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Optimization of the Shapes of Electrodes of Electrostatic Lenses

Frank H. Read

Emeritus Professor at School of Physics and Astronomy, University of Manchester, UK.

Abstract

We explore the benefits of using novel electrode shapes in electrostatic lenses. Two representative types of lenses are considered and significant improvements are obtained in their spherical aberrations.

Introduction

The majority of electrostatic lenses used as components of a wide variety of systems appear in the literature to be of the traditional designs, such as those dealt with in 1976 in the ‘standard’ book of lens properties [1]. The purpose of the present study is to show that considerable improvements in performance can be obtained by adding degrees of freedom, particularly by using lens elements that incorporate curved surfaces.

We shall consider two representative examples, double-aperture accelerating lenses and 3-tube einzel lenses, both having cylindrical rotation symmetry and therefore both dealt with by the CPO2D programs [2].

Constraints

As with all optimizations, the results are entirely dependent on the restraints imposed on the system. Without any restraints a system can be improved almost without limit, except of course the limits imposed by any theoretical limits. The following results are therefore arbitrary, in the sense that the restraints are arbitrary.

We shall be considering double-aperture accelerating lenses that have apertures of diameter D and 3-tube einzel lenses that have tubes of internal diameter D. All the results that involve distances scaled as D, so therefore in general the unit length D will not be mentioned (except in the description of the restraints below).

The restraints that we imposing in the present study are:
(1) the diameter of the outermost electrode (or containment electrode) is 5D,
(2) the working distance (from the last electrode to the image position), must be ≥ D,
(3) the image is real (so that the image size is not affected by putting an image electrode at the image position),
(4) the overall length S of the lens (first to last apertures) must be ≤ 6D,
(5) the electrode configuration is symmetrical about the center.
(6) only the 3rd.order spherical aberration is considered.

For the double aperture lenses we add the restraint:
(7) the accelerating ratio of energies is 10.

And for the 3-tube einzel lenses we add:
(8) the Gaussian focus must be at Q = 4D, measured from the physical center of the lens.
Of these constraints, number (5) is the least serious, and in fact is broken for the lens shown in Figure 1c, but is otherwise retained for simplicity.

Characterisation of the quality of the image

For an incoming parallel beam of diameter d_b the diameter of the disc of least confusion of the image can be shown to be

$$d_l = \frac{C_{s4}}{16 f_1^3} d_b^3$$

where C_{s4} (referred to the object) is the limit of $Cs_{obj}{*}M^4$ as $M \rightarrow 0$, as defined in Refs [1] and [2], and where we ignore higher-order aberrations. Throughout this study we use $d_b = 0.5{*}D$ and give the corresponding values of d_l/D in Table 1.

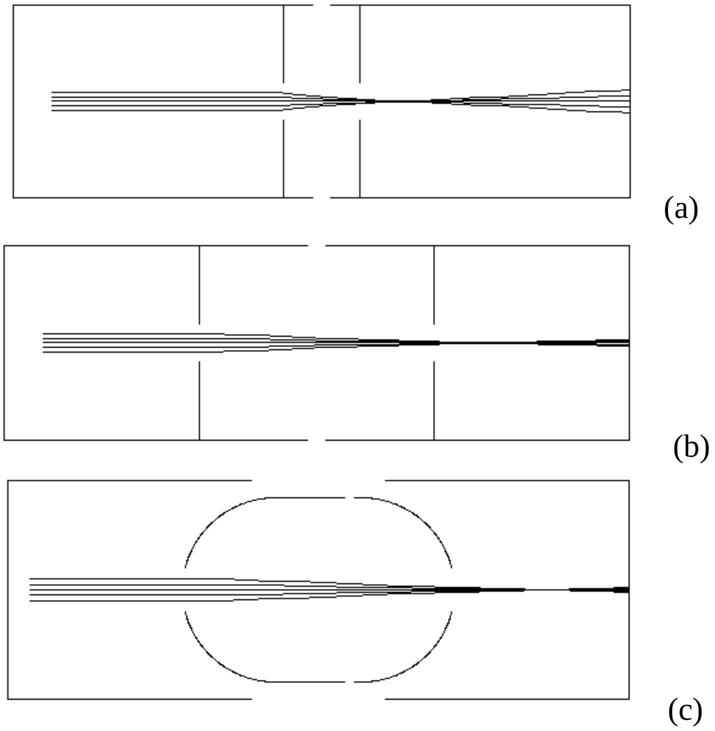

Figure 1. (a) The 'reference' 2-aperture lens, (b) The 2-element lens of flat apertures optimised for the aperture spacing, (c) A 2-element lens that includes parts with circular sections. The spot size in (c) is a factor of 8 smaller than that in (a).

Possible electrode shapes

We have considered electrodes that have curved sections, sometimes in combination with straight parts. There does not seem to be any technical limitations in manufacturing any reasonable type of curvature. A wide variety of curved shapes has been considered, including a range of conic sections (parabolic, elliptical and circular) and power-law dependences of the radial distance *r* on the axial distance *z*, or conversely *z* on *r*. These are easily modelled in the CPO program by means of the 'users equations' option [2] in which users simply type their equations into the databuilder.

Double-aperture lenses

These are accelerating lenses, as stated in restraint (7). Traditionally these lenses have flat electrodes and are characterised by the ratio β of the electrode spacing to the aperture radius. The two values of β considered in Ref. [1] were 1.0 and 2.0, of which β = 2.0 has the smaller aberrations. We therefore treat this lens as the 'reference' lens. It is illustrated in Figure 1a. The image position is at Q = 1.22 and so satisfies restraint (2).

This lens has the diameter of the disc of least confusion $d_l = 2.37\times10^{-3}$. The potential penetration at the image position is $\Delta V = 0.035$, which is small enough that the image would be almost unchanged if a target screen were to be put at $Q = 1.22$.

The next step is to optimize the lens by varying the spacing parameter β, subject of course to the chosen constraints. The resulting lens is shown in Figure 1b. The aberrations improve as the overall length S is increased, but we have to stop at $S = 6$ in order to satisfy restraint (4). The image position is at $Q = 4.556$ and so satisfies restraint (2). Clearly this is a considerable improvement on the reference lens. Note that although the overall length satisfies restraint (4), it is unusually large (and so perhaps restraint 4 is not strong enough). The value of d_l is 6.12×10^{-4}, which is almost a factor of 4 better than that of the 'reference' lens.

As a final step we consider electrodes that are not flat. After considering a wide variety of shapes for the curved parts it was found that elliptical sections seem to be the best, but that circular sections are the most convenient since they are only marginally worse.

In this optimization we again have to stop when the overall spacing S reaches the maximum value allowed by restraint (4). The resulting lens is shown in Figure 1b. The radius of the circular part is 2.1, which matches the radius of the straight section. The gap between the two halves of the lens has been allowed to be non-central in this optimization and is shifted by 0.7. The image position is at $Q = 5.201$ and so satisfies restraint (2). The other parameters are given in Table 1. The value of d_l is now 3.02×10^{-4}, which is a further improvement by a factor of 2 and so is almost a factor of 8 better than the 'reference' lens.

3-tube einzel lenses

The 'reference' lens is shown in Figure 2a. The inner and outer diameters of the tubes are 1 and 3, the length of the inner tube is 2.4 and the gaps between the tubes are 0.5 (all of which are of course rather arbitrary, but not untypical). A voltage of 2.766 on the inner element gives the object position specified in constraint 8.

The optimized geometry, subject to the constraints, is shown in Figure 2b. The inner and outer tube diameters have not been changed, except that rounded edges have been added to the outer tubes, as shown. Once again it has been found that circular sections are only marginally worse than the best curves of more complicated shapes and so are preferred for their convenience. The section here is an arc of a circle that is continuous at one end with the straight inner tube surface, has a radius of 1.625, and so meets the straight outer surface at an acute angle. The separation of the pointed ends of the outer tubes is 5. The length of the inner tube is short, 0.2, since it has been found consistently that the aberrations decrease as this length is decreased. The value of d_l is now 1.17×10^{-3}, which is a factor of 6 better than that of the 'reference' einzel lens. The voltage on the inner element is 5.477.

Conclusions.

Although the choice of constraints and lenses is rather arbitrary, and only two types of lens have been considered, some conclusions can be drawn.

Firstly, it is obvious that improvements in aberration properties can always be made when degrees of freedom are added to a design, but here we have found that the improvements can be surprisingly large, giving reductions in image sizes by factors of 6 and 8 for the present lenses.

Secondly, the source of the improvements for both types of lens studied here is clearly connected to the reduction in the strengths of the fields inside the lenses, which is achieved by extending the overall lengths of the lenses. In the case of the tube lens the shortness of the inner tube also contributes to this.

Thirdly, the most critical regions are those near to the entrance and exit positions, where the rate of change of field strength tends to be largest, and so it has been found that improvements are obtained by shielding these regions as much as possible. This is achieved for the aperture lens by using rounded parts that are concave to the centre of the lens, and for the tube lens by giving the edges of the tubes in those regions a large radius of curvature.

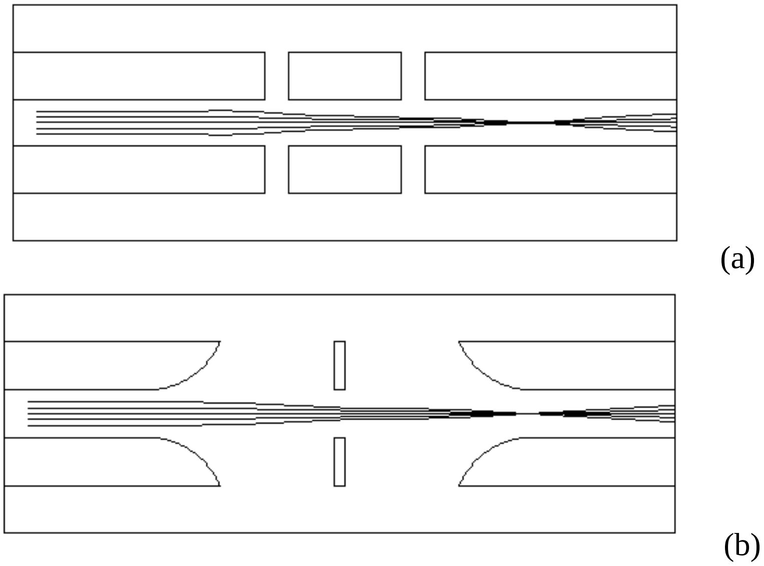

Figure 2. (a) The ‘reference 3-tube einzel lens. (b) 3-tube einzel lens that includes parts with circular sections. The spot size in (b) is a factor of 6 smaller than that in (a).

Lens	d_l	ΔV
2-aperture reference lens	2.37×10^{-3}	0.0350
2-aperture lens, optimised S	6.12×10^{-4}	0.0035
2-aperture lens, curved electrodes	3.02×10^{-4}	0.0064
3-tube einzel lens, reference	7.42×10^{-3}	0.0001
3-tube einzel lens, curved electrodes	1.17×10^{-3}	0.0026

Table 1. Values of the key parameter α, the diameter of least confusion d_{lc} and the potential penetration ΔV for the lenses considered.

References:

[1] E Harting and FH Read, “Electrostatic Lenses”, (Elsevier, Amsterdam), 1976.
[2] Available at www.electronoptics.com, including free demonstration version.