# Appendix C
## Coordinates and momenta

The space–time coordinates $(t, x, y, z) \equiv (t, \vec{x})$ are denoted by the *contravariant* four-vector $x$, which is defined as:[1]

$$x^\mu \equiv (t, x, y, z) \equiv (x^0, x^1, x^2, x^3) \ . \tag{C.1}$$

The *covariant* four-vector is defined as:

$$x^\mu \equiv (t, -x, -y, -z) \equiv (x_0, x_1, x_2, x_3) = g_{\mu\nu} z^\nu \ , \tag{C.2}$$

where:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{C.3}$$

The three-vector is also often denoted as:

$$\vec{x} \equiv \mathbf{x} \tag{C.4}$$

The momentum vector is defined in the same way:

$$p^\mu = (E, p_x, p_y, p_z) \tag{C.5}$$

The scalar products are:

$$\begin{aligned} x^2 &= x_\mu x^\mu = t^2 - \vec{x}^2 \ , \\ p_1 \cdot p_2 &= p_1^\mu p_{2,\mu} = E_1 E_2 - \vec{p}_1 \vec{p}_2 \ , \\ x \cdot p &= tE - \vec{x} \cdot \vec{p} \end{aligned} \tag{C.6}$$

The derivative operator is:

$$\partial_\mu \equiv \frac{\partial}{\partial x_\mu} \equiv \left( \frac{\partial}{\partial t}, -\vec{\nabla} \right) \equiv \left( \frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z} \right) . \tag{C.7}$$

The Dalembertian operator is:

$$\nabla^2 \equiv \partial x_\mu \partial x^\mu = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \ . \tag{C.8}$$

[1] We shall follow the notations of Bjorken–Drell and Landau–Lifchitz.



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The electromagnetic four-vector potential is:

$$A_\mu = (\Phi, \vec{A}) \tag{C.9}$$

The electromagnetic field strength is:

$$F_{\mu\nu} = \frac{\partial}{\partial x_\nu} A_\mu - \frac{\partial}{\partial x_\mu} A_\nu \tag{C.10}$$

The electromagnetic and magnetic fields are:

$$\mathbf{E} = (F^{01}, F^{02}, F^{03}) \qquad \mathbf{B} = (F^{23}, F^{31}, F^{12}) \tag{C.11}$$

The gluon field tensor is:

$$G^a_{\mu\nu} = \frac{\partial}{\partial x_\nu} A^a_\mu - \frac{\partial}{\partial x_\mu} A^a_\nu + g f_{abc} A^b_\mu A^c_\nu \tag{C.12}$$

where $A^a_\mu$ is the guon fields and $a = 1, 2, \ldots 8$ are the colour indices. The electromagnetic covariant derivative is:

$$D_\mu = \partial_\mu + ieA_\mu \tag{C.13}$$

The gluon covariant derivative acting on the quark colour componet $\alpha, \beta =$ red, blue, yellow is:

$$(D_\mu)_{\alpha\beta} \equiv \delta_{\alpha\beta}\partial_\mu - ig \sum_a \frac{1}{2} \lambda^a_{\alpha\beta} A^a_\mu \, , \tag{C.14}$$

where $\lambda^a_{\alpha\beta}$ are eight $3 \times 3$ colour matrices.

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