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Some analytical critical points in 3-space

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This thesis is concerned with the local phase portrait (that is, the configuration of solution curves near the origin) of systems of the form

$$\begin{aligned} \frac{dx_1}{dt} &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + f_1(x_1, x_2, x_3) \\ \frac{dx_2}{dt} &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + f_2(x_1, x_2, x_3) \\ \frac{dx_3}{dt} &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + f_3(x_1, x_2, x_3) \ . \end{aligned} \tag{1}$$

Here x_1, x_2, x_3, t are real variables, a_{ij} $(i, j = 1, 2, 3)$ are real numbers, and f_1, f_2, f_3 are real analytic functions whose power series expansions at the origin contain no term of degree less than two. In the early part of the thesis, a general method for studying the local phase portrait of (1) at the critical point at the origin is given. The ideas for this method are drawn from Gomory [2] and Bendixson [1]. In vague terms, the method is to "inflate" the critical point into a sphere, and to associate a system of differential equations outside and on the sphere. The associated system also has critical points, which may themselves be "inflated"; and so on. The method is applied to give the local phase portraits of (1) in three cases:

(i) the matrix (a_{ij}) has one zero eigenvalue, and two real non-zero eigenvalues of opposite sign;

(ii) the matrix (a_{ij}) has one zero eigenvalue, and two real distinct eigenvalues of the same sign;

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(iii) the matrix (a_{ij}) has one zero eigenvalue, and two complex conjugate eigenvalues with non-zero real parts.

References

[1] Ivar Bendixson, "Sur les courbes définies par des équations différentielles", *Acta Math.* 24 (1901), 1-88.

[2] Ralph E. Gomory, "Trajectories tending to a critical point in 3-space", *Ann. of Math.* 61 (1955), 140-153.

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