

## Terrestrial Planet Formation: The Solar System and Other Systems

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**Abstract.** Accretion of terrestrial planets and solid cores of jovian planets is discussed, based on the results of our  $N$ -body simulations. Protoplanets accrete from planetesimals through runaway and oligarchic growth until they become isolated. The isolation mass of protoplanets in terrestrial planet region is about 0.2 Earth mass, which suggests that in the final stage of terrestrial planet formation giant impacts between the protoplanets occur. On the other hand, the isolation mass in jovian planet region is about a few to 10 Earth masses, which may be massive enough to form a gas giant. Extending the above arguments to disks with various initial masses, we discuss diversity of planetary systems. We predict that the extrasolar planets so far discovered may correspond to the systems formed from disks with large initial masses and that the other disks with smaller masses, which are the majority of the disks, may form Earth-like planets.

### 1. Introduction

In the conventional scenario of the Solar system formation (e.g., Safronv 1969, Wetherill 1980, Hayashi et al. 1986), planets are formed through accretion of planetesimals with initial sizes  $\sim 1$ -10 km in a protoplanetary disk with a mass  $\sim 0.01$ - $0.02M_{\odot}$ . As a result of the accretion, terrestrial planets and solid cores of jovian planets are formed. As shown below, larger solid protoplanets are formed in outer region. If a solid protoplanet becomes large enough, pressure gradient of planetary atmosphere no more supports the atmosphere against planetary gravity and gas accretion onto the protoplanet (solid core) starts, so that a gas giant planet is formed (e.g., Mizuno 1980, Bodenheimer & Pollack 1986).

We apply the above model to other planetary systems with different initial disks and discuss diversity of planetary systems. We define an "initial" disk as the disk at the stage when planetesimals with  $\sim 1$ -10 km are born, which may correspond to disks around WTTS. Since disk masses of CTTS and WTTS show no clear dependence on stellar age up to  $10^7$  years (Beckwith & Sargent 1993), we consider the observed disk mass distribution of CTTS and WTTS (e.g., Beckwith & Sargent 1996) as "initial" disk masses for planet formation: inferred disk masses range from  $10^{-3}M_{\odot}$  to  $0.3M_{\odot}$  with a peak at  $\sim 0.03M_{\odot}$  (Beckwith & Sargent 1996).

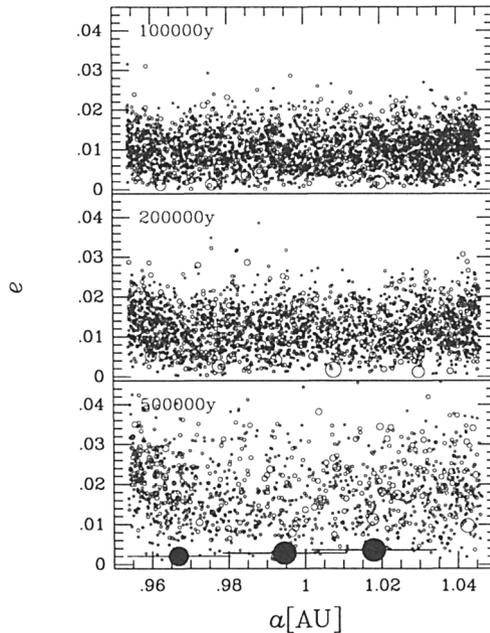


Figure 1. Snapshots of an  $N$ -body simulation of planetary accretion, starting from 4,000 planetesimals with  $2 \times 10^{23}$  g ( $\Sigma_s \simeq 10$  g cm $^{-2}$ ).  $e$  is orbital eccentricity and  $a$  is semimajor axis.

## 2. Runaway and Oligarchic Growth of Protoplanets

In the early stage of planetesimal accretion, the largest planetesimals grow more rapidly than the others and “run away” from the continuous mass distribution of planetesimals (e.g., Wetherill & Stewart 1989, Kokubo & Ida 1996). Runaway growth occurs as follows: (1) Dynamical friction makes velocity dispersion (orbital eccentricities and inclinations) of large planetesimals smaller than those of small planetesimals (e.g., Stewart & Wetherill 1988, Ida & Makino 1992), (2) gravitational focusing is more effective for larger planetesimals, and (3) as a result, larger planetesimals grow more rapidly than smaller planetesimals (e.g., Wetherill & Stewart 1989, Ohtsuki & Ida 1990, Kokubo & Ida 1996).

When a protoplanet becomes massive enough to pump up the velocity dispersion of small planetesimals in the vicinity of the protoplanet, runaway growth would slow down (Ida & Makino 1993). As a result, next runaway bodies formed in other regions catch up with the largest protoplanets, which results in formation of a two-component system of small number of similar-sized protoplanets and large number of small planetesimals (Kokubo & Ida 1998, 2000). In this system, “orbital repulsion” between protoplanets, which is caused by a coupling effect of distant perturbations between protoplanets and dynamical friction of small planetesimals (Kokubo & Ida 1995), results in almost equal spacing of protoplanets (Kokubo & Ida 1998, 2000). Such growth is called “oligarchic growth”.

$N$ -body simulations show that the average orbital distance between protoplanets is  $\sim 10$ - $15r_H$  (Kokubo & Ida 1998, 2000), where  $r_H$  is the Hill radius

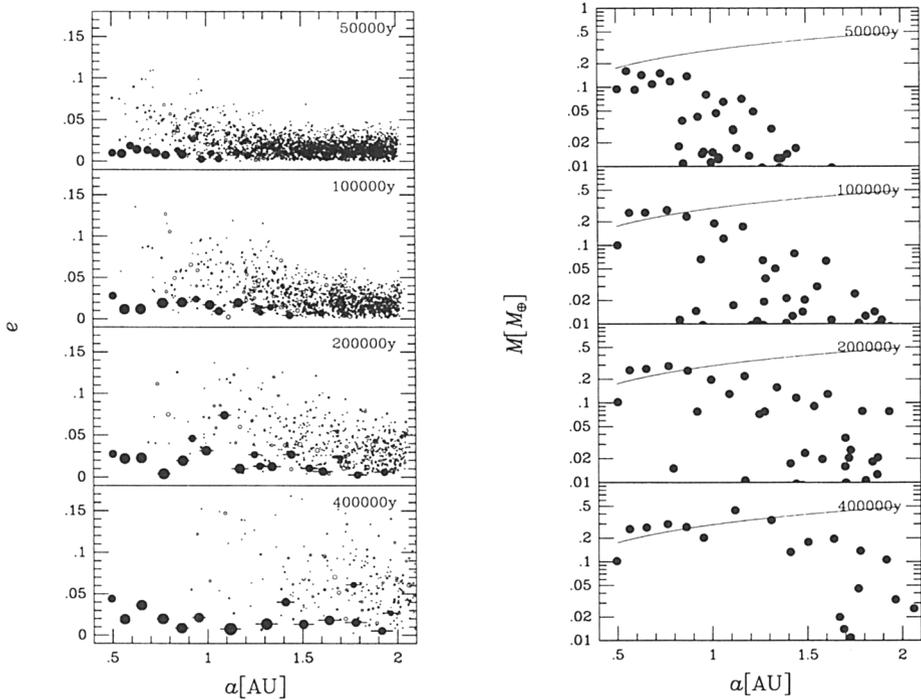


Figure 2. (left panel) Snapshots of an  $N$ -body simulation of planetary accretion in  $a$ - $e$  plane, starting from 10,000 planetesimals with  $2 \times 10^{24}$ g.  $\Sigma_s$  is given by Eq.(1). Physical radii are artificially enlarged by a factor 10. (right panel) Snapshots in  $a$ - $M$  plane. Analytical estimation of isolation masses given by Eq.(2) is shown by solid curves.

of a protoplanet (with mass  $M$  at semimajor axis  $a$ ) defined by  $(M/3M_{\odot})^{1/3}a$ . Figure 1 shows snapshots of a 3D  $N$ -body simulation, where  $e$  is orbital eccentricity and  $a$  is semimajor axis of planetesimals. (Orbital inclination is  $\sim 0.5e$  (in radian)). We calculated the region from 0.95AU to 1.05AU. We started from 4,000 planetesimals with  $2 \times 10^{23}$ g; mean surface density is  $\simeq 10 \text{ g cm}^{-2}$ . The sizes of circles are proportional to physical radii of planetesimals. We assume  $2 \text{ g cm}^{-3}$  as internal density of planetesimals. Hydrodynamic gas drag (Adachi et al. 1976) is included. If physical radii of planetesimals overlap, we create a merged body, neglecting fragmentation.

At 100,000 years, a few runaway bodies appear. At 500,000 years, three almost equal-sized large bodies (marked by filled circles) dominate the system. The bars attached to the large bodies express  $5r_H$  length in both sides. The separation distance between the bodies are almost  $\sim 10r_H$ .

### 3. Final Configuration of Protoplanet Systems

In Fig.2, we show snapshots of an  $N$ -body simulation of planetary accretion, starting from 10,000 planetesimals with  $2 \times 10^{24}$ g. We calculated the relatively

large region from 0.5AU to 2.0AU, where surface density of solid materials ( $\Sigma_s$ ) is given by

$$\Sigma_s = 10 \left( \frac{a}{1\text{AU}} \right)^{-3/2} \text{ gcm}^{-2} \simeq 1.5 \Sigma_{s,\text{min}}. \quad (1)$$

In the above,  $\Sigma_{s,\text{min}}$  is  $\Sigma_s$  in the minimum-mass disk model (Hayashi 1981). In this case, hydrodynamic gas drag is neglected and physical radii are artificially enlarged by a factor 10, so that accretion time scale is reduced by a factor about 10 (Kokubo & Ida 1996). Accretion proceeds from small  $a$  to large  $a$ , because spatial density and Keplerian frequency are higher at smaller  $a$ .

In this case, the orbital separation distance  $\Delta a$  between protoplanets is  $\sim 15r_H$ , which is slightly larger than the the result in Fig. 1. In Fig. 1, substantial amount of small planetesimals still remain because physical radii are not enlarged. If the simulation is continued until most planetesimals are accreted,  $\Delta a$  may increase to  $\sim 15r_H$ .

With  $\Delta a \sim 10\text{-}15r_H$  and a given  $\Sigma_s$ , we can estimate the final masses  $M_{\text{iso}}$  of protoplanets (isolation masses) that are masses when the protoplanets accrete all the solid materials between protoplanets. The estimated isolation masses are (Kokubo & Ida 1998)

$$M_{\text{iso}} \simeq \begin{cases} 0.2 \times \left( \frac{\Sigma_s}{\Sigma_{s,\text{min}}} \right)^{3/2} \left( \frac{a}{1\text{AU}} \right)^{3/4} \left( \frac{\Delta a}{15r_H} \right)^{3/2} M_{\oplus} & \text{[E]} \\ 5 \times \left( \frac{\Sigma_s}{\Sigma_{s,\text{min}}} \right)^{3/2} \left( \frac{a}{10\text{AU}} \right)^{3/4} \left( \frac{\Delta a}{15r_H} \right)^{3/2} M_{\oplus}, & \text{[J]} \end{cases} \quad (2)$$

where  $M_{\oplus}$  is Earth's mass,  $\Sigma_{s,\text{min}} = 7(a/1\text{AU})^{-3/2} \text{ gcm}^{-2}$  in the terrestrial planet region [E] inside snow boundary ( $a \lesssim 2.7\text{AU}$ ) and  $\Sigma_{s,\text{min}} = 1(a/10\text{AU})^{-3/2} \text{ gcm}^{-2}$  in the jovian planet region [J] beyond snow boundary ( $a \gtrsim 2.7\text{AU}$ ). Note that  $M_{\text{iso}}$  increases with  $a$ . As shown in Fig. 2, the above estimation agrees with the result of the  $N$ -body simulation. The corresponding orbital separation distances  $\Delta a$  between protoplanets are

$$\Delta a \simeq \begin{cases} 0.08 \times \left( \frac{\Sigma_s}{\Sigma_{s,\text{min}}} \right)^{1/2} \left( \frac{a}{1\text{AU}} \right)^{5/4} \left( \frac{\Delta a}{15r_H} \right)^{3/2} \text{ AU} & \text{[E]} \\ 2.6 \times \left( \frac{\Sigma_s}{\Sigma_{s,\text{min}}} \right)^{1/2} \left( \frac{a}{10\text{AU}} \right)^{5/4} \left( \frac{\Delta a}{15r_H} \right)^{3/2} \text{ AU}. & \text{[J]} \end{cases} \quad (3)$$

#### 4. Terrestrial Planet Formation

Equations (2) and (3) show that  $M_{\text{iso}}$  is significantly smaller than Earth's or Venus's mass and orbital separation distance between the present terrestrial planets is larger than the predicted  $10\text{-}15r_H$ . In order to complete terrestrial planets, further accretion among protoplanets would be necessary. The isolated protoplanets may be orbitally unstable on longer timescales. Orbital eccentricities may be pumped up by distant perturbations between protoplanets (Chambers et al. 1996), perturbations by Jovian planets (Chambers & Wetherill 1998,

Ito & Tanikawa 1999), or sweeping secular resonances during disk gas depletion (Nagasawa et al. 2000), so that orbit crossing would start.

Chambers & Wetherill (1998) pointed out that eccentricities of merged bodies are much higher than those of the present terrestrial planets. However, if we include damping effects by dynamical friction of residual planetesimals (Stewart & Wetherill 1988, Ida & Makino 1992) or tidal interactions with a gas disk (Ward 1993), the pumped-up eccentricities would decrease.

We performed  $N$ -body simulations of initially isolated protoplanets with  $0.2M_{\oplus}$ , including the damping forces corresponding to tidal interactions with a gas disk (Fig. 3). If the damping force is too strong, orbit crossing is suppressed before collisions between protoplanets occur enough to make planets as large as Earth or Venus. On the other hand, if the damping force is too weak, the pumped-up eccentricities are not reduced enough within disk life time  $T_{\text{disk}}$ ;  $T_{\text{disk}} \sim 10^7$  years (Strom et al. 1993, Zuckerman et al. 1995). With some range of damping strength, the pumped-up eccentricities decrease within  $T_{\text{disk}}$  after some planets become as large as Earth or Venus. Such a damping force may correspond to the tidal interactions with a residual gas disk with gas surface density  $\Sigma_g \sim 0.01\Sigma_{g,\text{min}}$  ( $\Sigma_{g,\text{min}}$  is  $\Sigma_g$  in the minimum-mass disk model), or equivalently, dynamical friction due to planetesimals with  $\Sigma_s \sim \Sigma_{s,\text{min}}$ , although more detailed calculations are needed. The results in Fig. 3 may be consistent with the terrestrial planet system in the Solar system.

## 5. Giant Planet Formation

Gas accretion onto a solid core starts when solid core accretion timescale (solid accretion works as heat source to support planetary atmosphere) becomes longer than contraction timescale  $T_c$  of planetary atmosphere due to the planetary gravity. When the core becomes isolated, solid accretion stops and instantaneous core accretion timescale becomes formally infinity. Hence gas accretion onto the core starts.  $T_c$  is given by (Ikoma et al. 2000)

$$T_c \sim 10^8 (M_{\text{iso}}/M_{\oplus})^{-2.5}. \quad (4)$$

If  $M_{\text{iso}}$  is relatively small, gas accretion proceeds slowly. The gas accretion becomes increasingly rapid as total planetary mass increases (e.g., Pollack et al. 1996, Ikoma et al. 2000). Equations (2) and (4) show that  $T_c$  may be short enough for the isolated cores in jovian planet region to become gas giants within  $T_{\text{disk}} \sim 10^7$  yr. Jupiter and Saturn would have formed in this way.

In Uranus and Neptune regions  $M_{\text{iso}}$  is large enough to start gas accretion. However, since they are located at large  $a$ , core accretion timescales ( $T_{\text{grow}}$ ) up to  $M_{\text{iso}}$  would well exceed  $T_{\text{disk}}$ . Thus they would have missed significant gas accretion.

"Orbital repulsion" would occur also during mass increase by gas accretion and it is expected that distances between gas giants also become  $10\text{--}15r_{\text{H}}$ , which is consistent with the orbital spacing of the present jovian planets (Kokubo & Ida 1998).

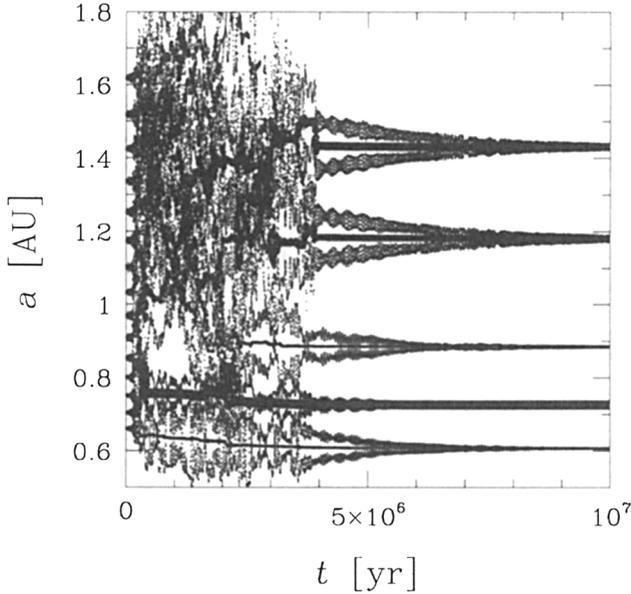


Figure 3. Accretional evolution of protoplanets. Initially, 15 protoplanets with  $M = 0.2M_{\oplus}$  and  $e, i = 0.001$  are distributed with  $\Delta a = 9r_{\text{H}}$ . Thick lines are semimajor axes; line width expresses the masses of the protoplanets. Thin lines express pericenter or apocenter, so that separation between the lines expresses orbital eccentricity. The damping force due to the tidal interactions with disk gas with  $\Sigma_{\text{g}} = 0.01\Sigma_{\text{g,min}}$  is included.

## 6. Diversity of planetary systems

As shown in Eq.(2), isolation masses depend on  $\Sigma_{\text{s}}$ . The results by Beckwith & Sargent (1996) suggest  $\Sigma_{\text{s}}$  would have distribution from  $\sim 0.1\Sigma_{\text{s,min}}$  to  $\sim 10\Sigma_{\text{s,min}}$ , if the  $a$ -dependence of  $\Sigma_{\text{s}}$  is similar to that of the minimum-mass disk model. (Note that  $\Sigma_{\text{s,min}}$  is the minimum surface density for our Solar system;  $\Sigma_{\text{s}}$  can be smaller than  $\Sigma_{\text{s,min}}$ .)

Let  $M_{\text{cr}}$  be the critical isolated core mass with  $T_{\text{c}} \simeq T_{\text{disk}}$  (Eqs. (2) and (4)). A gas giant is formed, if  $M_{\text{iso}} > M_{\text{cr}}$  and  $T_{\text{grow}} < T_{\text{disk}}$ .

In light disks,  $M_{\text{iso}}$  is small. In the case of  $\Sigma \sim 0.1\Sigma_{\text{s,min}}$ , for example,  $M_{\text{iso}} \sim 0.2M_{\oplus}$  even at  $a \sim 10\text{AU}$ . At smaller  $a$ ,  $M_{\text{iso}}$  is further smaller. At larger  $a$ ,  $T_{\text{grow}}$  would be longer than  $T_{\text{disk}}$ , since  $T_{\text{grow}}$  is proportional to  $\Sigma_{\text{s}}^{-1}$ . Therefore, gas giants would not be formed at all in light disks with, say,  $\Sigma_{\text{s}} \lesssim \Sigma_{\text{s,min}}/5$  (total disk mass  $M_{\text{disk}} \lesssim 0.003M_{\odot}$ ). Many relatively small solid planets would be formed ( $\Delta a$  is also small (Eq.3)).

On the other hand, in the case of  $\Sigma \sim 10\Sigma_{\text{s,min}}$ ,  $M_{\text{iso}} \simeq 6M_{\oplus}$  at 1AU, which is enough for gas accretion within  $T_{\text{disk}}$  (Eq.(4)). Also, in massive disks, a solid core accretion is so fast that  $M$  becomes  $\gtrsim M_{\text{cr}}$  within  $T_{\text{disk}}$  even at large  $a$ . Therefore, several gas giants would be formed in the regions from small  $a$

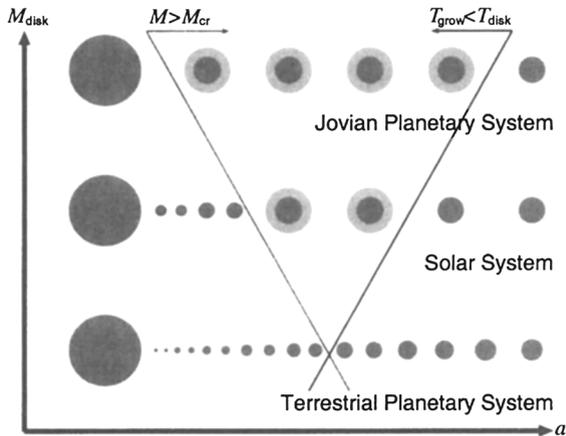


Figure 4. Schematic diagram of diversity of planetary system. For details, see text.

to large  $a$  in relatively massive disks with, say,  $\Sigma_s \gtrsim 5\Sigma_{s,\text{min}}$  (total disk mass  $M_{\text{disk}} \gtrsim 0.1M_{\odot}$ ).

In the case of  $\Sigma \sim \Sigma_{s,\text{min}}$ , a planetary system similar to the Solar system is expected. One or two gas giants are formed beyond snow boundary. Schematic diagram of predicted diversity of planetary systems is shown in Figure 4.

The several gas giants formed in massive disks might become orbitally unstable against long-term mutual distant perturbations. After ejection of some planet or a merging event, orbitally stable planets in eccentric orbits would remain, which may correspond to observed extrasolar planets in eccentric orbits (Weidenschilling & Marzani 1996, Lin & Ida 1997). Also, interactions between gas giants and a residual relatively massive disk may lead to significant orbital decay (type II migration) to a central star (e.g., Lin & Papaloizou 1993), which may correspond to extrasolar planets with short orbital periods (Lin et al. 1995).

At present, detection probability of extrasolar planets around sun-like stars is 5% or less. Most of extrasolar planets so far discovered have relatively small  $a$  and large masses. These extrasolar planetary systems may correspond to the planetary systems formed in significantly massive disks. The other disks with smaller masses, which are the majority of the disks, may form Earth-like planets, if planetary formation is not inhibited by other processes such as inhibition of planetesimal formation due to turbulence in a disk (e.g., Weidenschilling 1984) or rapid planet migration (type I migration) induced by tidal interactions with a disk (Ward 1986, 1997).

## 7. Summary

Terrestrial planets and solid cores of jovian planets accrete from planetesimals. The model of oligarch growth followed by runaway growth predicts the final masses of protoplanets:  $\sim 0.2M_{\oplus}$  in the terrestrial planet region and  $\sim 3\text{--}10M_{\oplus}$  in the jovian planet region. In the terrestrial planet region, long-term orbital instability of the protoplanets with some damping force may form a planetary

system similar to the present terrestrial planets. In the jovian planet region, comparison of  $T_{\text{grow}}$  and  $T_c$  with  $T_{\text{disk}}$  may explain the present jovian planets. Extending the above arguments, planetary systems with various initial disk masses are predicted: Massive disks ( $M_{\text{disk}} \gtrsim 0.1M_{\odot}$ ) may form systems similar to the extrasolar planets so far discovered, light disks ( $M_{\text{disk}} \lesssim 0.003M_{\odot}$ ) may form many Earth-like planets, and moderate disks may form systems similar to the Solar system.

## References

- Adachi, I., Hayashi, C., & Nakazawa, K. 1976, *Prog. Theor. Phys.*, 56, 1756
- Beckwith, S. V. W., & Sargent, A. I. 1993, in *Protostars and Planets III*, ed. E. H. Levy, & J. I. Lunine (Tucson: Univ. of Arizona Press), 521; 1996, *Nature*, 383, 139
- Bodenheimer, P., & Pollack, J. B. 1986, *Icarus*, 67, 391
- Chambers, J. E., & Wetherill, G. W. 1998, *Icarus*, 136, 30
- Chambers, J. E., Wetherill, G. W., & Boss, A. P. 1996, *Icarus*, 119, 261
- Hayashi, C. 1981, *Prog. Theor. Phys. Suppl.*, 70, 35
- Hayashi, C., Nakazawa, K., & Nakagawa, Y. 1985 in *Protostars and Planets II*, ed. D. C. Black & M. S. Matthews (Tucson: Univ. of Arizona Press), 1100
- Hut, P., & Makino, J. 1999, *Science*, 283, 501
- Ida, S., & Makino, J. 1992, *Icarus*, 96, 107; 1993, *Icarus*, 106, 210
- Ikoma, M., Nakazawa, K., & Emori, H. 2000, *ApJ*, 537, 1013
- Ito, T., & Tanikawa, K. 1999, *Icarus*, 139, 336
- Kokubo, E., & Ida, S. 1995, *Icarus*, 114, 247; 1996, *Icarus*, 123, 180; 1998, *Icarus*, 131, 171; 2000, *Icarus*, 143, 15
- Lin, D. N. C., & Ida, S. 1997, *ApJ*, 477, 781
- Lin, D. N. C., Bodenheimer, P., & Richardson, D. 1996, *Nature*, 380, 606
- Lin, D. N. C., & Papaloizou, J. C. B. 1993, in *Protostars and Planets III*, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. of Arizona Press), 749
- Mizuno, H. 1981, *Prog. Theor. Phys.*, 64, 544
- Nagasawa, M., Tanaka, H., & Ida, S. 2000, *AJ*, 119, 1480
- Ohtsuki, K., & Ida, S. 1990, *Icarus* 85, 499
- Pollack, J. B., Hubickyj, O., Bodenheimer, P., Lissauer, J. J., Podolak, M., Greenzweig, Y., *Icarus* 124, 62
- Ohtsuki, K., & Ida, S. 1990, *Icarus* 85, 499 Ohtsuki, K. & Ida, S. 1990, *Icarus* 85, 499
- Safronov, V. 1969, *Evolution of the protoplanetary cloud and formation of the earth and planets* (Moscow: Nauka Press)
- Stewart, G. R., & G. W. Wetherill 1988, *Icarus*, 74, 542
- Strom, S. E., Edwards, S., & Skrutski, M. F. 1993, in *Protostars and Planets III* eds. E.H. Levy & J.I. Lunine (Tucson: Univ. of Arizona Press), 837
- Ward, W. R. 1986, *Icarus*, 67, 164; 1993, *Icarus*, 106, 274; 1997 *Icarus*, 126, 261
- Weidenschilling, S. J. 1984, *Icarus*, 60, 553
- Weidenschilling, S. J., & Marzari, F. 1996, *Nature*, 384, 619
- Wetherill, G. W. 1980, *ARA&A*, 18, 77
- Wetherill, G. W., & Stewart, G. R. 1989, *Icarus*, 77, 330
- Zuckerman, B., Forveille, T., & Kastner, J. H. 1995, *Nature*, 373, 494