ON THE ADOPTION OF EMPIRICAL CORRECTIONS TO WOOLARD'S NUTATION THEORY

D. D. McCarthy, P. K. Seidelmann and T. C. Van Flandern
U. S. Naval Observatory
Washington, DC 20390

Abstract

Commission 4 of the International Astronomical Union has deferred the question of revisions to the constants and theory of nutation in anticipation that there might be recommendations from Symposium No. 78 in Kiev. The present rigid-Earth theory of nutation does not adequately represent current precise astronomical observations for the major nutation terms. Discrepancies between the presently adopted theory and observations can accumulate to 0.″1 in right ascension and significantly affect the determination of UT1 and materially influence the derivation of the new fundamental catalog of star positions and proper motions, FK5 There appears to be no obvious choice for a non-rigid-Earth model at present. The analysis of solid-Earth tides shows nutation coefficients in substantial agreement with astronomical observations and these values have been used in the reduction of radio interferometric and laser ranging observations. In the absence of a non-rigid-Earth model which can satisfy all requirements it is suggested that the coefficients found from the investigation of solid-Earth tides be adopted as a working standard until such a model can be adopted as a basis for nutation.

The nutation of the Earth's axis in space may be described by a dynamical theory of the motion and numerical constants relating the theory to the observed motion. These constants may be determined directly by observations or by a combination of other well-known astronomical constants. These include the ratio of the masses of the Moon and the Earth and the dynamical ellipticity, (C-A)/C, where C and A are the moments of inertia about the polar axis and an equatorial axis, respectively. For a rigid-Earth theory of nutation, a single constant of nutation in obliquity can be adopted. For a non-rigid-Earth theory of nutation a single constant of nutation is not adequate and the meaning of the corresponding term is not the same.

E. P. Fedorov, M. L. Smith and P. L. Bender (eds.), Nutation and the Earth's Rotation, 117–124.

The IAU adopted the rigid-Earth theory of Woolard (1953) along with the constants listed in his classical work. These constants are related by the rigid-Earth theory to a value for the coefficient of the principal term in obliquity. The value used by Woolard was based on observational results and not on the theoretical value determined from other astronomical constants. The discrepancy between the theoretical and observational values of this coefficient has led to investigations of the theory of nutation for a non-rigid Earth (Jeffreys & Vicente, 1957a and 1957b; Molodensky, 1961; Pederson, 1967; Kakuta, 1970). Another source of the nutation coefficients is the study of solid Earth tides (Melchior, 1972).

Recent astronomical observations indicate that the currently adopted description of nutation is no longer adequate for precise investigations. This paper will be limited in scope to the four main nutational terms, namely the principal (18.6 year) and semi-monthly terms due to the action of the Moon and the annual and semi-annual terms due to the action of the Sun. We shall not attempt to discuss the methods involved in the various determinations of the nutational coefficients, either theoretical or observational. However, a comparison of the numerical values obtained in these different ways illustrates the options which we feel are viable for any modification of the current theory or numerical coefficients describing the nutation of the Earth's axis.

In the discussion of specific numerical values, some important facts must be kept in mind. The theoretical value of the rigid-Earth coefficient of the principal term in obliquity, and hence all of the theoretical coefficients, are directly dependent on the dynamical ellipticity of the Earth, which may be determined from the constant of lunisolar precession. A change in this constant will lead to changes in the theoretical nutation coefficients. Secondly, comparison between observations and theory may be confused by questions as to whether the coefficients refer to the description of the nutation of the Earth's instantaneous axis of rotation, to the axis of figure, or to the Eulerian position of the pole. Specifically, the question is whether the theory contains the additional terms for the axis of figure given by Woolard (1953) in his expression (54). In this paper all coefficients refer to the nutation of the instantaneous rotational axis used by Woolard, because the majority of the observations were published in terms of that rotational axis. For the complete reduction of time and latitude observations, the last six terms of each of Woolard's expressions (70) must be included in the theory.

Astronomical determinations of the coefficients along with their mean errors are shown in Tables 1 and 2. The values of the coefficients of the principal term obtained since 1920 are shown in Table 1. Table 2 shows the values of the semi-annual and semi-monthly coefficients determined from astronomical observations. Systematic errors have prevented accurate astronomical determinations of the coefficients of the annual term.

https://doi.org/10.1017/S0074180900031909 Published online by Cambridge University Press

Table 1

Astronomical Determinations of the Coefficients of the Principal Nutation (units are arcseconds)

Obliquity	Longitude	Reference
9.214 ± 0.004	6.837 ± 0.004	Wako & Yokoyama, 1976
9.212 ± 0.001	6.838 ± 0.001	Wako & Yokoyama, 1976
9.1980 ± 0.0018	6.8522 ± 0.0031	Fedorov, 1963
9.206 ± 0.004	6.858 ± 0.004	McCarthy, 1972
9.1970 ± 0.0010	6.8476 ± 0.0010	Tarady, 1969
9.200 ± 0.004	6.826 ± 0.004	Yokoyama, 1975
9.212 ± 0.005	6.838 ± 0.005	Yokoyama, 1975
9.212 ± 0.003	6.831 ± 0.003	Yokoyama, 1975
9.210	6.832	Guinot & Feissel, 1975
	6.819	Billaud, 1975
9.208 ± 0.007	6.846 ± 0.007	Jaks, 1977
9.1985 ± 0.0051		Hattori, 1951
9.2073 ± 0.0041		Hattori, 1951
9.1967 ± 0.0043		Hattori, 1951
9.1955 ± 0.0034		Hattori, 1951
9.2108 ± 0.0019		Kulikov, 1949
9.206 ± 0.0104		Morgan, 1943
9.2066 ± 0.0062		Jones, 1939
9.2066 ± 0.0082		Jackson, 1930
9.2069 ± 0.0030		Przybyllok, 1920
9.2050 ± 0.0017	6.8409 ± 0.0025	MEAN VALUE

Table 2

Astronomical Determinations of the Coefficients of the Semi-Annual and Semi-Monthly Nutation (units are arcseconds)

Obliquity	Longitude	Reference
Semi-Annual		
0.578 ± 0.004	0.533 ± 0.0004	Fedorov (1958, 1959)
Semi-Monthly		
0.0922 ± 0.0016	0.0845 ± 0.0015	McCarthy (1976)
0.0944 ± 0.0015		Gubanov (1969)
0.0894 ± 0.0010	0.0866 ± 0.0010	Fedorov (1958, 1959)
0.0912 ± 0.0015	0.0859 ± 0.0010	MEAN VALUE

Various values for the coefficients of the nutation terms are compared in Table 3. Woolard's values are those based on the rigid-Earth theory with a nutation constant of 9''2100. Astronomical determinations of the coefficients along with their mean errors are shown. These are the weighted means of the determinations as given in Tables 1 and 2. Column three of Table 3 shows the coefficients which would result if the rigid-Earth theory were to be kept and the coefficients were adjusted to agree with the astronomical determination of the principal coefficient in obliquity. Melchior's determinations of the coefficients from the investigation of solid-Earth tides are shown in column 4.

Theoretical values of the coefficients are shown for various non-rigid-Earth models in the remainder of Table 3. Using currently-adopted constants, a theoretical value for the principal nutation coefficient in obliquity for a rigid Earth is found to be 9''2273. This value can then be used to find the remainder of the constants in the table. A change in the value of the luni-solar precession of +1''10 per century will lead to a value of 9''2293. The resulting coefficients are shown for both values of the principal nutation coefficient in obliquity. The Earth models represented are as follows:

JVI - Jeffreys & Vicente (1957a)
JVII - Jeffreys & Vicente (1957b)
MI, MII - Molodensky (1961)
P - Pederson (1967)
K - Kakuta (1970)

Table 3 shows the disagreement between the recent observational results and the currently adopted constants. Also, from Table 3 it can be seen that a change only in the principal nutation coefficient in obliquity will not suffice to improve the agreement with astronomical observations if the current rigid-Earth theory is maintained.

The coefficients of Melchior are seen to be in reasonable agreement with astronomical results. The worst agreement is in the semi-annual terms which are based on one astronomical determination. Without considering the annual term the best theoretical fit to the astronomical and solid-Earth tide determinations is obtained when the rigid-Earth constant 9.2293 is used. Thus it appears that, in general, theoretical nutation coefficients will be in better agreement with observational results when an improved value for the principal nutation coefficient in obliquity determined with an improved value of luni-solar precession is used. With a change of +1.10 in the luni-solar precession the MI model is seen to be the best theoretical fit to both the astronomical observations and Melchior coefficients. However, the JVI and P models are also in reasonable agreement. Statistically, there is no significant difference in the fit of Melchior's coefficients, MI, JVI, or P models to the astronomical observational results, although the Melchior coefficients are the best overall. Looking at individual

Table 3. Nutation coefficients. N is the coefficient of the principal (18.6 year) term in obliquity for a rigid Earth. The upper line of theoretical values for each component is for N=9.2273, and the lower line is for N=9.2293 (units are arcseconds).

Term	Woolard	Astr. Obs.	Rigid Earth: N=9.2050	Melchior	Theoretical Values: Rigid Earth	JVI	JVII	MI	MII	P	K
18.6 yr Obliquity	9.2100	9.2050 ±.0017	9.2050	9.2014	9.2273 9.2293	9.1972 9.2044	9.2145 9.2217	9.1951 9.2023	9.1985 9.2057	9.1974 9.2045	9.1766 9.1838
18.6 yr Longitude	6.8584	6.8409 ±.0025	6.8547	6.8408	6.8713 6.8728	6.8327 6.8397	6.8556 6.8626	6.8312 6.8382	6.8356 6.8426	6.8343 6.8412	6.8083 6.8153
1.0 yr Obliquity				0.0056							
1.0 yr Longitude	-0.0502		-0.0503	-0.0579	-0.0500 -0.0500						
0.5 yr Obliquity	0.5522	0.578 ±.004	0.5528	0.5724	0.5500 0.5501	0.5706 0.5707	0.5377 0.5378	0.5710 0.5711	0.5686 0.5687	0.5700 0.5701	0.5773 0.5775
0.5 yr Longitude	0.5066	0.533 ±.004	0.5072	0.5230	0.5046 0.5047	0.5212 0.5213	0.4859 0.4860	0.5216 0.5217	0.5196 0.5197	0.5206 0.5207	0.5281 0.5283
13.7 days Obliquity	0.0884	0.0912 ±.0015	0.0844	0.0906	0.0886 0.0886	0.0914 0.0914	0.0912 0.0912	0.0913 0.0913	0.0908 0.0908	0.0914 0.0914	0.0945 0.0945
13.7 days Longitude	0.0811	0.0859 ±.0010	0.0811	0.0828	0.0813 0.0813	0.0835 0.0835	0.0835 0.0835	0.0834 0.0834	0.0831 0.0831	0.0837 0.0837	0.0866 0.0866

terms it can be seen that the P model gives the best agreement while K fits the semi-annual term best. There is no obvious choice for the best Earth model.

The question now is, what action should be taken by the IAU concerning a theory for nutation? Four apparent options will be discussed. The first option is to retain the present theory and constant of nutation. This has the advantage of maintaining continuity with the past. It has the disadvantage that the observational data indicates that significant corrections are required. In practice the determination of UT1 based on optical observations, and lunar laser ranging and radio interferometric reductions, are sufficiently accurate that their usefulness is degraded with the present theory of nutation. If this first option were followed, it might result in degradation of other efforts as well, such as the construction of the FK5. Moreover, there is the likelihood that numerous different nutation coefficients would be used in some critical applications, due to the absence of a suitable standard theory. In view of the fact that corrections to the constant of precession and other astronomical constants have already been adopted by the IAU, and corrections to the nutation theory could be introduced at the same time as the other astronomical constants, it appears that the retention of the current theory and constant of nutation would be ill-advised.

If we decide that the current constant of nutation should be changed, then the question becomes, what should the change be? The second option available is to merely change the constant of nutation, but to retain the present theory. Table 3 shows the coefficients of the terms if such a change were to be made. This is a rather easy change to make, and would be consistent with the change in the constant of precession. But the observational data indicate that this change would also be inadequate. Therefore, since this would fail to satisfy the needs of the optical astronomical users, the requirements for the determination of UT1, and the reduction of laser ranging data and radio interferometric data, we feel that this option should also be rejected.

The third option is to adopt a complete new non-rigid-Earth model for the theory of nutation. This approach appears to be the most desirable for a new theory of nutation. Such a theory should be based on the newly-adopted IAU constants for the Earth, should be compatible with the observational data to the accuracy of the observations, and should be theoretically consistent with other observed parameters. It is hoped that the papers presented at this symposium will indicate whether a suitable theory is available which meets the above cited criteria.

The fourth option is to adopt the nutation coefficients based on a study of the solid-Earth tides by Melchior. These coefficients appear to be reasonably consistent with the astronomical observational data, and are already being used in some cases where greater accuracy is required. Although theoretical relationships between coefficients are lacking in this approach, it is similar to a complete theory in which eight parameters are adjusted.

In summary, it appears that at a time when we are adopting new astronomical constants, including a new precession constant, a new dynamical time scale, and new methods for reduction to apparent place, it would be a mistake not to correct the theory and constant of nutation, which are known to be at variance with the observational data. It should be recognized that while a single constant of nutation is sufficient for a rigid-Earth theory of nutation, it is not satisfactory for a non-rigid-Earth model. The adoption of a theory and constants of nutation based on the non-rigid-Earth model, and satisfying the observational data, would be desirable. If that is not possible, or if no theory can be agreed upon, we urge the adoption of the corrections based on solid-Earth tides, so as to provide a working standard for determination of UT1, the reduction of lunar laser ranging data and radio interferometric data, the construction of FK5, and other new high-precision requirements of the future.

References

Billaud, G.: 1975, Private communication in Wako & Yokoyama (1976).

Fedorov, E. P.: 1958, Nutation as Derived from Latitude Observations, IAU Moscow, as reported in Jeffreys (1958).

Fedorov, E. P.: 1959, Astron. J., 64, 81.

Fedorov, E. P.: 1963, Nutation and Forced Motion of the Earth's Pole (The MacMillan Co., New York).

Gubanov, V. A.: 1969, Soviet Astronomy - A. J., 13, 529.

Guinot, B. and Feissel, M.: 1975, Private communication in Wako & Yokoyama (1976).

Hattori, T.: 1951, Pub. Astron. Soc. Japan, 3, 126.

Jackson, J.: 1930, Monthly Notices Roy. Astron. Soc., 90, 733.

Jaks, W.: 1977, Pub. Inst. Geophys. Polish Acad. Science, F 2 (108), 75.

Jeffreys, H. and Vicente, R. O.: 1957a, Monthly Notices Roy. Astron. Soc., 117, 142.

Jeffreys, H. and Vicente, R. O.: 1957b, Monthly Notices Roy. Astron. Soc., 117, 162.

Jeffreys, H.: 1958, Monthly Notices Roy. Astron. Soc., 119, 7.

Jones, H. S.: 1939, Monthly Notices Roy. Astron. Soc., 99, 211.

Kakuta, C.: 1970, Pub. Astron. Soc. Japan, 22, 199.

Kulikov, K. A.: 1949, Astron. J. USSR, 26, 165.

McCarthy, D. D.: 1972, Analysis of Washington Latitude Variations from 1915 to 1970 Using the Photographic Zenith Tube, Dissertation, University of Virginia, Charlottesville, Virginia.

McCarthy, D. D.: 1976, Astron. J., 81, 482.

Melchior, P.: 1972, "Past and Future of Research Methods in Problems of the Earth's Rotation," in Rotation of the Earth, IAU Symposium No. 48, P. Melchior and S. Yumi, eds. (D. Reidel Publishing Co., Dordrecht, Holland).

Molodensky, M. S.: 1961, Commun. Obs. Royal Belgique, No. 188.

Morgan, M. R.: 1943, Astron. J., 50, 125.

Pederson, G. H.: 1967, The Effect of the Fluid Core on Earth Tides, Thesis, University of Waterloo, Ontario.

Przybyllok, E.: 1920, Zentral Bureau der Internationalen Erdwessung Neue Folge der Veroffentlichungen, No. 36.
Tarady, V. K.: 1969, Astrometrija i Astrofiz, No. 2, 7.
Wako, Y. and Yokoyama, K.: 1976, "Nutation Terms Derived from Time and Latitude Observations," paper presented to XVI General Assembly of IAU.
Woolard, E. W.: 1953, Astron. Papers Prepared for the Use of the American Ephemeris and Nautical Almanac, 15, Part 1.
Yokoyama, K.: 1975, private communication in Wako & Yokoyama (1976).