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On the predictability horizon in Impact Monitoring of Near Earth Objects

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Abstract. The Impact Monitoring (IM) of Near Earth Objects (NEOs) is a fundamental part of the planetary defense strategy. Current NEO IM systems (Aegis, NEODyS and Sentry) scan the Confidence Region (CR) of each observed object looking for Virtual Impactors (VIs) with a time horizon of about 100 years. This procedure is performed regardless of the uncertainty with which the orbit of the object is known, and without considering whether a scattering encounter is present in the propagation time span. In view of the likely future increase of the IM workload due to higher future NEO discovery rates, it might be more reasonable to adapt the predictability horizon of the impacts to each object, taking into account the orbit uncertainty and the close encounters experienced. In this paper we discuss the problem of estimating a reasonable predictability horizon when multiple close encounters are present and start to address the problem proposing a formal mathematical definition of scattering encounter.

Keywords. Near Earth Objects, Impact Monitoring, dynamics, close encounters, scattering encounter

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1. Introduction

The Impact Monitoring (IM) of Near Earth Objects (NEOs) is a relatively young field of research, considering that 25 years ago precise algorithms to compute the probability of an impact with the Earth did not exist; CLOMON, the first automatic IM system, was born in 1999 (Milani *et al.* 1999, 2000a; Chesley and Milani 2000; Tommei 2019, 2021).

Recent years have seen a strong interest in asteroid hazard:

• since 2015 three systems for the detection of imminent impactors (small asteroidal objects detected a few days before the possible impact with the Earth) have been developed: Scout at JPL/NASA (Farnocchia *et al.* 2015), NEORANGER at University of Helsinki (Solin and Granvik 2018) and NEOScan at University of Pisa/SpaceDyS (Spoto *et al.* 2018);

• the year 2020 saw the increase of IM operational systems: in addition to the two historical systems, CLOMON2 at University of Pisa/SpaceDyS and Sentry at JPL/NASA (Milani *et al.* 2005a; Roa *et al.* 2021), the European Space Agency (ESA) started its own system, Aegis;

• on 2022, September 26th, the DART spacecraft successfully impacted on Dimorphos, the moon of the Near Earth Asteroid (NEA) (65803) Didymos, showing the capability of deflecting a small body (Cheng *et al.* 2018, 2023; Rivkin and Cheng 2023);

• the European Space Agency (ESA) will launch in 2024 the Hera mission which will visit the Didymos system to investigate the effects of the DART impact and to characterize the properties of the system (Michel *et al.* 2022).

The IM science, in addition to being useful for the planetary defense, is a fascinating field of research involving astronomy, physics, mathematics and computer science. Current and future challenges in this activity include the continuous need to check and upgrade, when necessary, the dynamical model, including non-gravitational perturbations where appropriate, the extension of the time span covered by IM for objects with very well determined orbits and, last but not least, the likely huge increase of the workload that will be caused by future large-scale surveys. The related issue of the so called imminent impactors (Mochi and Tommei 2021) is out of the scope of the present paper.

Here we are interested in the dynamics of NEOs influenced by multiple close encounters with the Earth. This is the main reason why the long term computation of an orbit of a NEO is not a simple task.

In Section 2 we briefly summarize the problems arising in the determination of the orbits of NEOs and the solutions adopted to handle the orbital uncertainty. Section 3 is devoted to the problem of the predictability horizon after multiple close encounters; in particular, we give a formal mathematical definition of scattering encounter. In Section 4 we draw some conclusions.

2. NEO dynamics

As already said, the computation of the orbit of a NEO is not a simple task because, in many cases, the observations are few and the dynamics is strongly affected by close encounters with the Earth and other planets. Even if a preliminary orbit is available, the uncertainty can be large and the best way to proceed is to consider a set of orbits belonging to a Confidence Region (CR, a subset of the 6-dimensional space of the orbital elements) where the astrometric residuals are acceptable. Such orbits, obtained with an appropriate sampling of the CR, are called Virtual Asteroids (VAs, Milani *et al.* 2000b); they can have a very low Minimum Orbit Intersection Distance with our planet (MOID, Gronchi 2005; Gronchi and Tommei 2007) and thus be dangerous. The goal of IM is to understand whether the CR contains some Virtual Impactor (VI), that is a subset of initial conditions bringing to an impact with the Earth in the future.

Under appropriate conditions, it may happen that the CR contains multiple VIs at the same date in the future. What happens most often in this case is that each VI has a dynamical path to its collision different from those of the other VIs.

For example, when (99942) Apophis was discovered and the timing of its 13 April 2029 approach to the Earth was still rather badly constrained from the then available observations, there were at least three separate VIs for 13 April 2036, that is, after the Earth would have made 7 revolutions about the Sun following the 2029 encounter:

• via the 6/7 mean motion resonance, with Apophis making 6 revolutions about the Sun (Chesley 2006);

• via the 5/7 resonance, with Apophis making 5 revolutions;

• and via the 4/7 resonance, with Apophis making only 4 revolutions.

Clearly, between 2029 and 2036 the CR has wound up itself about the Sun multiple times, and the three VIs define disconnected collision subsets; in this case each VI is a connected component of the CR.

Once a *representative of a VI*, i.e. a particular initial condition that leads to a collision, has been identified, an Impact Probability (IP) needs to be computed: in general terms, the IP of a VI is proportional to the volume of the VI in the orbital elements space. If we are looking for small IPs, the sampling should be very dense; but the real problem is how to ensure completeness to the VI search, taking into account the computational costs

(Del Vigna *et al.* 2019a). One class of methods, including Monte Carlo (MC) and statistical ranging, uses random sampling of the CR to study the probabilistic distributions of the orbits through the swarm of VAs.

When we have to manage a large catalog of objects and small probabilities with a small number of VAs, it is more cost-effective to sample the CR with a geometrical object, such as a smooth manifold. At the end of 1990s in Pisa a class of 1-dimensional sampling methods based on the Line Of Variations (LOV) was developed (Milani *et al.* 2005b): the LOV is a differentiable curve, which can represent, in some cases, the spine of the CR; it is sampled generating a swarm of indexed VAs, allowing the interpolation between consecutive VAs. The sampling could be done uniformly using the curve parameter; recently, the possibility to collect points at fixed step in probability has been introduced (Del Vigna *et al.* 2020).

An important and expensive (in terms of computational costs) step in IM is the propagation of the VAs, a set of orbits of the order of thousands, for 100 years or more. Therefore, in the light of the expected increase in NEO discoveries (Jones *et al.* 2018; Cibin *et al.* 2016), we wonder whether it is possible to identify an algorithm that tailors the propagation horizon to each object. In the NEO population there are objects with very different types of motion, and for which the OD procedure results in very different orbital uncertainties. If the CR is either large due to poor observational record, or becomes large due to intervening encounters, there is the possibility for the VAs to follow different dynamical paths, going through different sequences of close encounters, so that the predictability horizon (after which the dynamics will be even more chaotic, Spoto and Milani 2016) becomes short.

An algorithm that would allow to identify the close encounter that causes the subsequent unpredictability of the orbit (the so called *scattering encounter*) would be useful both in the daily IM operations and also for the long term IM, hat involves the analysis of objects with a very well constrained orbits, including, if needed, also the determination of non-gravitational accelerations, like the Yarkovsky effect (Bedini and Tommei 2023). But before designing such an algorithm it is necessary to give a formal definition of scattering encounter, that will be presented in the next section.

3. The scattering encounter

In order to discuss the dynamics at close encounters, it is useful to introduce the analytical theory of close encounters and the b-plane (Öpik 1976; Greenberg *et al.* 1988; Valsecchi *et al.* 2003). The b-plane is the plane centred on the planet and perpendicular to the pre-encounter planetocentric velocity vector $\vec{U}$. In the theory, the encounter is modelled as an instantaneous rotation of $\vec{U}$, taking place when the small body crosses the b-plane; the crossing point has coordinates ξ, ζ. The magnitude of $\vec{U}$, together with the angles θ, ϕ defining its direction, are all given by simple functions of a, e, i, the pre-encounter semimajor axis, eccentricity and inclination of the orbit of the small body; the encounter rotates $\vec{U}$ into $\vec{U}'$, whose magnitude is the same, but its direction is given by the angles θ', ϕ' (θ', ϕ' are explicit functions of ξ, ζ), allowing the computation of the post-encounter values a', e', i' of the small body orbit. The two b-plane coordinates are chosen so that ξ is the local MOID, while ζ is related to the timing of the encounter, i.e. to whether the small body passes by the planet earlier or later with respect to the closest possible distance associated to the local MOID.

Using the so-called wire approximation (Valsecchi *et al.* 2003), we consider a continuous stream of particles, all with the same U, θ, ϕ, ξ (i.e. all with the same a, e, i and MOID), differing only for ζ; in this way, we can model the projection of the LOV onto the b-plane, and, using simple analytical formulae, we can identify the regions on the b-plane of a certain encounter that lead to post-encounter orbits characterized by a a given

Table 1. The encounter sequences of (410777) 2009 FD and (101955) Bennu; for details, see the text.

(410777) 2009 FD				(101955) Bennu			
Date	$b(r_\oplus)$	$\sigma_\zeta(r_\oplus)$	$\dot\sigma_\zeta(r_\oplus/y)$	Date	$b(r_\oplus)$	$\sigma_\zeta(r_\oplus)$	$\dot\sigma_\zeta(r_\oplus/y)$
2015/10/29.4	983.7	0.0021		2005/09/20.4	777.9	0.0002	
2063/03/29.3	304.5	0.2158	$4.5{\cdot}10^{-3}$	2054/09/30.0	922.8	0.0017	$3.1{\cdot}10^{-5}$
2064/10/26.4	625.1	0.2428	$1.8{\cdot}10^{-2}$	2060/09/23.0	117.6	0.0024	$1.1{\cdot}10^{-4}$
2136/03/28.3	511.3	18.30	$2.5{\cdot}10^{-1}$	2080/09/22.0	365.4	1.15	$5.7{\cdot}10^{-2}$
2185/03/29.4	210.4	208.7	3.9	2135/09/25.4	47.2	12.5	0.21

semimajor axis and thus by a given period (Valsecchi *et al.* 2018). That is, we can find the regions of the pre-encounter b-plane leading to specific *resonant returns*.

In recent years there were, among others, a couple of successful attempts of long term IM for two remarkable objects, (410777) 2009 FD (Spoto *et al.* 2014) and (101955) Bennu (Chesley *et al.* 2014)†. In both cases the OD involved solving for seven parameters, the six orbital elements and the Yarkovsky acceleration, that is essential in the study of the long term dynamics of NEOs.

Table 1 summarizes the sequences of encounters with the Earth within 0.5 au for the two NEAs, that are presented side by side: on the left 2009 FD, on the right Bennu. In both cases there are 4 intervening encounters before the final, scattering one; the Table lists the date of closest approach for the nominal VA, the impact parameter $b=\sqrt{\xi^2+\zeta^2}$, the 1-σ uncertainty along ζ, in Earth radii, as well as its variation in time in the time span between the encounters, in Earth radii per year.

The encounters of 2009 FD take place alternatively at either node, and only the scattering encounter of 2185, in which also an Earth collision was possible at the time of Spoto *et al.* (2014), is a resonant return from 2136‡. In the case of Bennu, all the encounters take place at the same node, and the scattering encounter of 2135 does not involve an Earth collision; nevertheless, the scattering of nearby VAs in 2135 is so strong that the exploration of the subsequent impact hazard has to be done through statistical means (Chesley *et al.* 2014), a feature that has not changed with the new data acquired after 2014 (Farnocchia *et al.* 2021). For both asteroids, the sequence of encounters preceding the scattering one leads to a very large increase of σ_ζ, of about five orders of magnitude in less than two centuries.

What do these two examples tell us? Essentially that, if we want to perform a long term propagation, it is crucial to analyze the close encounters and, in particular, identify the scattering encounter; for very well determined orbits, like the ones just discussed, it is an encounter after which the impact hazard can only be computed statistically, since the dynamics of the object has become chaotic. Since for an encounter to be considered a scattering one depends on the subsequent uncertainty of the orbit, it can happen that for not well constrained orbits also a not so deep encounter could turn out to be a scattering one.

We now propose a topological definition of scattering encounter, which will help us to develop an operational definition.

Definition 1. A close encounter of a minor body with the Earth is said to be a **scattering encounter** if the post-encounter CR ceases to be simply connected.

† In both cases, subsequent observations have led to the exclusion of many VIs (Del Vigna *et al.* 2019b; Farnocchia *et al.* 2021).

‡ The value of σ_ζ in the Table refers to the nominal VA, and differs from the stretching given in Table 2 of Spoto *et al.* (2014) since the latter refers to the 2185 VI, at about 1 σ_ζ from the nominal; as Fig. 6 of Spoto *et al.* (2014) shows, on the b-plane of 2185 the stretching varies appreciably along the LOV.

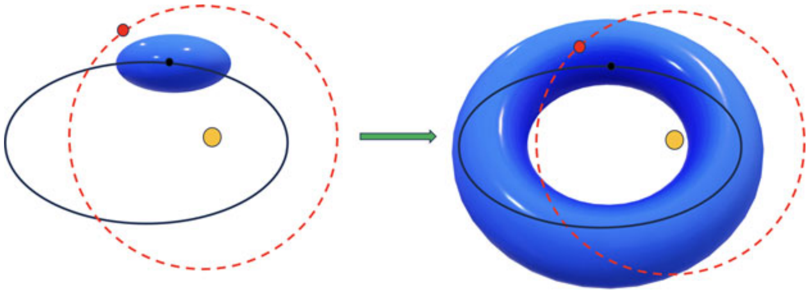

Figure 1. 3D representation of the effect of a scattering encounter.

Since it is not easy to visualize the situation in dimension 6, in Fig. 1 we have sketched the situation in dimension 3. The dashed red line is the Earth orbit, while the black continuous line is an hypothetical minor body orbit; on the left we have the nominal orbit of the asteroid and its CR, sketched with a blue ellipsoid before a close approach; we know that the geometrical shape could be more complex, like a bananoid for example (Vavilov 2020), but in any case a simply connected object. Due to encounters the CR grows, stretching along the orbit, until after the scattering encounter it changes completely the topology: from a simply connected confidence region, homotopic to a point, it becomes an object having a non-trivial fundamental group, like the torus in the figure. The underlying meaning is essentially that the uncertainty region, after a scattering encounter, contains orbits having periods in different resonances with the Earth, due to the large variation of the post-encounter semimajor axis throughout the CR.

4. Conclusions and future work

In this paper we introduced a topological formal definition for the scattering encounter of a minor body with the Earth (but the definition can be applied to close encounters with other planets) and we highlighted the need of finding an operational definition to be implemented in the IM automatic systems; this would be an important step to optimize the pipeline of such systems in view of the expected increase of their workload due to the forthcoming more capable NEA surveys.

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