| \le \rho$ holds, where ρ is a given positive scalar.

DEFINITION 4. Let P be an $n \times n$ symmetric matrix. We write $P < 0$ ($P > 0$), if P is negative (positive) definite; and $P \le 0$ ($P \ge 0$), if P is negative (positive) semi-definite.

The following lemmas will be used in the proofs of the later theorems.

LEMMA 2.1. *Let P be an $n \times n$ symmetric matrix, then for any number $\sigma_\alpha > 0$, it follows that*

$$2X^T P C_\alpha W \le \frac{1}{\sigma_\alpha} X^T P C_\alpha C_\alpha^T P X + \sigma_\alpha W^T W,$$

where C_α is any matrix of appropriate dimensions, and X, W are any vectors of appropriate dimensions.

PROOF. Since $\left\|(1/\sqrt{\sigma_\alpha}) C_\alpha^T P X - \sqrt{\sigma_\alpha} W\right\|^2 \ge 0$, namely,

$$\left(\frac{1}{\sigma_\alpha} C_\alpha^T P X - \sqrt{\sigma_\alpha} W\right)^T \left(\frac{1}{\sigma_\alpha} C_\alpha^T P X - \sqrt{\sigma_\alpha} W\right) \ge 0,$$

then

$$\frac{1}{\sigma_\alpha} X^T P C_\alpha C_\alpha^T P X - X^T P C_\alpha W - W^T C_\alpha^T P X + \sigma_\alpha W^T W \ge 0.$$

Since $X^T P C_\alpha W = W^T C_\alpha^T P X$, it follows that

$$2X^T P C_\alpha W \le \frac{1}{\sigma_\alpha} X^T P C_\alpha C_\alpha^T P X + \sigma_\alpha W^T W. \qquad \square$$

LEMMA 2.2 (Schur complement [1]). *The following three inequalities are equivalent*:

(1) $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \leq 0$;
(2) $A \leq 0$, *and* $C - B^T A^{-1} B \leq 0$;
(3) $C \leq 0$, *and* $A - BC^{-1}B^T \leq 0$,

where A, B, C *are real constant matrices of appropriate dimensions, and* A, C *are symmetric.*

LEMMA 2.3. *Let* A, B *be symmetric matrices. If* $A > 0$, *then the following two inequalities are equivalent*:

(1) $B \leq 0$;
(2) $ABA \leq 0$.

LEMMA 2.4. *If* $P_{n\times n}$ *is a positive definite matrix, for any vector* X, *then*

$$\lambda_{\min}\|X\|^2 \leq X^T P X \leq \lambda_{\max}\|X\|^2,$$

where $\lambda_{\min}$ *is the minimal eigenvalue of* $P_{n\times n}$ *and* $\lambda_{\max}$ *is the maximal eigenvalue of* $P_{n\times n}$.

LEMMA 2.5 ([8]). *If* A *is a stable matrix, then for any constant* $\beta > \max_{1\leq i\leq n}\{\operatorname{Re}\lambda_i\}$, *where* λ_i *is the eigenvalue of* A, *there exists an* $N > 0$ *such that*

$$\|X(t)\| \leq N e^{\beta(t-t_0)}\|X(t_0)\|,$$

where $X(t)$ *is any solution of the linear system* $\dot{X} = AX$, $X(0) = X_0$.

3. Main results

Consider the uncontrolled system

$$\begin{cases} \dot{X}(t) = A_\alpha X(t) + C_\alpha W(t), & t \neq t_k; \\ \Delta X(t_k) = X(t_k^+) - X(t_k^-) = E_k X(t_k), & t = t_k; \\ Y(t) = D_\alpha X(t). \end{cases} \tag{3.1}$$

Let λ_α be the eigenvalue of A_α. Denote by N_α the constant N with respect to $\bar{\beta}$ ($\bar{\beta} > \max_{\alpha\in P}\{\operatorname{Re}(\lambda_\alpha)\}$) in Lemma 2.5. Let $\overline{N} = \max_{\alpha\in P}\{N_\alpha\}$. Denote by Δt the dwell time (that is, the switching interval).

THEOREM 3.1. *For system* (3.1), *let ρ be a given positive scalar. If for any α, there exists a positive definite matrix Q_α and a positive number σ_α such that*

$$\begin{bmatrix} Q_\alpha A_\alpha^T + A_\alpha Q_\alpha + \sigma_\alpha Q_\alpha & C_\alpha \\ C_\alpha^T & -\sigma_\alpha I \end{bmatrix} \le 0, \tag{3.2}$$

and if for any switching signal, there exists μ $(0 < \mu < 1)$ such that the dwell time Δt satisfies

$$\Delta t \ge \frac{1}{2\bar{\beta}} \ln\left(\frac{\mu\bar{\lambda}_{\min}}{\bar{N}^2\bar{\lambda}_{\max}}\right), \tag{3.3}$$

where

$$\bar{\lambda}_{\min} = \inf_{\alpha\in P}\{\lambda\} \quad \textit{and} \quad \bar{\lambda}_{\max} = \sup_{\alpha\in P,\ k\in\mathbb{N}}\{\lambda_{\alpha,k}\},$$

where λ is the minimal eigenvalue of Q_α^{-1} and $\lambda_{\alpha,k}$ is the maximal eigenvalue of $(I+E_k)^T Q_\alpha^{-1}(I+E_k)$ $(\alpha \in P,\ k \in \mathbb{N})$, then system (3.1) *is internally stable.*

PROOF. For any subsystem α, let $V_\alpha(x) = X^T P_\alpha X$, where $P_\alpha = Q_\alpha^{-1}$. By Lemma 2.1, we obtain

$$\begin{aligned}
\dot{V}_\alpha(x) &= X(t)^T P_\alpha \dot{X}(t) + \dot{X}(t)^T P_\alpha X(t) \\
&= X(t)^T P_\alpha\big(A_\alpha X(t) + C_\alpha W(t)\big) + \big(A_\alpha X(t) + C_\alpha W(t)\big)^T P_\alpha(t) \\
&= X(t)^T(P_\alpha A_\alpha + A_\alpha^T P_\alpha)X(t) + 2X(t)^T P_\alpha C_\alpha W(t) \\
&\le X(t)^T(P_\alpha A_\alpha + A_\alpha^T P_\alpha)X(t) + \frac{1}{\sigma_\alpha}X(t)^T P_\alpha C_\alpha C_\alpha^T P_\alpha X(t) \\
&\quad + \sigma_\alpha W(t)^T W(t) \\
&\le X(t)^T\left(P_\alpha A_\alpha + A_\alpha^T P_\alpha + \frac{1}{\sigma_\alpha}P_\alpha C_\alpha C_\alpha^T P_\alpha + \sigma_\alpha P_\alpha\right)X(t) \\
&\quad - \sigma_\alpha\big(X(t)^T P_\alpha X(t) - W(t)^T W(t)\big).
\end{aligned}$$

By inequality (3.2) and Lemma 2.2, we have

$$Q_\alpha A_\alpha^T + A_\alpha Q_\alpha + \frac{1}{\sigma_\alpha}C_\alpha C_\alpha^T + \sigma_\alpha Q_\alpha \le 0.$$

By Lemma 2.3, we obtain

$$P_\alpha(Q_\alpha A_\alpha^T + A_\alpha Q_\alpha + \frac{1}{\sigma_\alpha}C_\alpha C_\alpha^T + \sigma_\alpha Q_\alpha)P_\alpha \le 0,$$

namely,

$$P_\alpha A_\alpha + A_\alpha^T P_\alpha + \frac{1}{\sigma_\alpha}P_\alpha C_\alpha C_\alpha^T P_\alpha + \sigma_\alpha P_\alpha \le 0.$$

Moreover, for all $X(t) \notin \Omega_{P_\alpha}$, it follows that $X(t)^T P_\alpha X(t) > 1$. Thus, for all $W(t) \in \Phi$, we obtain $X(t)^T P_\alpha X(t) - W(t)^T W(t) > 0$. So

$$\dot{V}_\alpha(x) < 0, \quad \text{for all } X(t) \notin \Omega_{P_\alpha} \text{ and } W(t) \in \Phi. \tag{3.4}$$

From inequality (3.4), we see that for all $X(0) \notin \Omega_{Q_\alpha^{-1}}$, all state trajectories $X(t)$ of the subsystem are attracted to $\Omega_{Q_\alpha^{-1}}$, and for all $X(0) \in \Omega_{Q_\alpha^{-1}}$, $X(t)$ always remains in $\Omega_{Q_\alpha^{-1}}$. Therefore $\Omega_{Q_\alpha^{-1}}$ is a robust attractor of the subsystem α which is internally stable.

Next we prove that the whole impulsive switched system (3.1) is internally stable. Let $\Psi_{t_\alpha} = \{$the time when the subsystem α is active$\}$,

$$\varepsilon_\alpha(t) = \begin{cases} 1, & t \in \Psi_{t_\alpha}, \\ 0, & t \notin \Psi_{t_\alpha}, \end{cases} \quad \text{and} \quad V(X(t)) = \sum_{\alpha \in P} \varepsilon_\alpha(t) X(t)^T P_\alpha X(t).$$

We consider two sequential switches. Assume that system (3.1) switches into the subsystem $\alpha = i$ at time $t = t_{k-1}$, and into any subsystem $\alpha = j$ at time $t = t_k$ in succession.

At the switching points, by Lemma 2.4, we have

$$\begin{aligned} V&\big(X(t_k^+)) - \mu V(X(t_{-1}k^+)\big) \\ &= X(t_k^+)^T P_j X(t_k^+) - \mu X(t_{k-1}^+)^T P_i X(t_{k-1}^+) \\ &= X(t_k^-)^T (I + E_k)^T P_j (I + E_k) X(t_k^-) - \mu X(t_{k-1}^+)^T P_i X(t_{k-1}^+) \\ &\le \bar{\lambda}_{\max} \|X(t_k^-)\|^2 - \mu \bar{\lambda}_{\min} \|X(t_{k-1}^+)\|^2. \end{aligned}$$

When $W(t) = 0$, by condition (3.3) and Lemma 2.5, we obtain

$$\begin{aligned} V&(X(t_k^+)) - \mu V(X(t_{k-1}^+)) \\ &\le \bar{\lambda}_{\max} N^2 e^{2\beta_i \Delta t_k} \|X(t_{k-1}^+)\|^2 - \mu \bar{\lambda}_{\min} \|X(t_{k-1}^+)\|^2 \\ &\le (\bar{\lambda}_{\max} N^2 e^{2\beta_i \Delta t_k} - \mu \bar{\lambda}_{\min}) \|X(t_{k-1}^+)\|^2 \le 0. \end{aligned}$$

Furthermore, for $t \in (t_{k-1}, t_k]$, $V(X(t)) = V_{\alpha=i}(x)$ along the solution of system (3.1) is strictly decreasing, and so

$$0 \le V(X(t)) \le V(X(t_{k-1}^+)) \le \mu V(X(t_{k-2}^+)) \le \cdots \le \mu^{n-1} V(X(0)).$$

Since $\lim_{n\to\infty} \mu^{n-1} V(X(0)) = 0$, we see that $\lim_{n\to\infty} V(X(t)) = 0$, namely, $\lim_{n\to\infty} \|X(t)\| = 0$. So system (3.1) is internally stable. □

REMARK. The positive matrix Q_α can be obtained by solving the linear matrix inequality (LMI)

$$Q_\alpha A_\alpha^T + A_\alpha Q_\alpha + \frac{1}{\sigma_\alpha} C_\alpha C_\alpha^T + \sigma_\alpha Q_\alpha \le 0.$$

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THEOREM 3.2. *For system* (3.1), *if the conditions of Theorem* 3.1 *hold, and furthermore, if for all* $i, j \in P$ *and* $k = 1, 2, \ldots$, *the following inequalities hold*:

$$\begin{bmatrix} -Q_i & Q_i(I+E_k)^T \\ (I+E_k)Q_i & -Q_j \end{bmatrix} \le 0, \tag{3.5}$$

$$\begin{bmatrix} -\rho^2 Q_\alpha & Q_\alpha D_\alpha^T \\ D_\alpha Q_\alpha & -I \end{bmatrix} \le 0, \tag{3.6}$$

then system (3.1) *has* ρ-*performance.*

PROOF. By Theorem 3.1, we see that system (3.1) is internally stable.

Next, we analyse the ρ-performance of the system. By condition (3.5) and Lemma 2.2, we have

$$-Q_i + Q_i(I+E_k)^T P_j (I+E_k) Q_i \le 0.$$

By Lemma 2.3, we obtain

$$-P_i + (I+E_k)^T P_j (I+E_k) \le 0.$$

For any switching signal, assume that at the switching point k, system (3.1) switches into subsystem j from subsystem i. Then we have

$$\begin{aligned} &V\big(X(t_k^+)\big) - V\big(X(t_k^-)\big) \\ &\quad = X(t_k^+)^T P_j X(t_k^+) - X(t_k^-)^T P_i X(t_k^-) \\ &\quad = X(t_k^-)^T\big((I+E_k)^T P_j (I+E_k) - P_i\big)X(t_k^-) = 0. \end{aligned} \tag{3.7}$$

When $X(0) = 0$, since $\Omega_{Q_\alpha^{-1}}$ is the attractor of subsystem α, for all $t \in (0, t_1]$, $X(t)^T P_\alpha X(t) \le 1$. By inequality (3.7), we obtain

$$X(t_1^+)^T P_\alpha X(t_1^+) \le X(t_1^-)^T P_\alpha X(t_1^-) \le 1.$$

Therefore, for all $t \in (t_1, t_2]$, $X(t)^T P_\alpha X(t) \le 1$. Similarly, for all $t \in (t_{k-1}, t_k]$, $X(t)^T P_\alpha X(t) \le 1$. By Lemma 2.2, condition (3.6) is equivalent to

$$-\rho^2 Q_\alpha + Q_\alpha D_\alpha^T D_\alpha Q_\alpha \le 0.$$

By Lemma 2.3, we have

$$P_\alpha(-\rho^2 Q_\alpha + Q_\alpha D_\alpha^T D_\alpha Q_\alpha) P_\alpha \le 0,$$

namely, $-\rho^2 P_\alpha + D_\alpha^T D_\alpha \le 0$. Then

$$\|Y(t)\| = \|D_\alpha X(t)\| = X(t)^T D_\alpha^T D_\alpha X(t) \le \rho^2 X(t)^T P_\alpha X(t) \le \rho^2.$$

So system (3.1) has ρ-performance. □

Now we consider the controlled system (2.1). Let a state-feedback controller $U_\alpha = K_\alpha X(t)$, then system (2.1) turns into

$$\begin{cases} \dot{X}(t) = (A_\alpha + B_\alpha K_\alpha)X(t) + C_\alpha W(t), & t \neq t_k, \\ \Delta X(t_k) = X(t_k^+) - X(t_k^-) = E_k X(t_k), & t = t_k, \\ Y(t) = D_\alpha X(t). \end{cases} \tag{3.8}$$

THEOREM 3.3. *For system* (3.8), *let ρ be a given positive scalar. If for any α, there exists a matrix G_α, a positive definite Q_α and a positive number σ_α such that*

$$\begin{bmatrix} Q_\alpha A_\alpha^T + A_\alpha Q_\alpha + \sigma_\alpha Q_\alpha + G_\alpha^T B_\alpha^T + B_\alpha G_\alpha & C_\alpha \\ C_\alpha^T & -\sigma_\alpha I \end{bmatrix} \leq 0, \tag{3.9}$$

and if for any switching signal, there exists μ $(0 < \mu < 1)$ such that the dwell time Δt satisfies

$$\Delta t \geq \frac{1}{2\bar{\beta}} \ln\left(\frac{\mu\bar{\lambda}_{\min}}{\bar{N}^2\bar{\lambda}_{\max}}\right), \tag{3.10}$$

where $\bar{\lambda}_{\min}$ is the minimal eigenvalue of Q_α^{-1}, and $\bar{\lambda}_{\max}$ is the maximal eigenvalue of $(I + E_k)^T Q_\alpha^{-1}(I + E_k)$ $(\alpha \in P,\ k = 1, 2, \ldots)$, then system (3.8) *is internally stable and the state-feedback matrix $K_\alpha = G_\alpha Q_\alpha^{-1}$.*

PROOF. Let $G_\alpha = K_\alpha Q_\alpha$. Condition (3.9) turns into

$$\begin{bmatrix} Q_\alpha A_\alpha^T + A_\alpha Q_\alpha + \sigma_\alpha Q_\alpha + Q_\alpha K_\alpha^T B_\alpha^T + B_\alpha K_\alpha Q_\alpha & C_\alpha \\ C_\alpha^T & -\sigma_\alpha I \end{bmatrix} \leq 0,$$

namely,

$$\begin{bmatrix} (A_\alpha + B_\alpha K_\alpha)Q_\alpha + Q_\alpha(A_\alpha^T + K_\alpha^T B_\alpha^T) + \sigma_\alpha Q_\alpha & C_\alpha \\ C_\alpha^T & -\sigma_\alpha I \end{bmatrix} \leq 0. \tag{3.11}$$

Then the conditions (3.2) and (3.3) of Theorem 3.1 are satisfied by inequalities (3.10) and (3.11), so the desired results follow. □

THEOREM 3.4. *If the conditions of Theorem* 3.3 *hold, and furthermore, if for all $i, j \in P$ and $k = 1, 2, \ldots$, the following inequalities hold*:

$$\begin{bmatrix} -Q_i & Q_i(I + E_k)^T \\ (I + E_k)Q_i & -Q_j \end{bmatrix} \leq 0, \tag{3.12}$$

$$\begin{bmatrix} -\rho^2 Q_\alpha & Q_\alpha D_\alpha^T \\ D_\alpha Q_\alpha & -I \end{bmatrix} \leq 0, \tag{3.13}$$

then system (3.8) *has ρ-performance.*

PROOF. By Theorem 3.3, we see that system (3.8) is internally stable. Since conditions (3.12) and (3.13) are the same as conditions (3.5) and (3.6), by Theorem 3.2, we see that system (3.8) has ρ-performance. $\square$

4. Numerical examples

Consider the following impulsive switched system with disturbance. Assume that $P = \{1, 2\}$ and that we switch between two linear systems:

$$\begin{cases} \dot{X}(t) = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix} X(t) + \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix} U_1 + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} W(t), & t \neq t_k, \\ \Delta X(t_k) = X(t_k^+) - X(t_k^-) = \begin{bmatrix} -1.8 & 0 \\ 0 & -1.8 \end{bmatrix} X(t_k), & t = t_k, \\ Y(t) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} X(t) \end{cases}$$

and

$$\begin{cases} \dot{X}(t) = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} X(t) + \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} U_1 + \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix} W(t), & t \neq t_k, \\ \Delta X(t_k) = X(t_k^+) - X(t_k^-) = \begin{bmatrix} -1.8 & 0 \\ 0 & -1.8 \end{bmatrix} X(t_k), & t = t_k, \\ Y(t) = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix} X(t). \end{cases}$$

Obviously, if there is no state-feedback controller, the system is unstable. Solving inequalities (3.9), (3.12) and (3.13) for given $\rho = \sqrt{2}/2$ and $\sigma_1 = \sigma_2 = 1$ we obtain

$$G_1 = \begin{bmatrix} -0.4574 & 2.2230 \\ -0.8349 & -0.2370 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.4752 & 0 \\ 0 & 0.4925 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 2.6329 & -1.5129 \\ -2.7058 & 0.7452 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.6656 & 0 \\ 0 & 0.3427 \end{bmatrix}.$$

By Theorem 3.2, $K_\alpha = G_\alpha Q_\alpha^{-1}$ and $U_\alpha = K_\alpha X(t)$ so the above system transforms into the following:

$$\begin{cases} \dot{X}(t) = \begin{bmatrix} -2.5138 & 1.0374 \\ -1.0374 & -2.5138 \end{bmatrix} X(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} W(t), & t \neq t_k, \\ \Delta X(t_k) = X(t_k^+) - X(t_k^-) = \begin{bmatrix} -1.8 & 0 \\ 0 & -1.8 \end{bmatrix} X(t_k), & t = t_k, \\ Y(t) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} X(t) \end{cases}$$

and

$$\begin{cases} \dot{X}(t) = \begin{bmatrix} -2.0654 & 3.1748 \\ -3.1748 & -2.0654 \end{bmatrix} X(t) + \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix} W(t), & t \neq t_k, \\ \Delta X(t_k) = X(t_k^+) - X(t_k^-) = \begin{bmatrix} -1.8 & 0 \\ 0 & -1.8 \end{bmatrix} X(t_k), & t = t_k, \\ Y(t) = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix} X(t). \end{cases}$$

It is internally stable for all switching signals and has ρ-performance. Assume the switching interval is two seconds and

$$W(t) = \begin{bmatrix} \frac{\sqrt{2}}{2} \sin(\pi t) & \frac{\sqrt{2}}{2} \cos(2\pi t + 2) \end{bmatrix}^T.$$

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We consider the switching signal $s = \{1, 2, 1, 2, \dots\}$, $s \in S$. Figure 1 shows the state response of the above system without disturbance. Figure 2 shows the state response of the system with disturbance $W(t)$. Figure 3 shows the state response of the system with disturbance $W(t)$ from the initial state $X(0) = 0$.

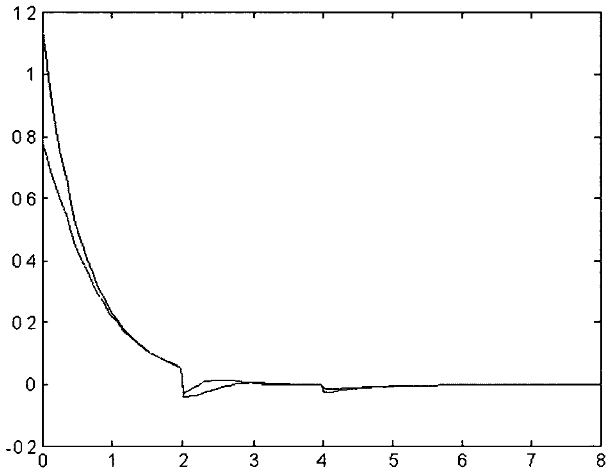

FIGURE 1. The state response of the controlled system without disturbance.

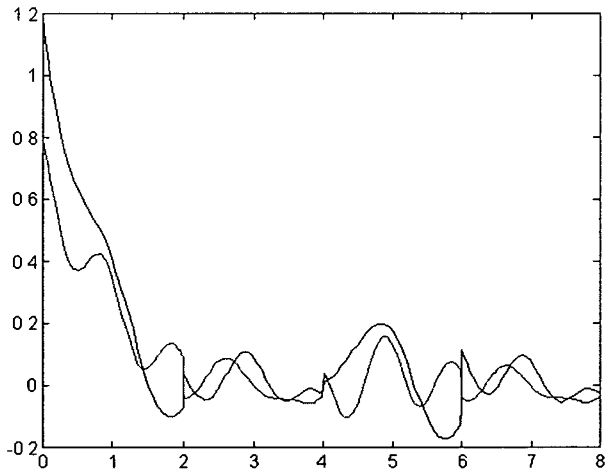

FIGURE 2. The state response of the controlled system with disturbance $W(t)$.

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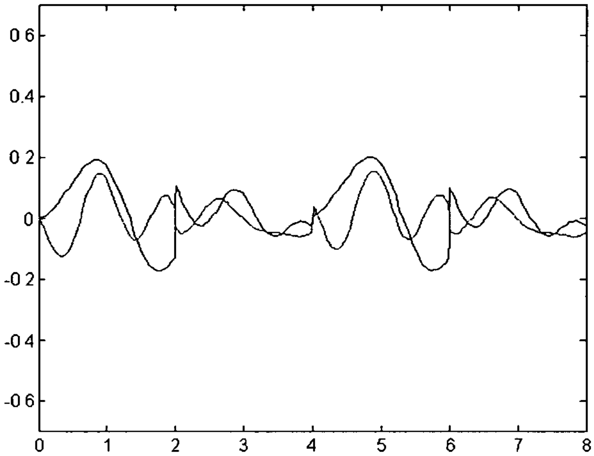

FIGURE 3. The state response of the controlled system with disturbance $W(t)$ from the initial state $X(0) = 0$.

5. Conclusions

In this paper, we have presented a sufficient condition for the robust stability of a class of impulsive switched systems with persistent bounded disturbance. We have shown that if the switched system is composed of a finite family of asymptotically stable subsystems and every switching state can last a sufficiently long time, then the system is internally stable. Furthermore, we discuss ρ-performance of the system, and some results have been derived.

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