CORRESPONDENCE.

INSTITUTE OF ACTUARIES' TEXT-BOOK, PART II.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—A number of correspondents, for the most part students who have been studying for their examinations, have called my attention to sundry errata in Part II of the *Text-Book*. A table of these is appended, which it may be hoped is nearly complete. The majority of the errata are of trivial importance, and of such a kind that the context suggests at once the correction; and in a number of instances the error occurs only in some copies of the book, other copies being correct.

I may, perhaps, be permitted to take this opportunity of pointing out and removing an ambiguity which occurs in Arts. 92 and 93 of Chap. xviii, on Policy-Values, the more so, as the matter is one of some practical importance. These articles deal with the valuation, in groups, of policies under which the premiums are payable more frequently than once a year. In Art. 92 it is assumed that the ordinary annual premium, P_x, is to be valued; while in Art. 93 there is substituted for it $P_x^{(m)}$, namely, the premium per annum payable at intervals m times throughout the year. In consequence of this change in the premiums valued, there is a change in the nature of the policy, which, however, is not brought out in the articles in question. It would be well, therefore, to add an explanatory sentence to each of the articles as follows:

To Art. 92 add:

> "When valuation in groups is resorted to, then, according to "the method of this article, the assumption is tacitly made that "the premiums are really yearly premiums payable by instal-"ments, so that those instalments for the current year, unpaid "at the time of death, will fall to be deducted from the sum "assured on settlement of the claim."

To Art. 93 add:

> "According to this method, the periodical premiums are "really premiums in themselves, and not merely instalments of "yearly premiums as in Art. 92."

The rationale of the Addendum to Art. 92 may be thus explained:

Considering a little more minutely the question discussed in Art. 92, we may first ascertain what net premium must, under such conditions, be charged. The premium not being all payable at the beginning of the year, but by m equal instalments throughout the year, there will be a loss of interest on all the instalments except the first; but because the unpaid instalments for the year of death are to be deducted on settlement of the claim, there will, in the end, be as many full years' premiums paid as if premiums were payable annually

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in advance. The first instalment will not be deferred at all; the second will be deferred $\frac{1}{m}$ of a year; the third, $\frac{2}{m}$ of a year; and so on; and the last, $\frac{m-1}{m}$ of a year; and it will come to the same thing on the average, if the whole premium be deferred, as regards interest but not as regards mortality, by $\frac{m-1}{2m}$ of a year. Therefore, if P'_x by the total premium per annum, we must have

$$v^{\frac{m-1}{2m}} \times P'_x(1+a_x) = A_x;$$

whence $$P'_x = (1+i)^{\frac{m-1}{2m}} \times P_x.$$

Passing now to the policy-value: At any time the annuity for finding the value of the future premiums (leaving for the moment out of account the unpaid instalments for the current insurance year), will be the same as if the premiums were payable annually in advance, except that, because of the conditions of the case, the premiums must be discounted for $\frac{m-1}{2m}$ of a year more. That is, the value of the premiums payable annually being $P_x(\frac{1}{2}+a_{x+n})$, the value of the premiums by instalments must be $P'_x v^{\frac{m-1}{2m}}(\frac{1}{2}+a_{x+n})$; and this becomes, when we write P'_x in terms of P_x, $P_x(\frac{1}{2}+a_{x+n})$, as before. To this, however, we must now add the actual amount of unpaid instalments for the current insurance year, as these are certain to be received, and in their case the operation of discount is insignificant. On the average, the actual amount of outstanding instalments is $\frac{m-1}{2m}P'_x$, or, what is practically the same thing, $\frac{m-1}{2m}P_x$. Therefore, the total value of the future premiums is

$$P_x(\tfrac{1}{2}+a_{x+n}) + \frac{m-1}{2m}P_x, \ = P_x\left(\frac{2m-1}{2m}+a_{x+n}\right), \text{ as in Art. 92.}$$

I remain, &c.,

London,
18 *May* 1889.

GEORGE KING.

ERRATA.

In the first of the following columns is given the number of the page, and in the second that of the line. The letters *t* and *b* denote that the lines are to be counted from the top or bottom of the page, respectively. The third column gives the error, and the fourth the correction.

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9	11 *t*	·64496	·64541
,,	16 *t*	·64496	·64541
15	11 *b*	*After the algebraical expression insert* times	
25	12 *b*	d_{+2}	d_{+1}
33	16 *t*	$_{n\|}\mathring{e}$	$_{n\|}\mathring{e}_x$
37	3 *b*	*After* respectively, *insert* and, after reduction, replacing *m* by *t*,	
46	9 *t*	*Reverse the signs of the two expressions*	
,,	16 *t*	*The expression should be preceded by + instead of −*	
50	14 *b*	greater	less
61	23 *t*	*After* to be	*Insert* approximately
116	13 *t*	Art. 40	Art. 38
124	14 *b*	A^3	$A^3{}_x$
131	13 *t*	Art. 48	Art. 49
134	1 *t*	$a_{\overline{xxx\ldots m)}}$	$a_{\overline{xxx\ldots(m)}}$
,,	17 *t*	$a_{\overline{wxyz}}$	$a_{\overline{wxyz}}^{\;3}$
136	6 *b*	A *numerator*	$\|_n A$
144	Heading	Chapter VI	Chapter VII.
146	4 *t*	$=v^n[$	$=\Sigma v^n[$
159	3 *t*	equation 36	equation 37
170	1 *b*	(14)	*Delete* . . . (14)
171	2 *t*	*After* &c.	*Insert* . . . (14)
,,	4 *t*	$\frac{d\log D_x}{D_x}$	$\frac{d\log D_x}{dx}$
184	8 *b*	$A_{x\overline{n}\|}$	$\bar{A}_{x\overline{n}\|}$
185	10 *t*	2 Geo. II.	11 Geo. II.
187	10 *b*	$a_x=$	$\mathring{a}_x=$
,,	8 *b*	$\frac{i^2}{8}$	$\frac{i^2}{16}$
,,	7 *b*	$\frac{i-i^2}{12}$	$\frac{i-\frac{1}{4}i^2}{12}$
,,	5 *b*	$A_x\frac{i-i^2}{12}$	$A_x\frac{i-\frac{1}{4}i^2}{12}$
191	4 *t*	$a_x=$	$\mathring{a}_x=$
192	14 *t*	$\mathring{a}^{(m)}$	$\mathring{a}_x^{(m)}$
196	10 *t*	$A^{1}_{x\overline{1}\|}$	$\bar{A}^{1}_{x\overline{1}\|}$
,,	14 *t*	$A_{\overset{1}{\overline{x+1}}:\overline{1}\|}$	$\bar{A}_{\overset{1}{\overline{x+1}}:\overline{1}\|}$
200	5 *b*	a_{xy}	a_{yz}
212	11 *b*	$\log {}_tp_x$	$_tp_x$ *Delete* log
217	16 *t*	$\bar{a}_{12}$	$\bar{a}_{12:20}$

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219	2 *t*	$\log v$	$\log v^t$
222	4 *t*	*The expression should be preceded by + instead of −*	
225	3 *t*	$l_{x+n-\frac{1}{2}}$ *numerator*	$l_{y+n-\frac{1}{2}}$
226	1 *t*	values	value
231	6 *t*	$\bar{a}_{xy}$	$\bar{a}_{xz}$
”	13 *t*	$\bar{a}_{xy}$	$\bar{a}_{xz}$
237	1 *b*	$\overline{A}^2_{xyz}$	$\overline{A}^2_{x\underset{1}{y}z}$
239	5 *b*	6 (Col. 1)	56
252	2 *b*	formula 14	formula 13
268	19 *t*	$A^3_{\underset{1}{3:45:60}}$	$A^3_{\underset{1}{30:45:60}}$
”	1 *b*	(x)	(w)
269	3 *t*	${}^t\bar{a}_x\,.\,dt$	${}^t\bar{a}_w\,.\,dt$
282	1 *b*	end of t years	end of mt years
284	1 *t*	$N_{x+t-1}+N_{x+2t-1}+\&c.+N_{x+mt-1}$	$\pi(N_{x+t-1}+N_{x+2t-1}+\&c.+N_{x+mt-1})$
294	1 *t*	*Delete from the top of the page to the end of Art.* 70, *and substitute the following*:—	

$\pi w\dfrac{M'_{x+1}}{D'_x}$; and in respect of the third, $\pi w^2\dfrac{M'_{x+2}}{D'_x}$, and so on, when we write w for $\dfrac{1}{1+j}$; and the total value of the return will be $\pi\dfrac{M'_x+wM'_{x+1}+w^2M'_{x+2}+\&c.}{D'_x}$.

We therefore have

$$\text{Benefit Side}=\frac{M_x}{D_x}+\pi\frac{M'_x+wM'_{x+1}+w^2M'_{x+2}+\&c.}{D'_x}$$

$$\text{Payment Side}=\pi\frac{N_{x-1}}{D_x}$$

$$\text{Whence}\qquad \pi=\frac{M_x\div D_x}{\dfrac{N_{x-1}}{D_x}-\dfrac{M'_x+wM'_{x+1}+w^2M'_{x+2}+\&c.}{D'_x}}\quad . \quad . \quad (45)$$

70. In practice J will never be such a rate of interest as is to be found in ordinary tables, and therefore in formula 44 the value of A'_x will always have to be specially computed, and the most convenient course will be to use a formula of approximate summation. Any of the formulas of approximate summation given in Chap. xxiv may be employed. In the case of formula 45, the negative term in the denominator can be computed by constructing the commutation column of M'_x. This term may, however, be put in the form $\dfrac{C'_x+C'_{x+1}(1+a_{\overline{1}|})+C'_{x+2}(1+a_{\overline{2}|})+\&c.}{D'_x}$, to the calculation of which Lubbock's formula may be applied.

297	14 *t*	R*i denominator*	$R_x i$

298	9 *t*	$\frac{R'_x}{D'_x}$	$\frac{M'_x + wM'_{x+1} + w^2M'_{x+2} + \&c.}{D'_x}$
,,	11 *t*	*Instead of equation* 54, *write* $\pi = \dfrac{\dfrac{M_x}{D_x} + c\,\dfrac{M'_x + wM'_{x+1} + w^2M'_{x+2} + \&c.}{D'_x}}{\dfrac{N_{x-1}}{D_x} - (1+\kappa)\,\dfrac{M'_x + wM'_{x+1} + w^2M'_{x+2} + \&c.}{D'_x}}$	
312	8 *t*	*The whole of the expression in each of these lines should have* $\bar{A}_x$ *for a factor, and not merely the first term. Therefore, insert the necessary brackets.*	
,,	10 *t*		
,,	13 *t*	$\frac{f}{1-\bar{A}_y}$	$\frac{\bar{A}_x f}{1-\bar{A}_y}$
313	2 *b*	$\frac{f}{1-\bar{A}_y}$	$\frac{\bar{A}_x f}{1-\bar{A}_y}$
318	4 *t*	Col. 10	Col. 9
319	2 *b*	·00155 (Col. 9)	·00115
323	6 *t*	*Number omitted*	*Insert* (7)
,,	8 *t*	*Ditto*	*Insert* (8)
330	5 *t*	$P_{x+n}=$	$P_{x+n}+d=$
336	20 *t*	$\frac{P'}{1{\cdot}05}$	$\frac{P}{1{\cdot}05}$
347	1 *t*	formula 31	formula 32
351	13 *b*	Art. 80	Art. 82
356	11 *b*	formula 55	formula 57
362	8 *b*	5485·600	5489·602
,,	6 *b*	253·024	253·208
,,	5 *b*	5253·024	5253·208
,,	4 *b*	236·861	236·394
,,	3 *b*	253·024	253·208
,,	2 *b*	489·885	489·602
,,	1 *b*	5253·024	5253·208
364	11 *t*	P'	P'_1
367	5 *t*	*Numerator* of second expression − *(minus)*	+ *(plus)*
371	14 *b*	$A_{x(i}$	$A_{x(i)}$
380	4 *b*	Houghton	Haughton
389	9 *b*	of p_x	of $\log p_x$
393	Table	(*Age* 14) ·99990	$\bar{1}{\cdot}99990$
398	Table	(*Age* 93) 1·471668	1·471667
405	8 *t*	Art. 66	Art. 67

410	4 *t*	log vp_x	log vp_{xy}
414	5 *b*	*Cancel Art.* 94, *as the method does not apply to De Morgan's form of Commutation Columns.*	
416	11 *b*	$C^1_{x:y-1}=C^1_{xy}-C^1_{xy}\times\Delta l_{y-\frac{1}{2}}$	$C^1_{x:y-1}=C^1_{xy}-C_x\times\Delta l_{y-\frac{1}{2}}$
,,	10 *b*	$=C^1_{xy}+C^1_{xy}\times d_{y-\frac{1}{2}}$	$=C^1_{xy}+C_x\times d_{y-\frac{1}{2}}$
,,	8 *b*	$C_{\overset{1}{x-1}:y}=C^1_{xy}-C^1_{xy}\times\Delta d_{x-1}$	$C_{\overset{1}{x-1}:y}=C^1_{xy}-v^{y+1}l_{y+\frac{1}{2}}\times\Delta d_{y-1}$
,,	7 *b*	} *Cancel these two lines*	
,,	6 *b*		
,,	5 *b*	C^1_{xy}	C_x, or $v^{y+1}l_{y+\frac{1}{2}}$
,,	3 *b*	or $(1+i)^{\frac{1}{2}}d_{y-\frac{1}{2}}$	*Delete* or $(1+i)^{\frac{1}{2}}d_{y-\frac{1}{2}}$
417	8 *t*	$[t]\{\log \Pi_{xy}-\log A_{\overset{1}{x+1}:\overline{y+1}}\}$	$[t]\{\log A_{\overset{1}{x+1}:\overline{y+1}}-\log \Pi_{xy}\}$
,,	11 *t*	$\beta-\alpha$	$\alpha-\beta$
,,	15 *t*	$\beta_{-1}-\alpha_{-1}$	$\alpha_{-1}-\beta_{-1}$
430	1 *b*	kx	$kx^{(m)}$
434	3 *b*	23	20
441	14 *b*	Art. 20	Art. 17
453	3 *b*	$n=\frac{1}{2}$	$n=-\frac{1}{2}$
460	15 *b*	Art. 25	Art. 22
,,	6 *b*	Art. 28	Art. 25
462	13 *t*	$\Delta^3 0^{3'}$	$\Delta^3 0^3$
463	2 *t*	Art. 29	Art. 26
468	5 *b*	$u_{(3n-1)}$	$u_{(3n-1)t}$
470	12 *b*	u_{mn-1}	$u_{(m-1)n}$
,,	6 *b*	$480n$	$480n^3$
471	5 *t*	McLachlan	McLauchlan
473	5 *b*	19·9095	19·9051
474	21 *t*	$\times=n$	$\times n=$
478	6 *b*	$\frac{1}{3}$	$\frac{1}{\lfloor\underline{3}}$
479	9 *b*	*After* $\frac{\Delta^3 u_x}{n^3}$	*Insert* approximately
482	2 *t*	n_{5n}	u_{5n}
,,	6 *t*	—03724	—·03724
,,	9 *t*	v^t	log v^t
,,	7 *b*	·0004	·0007
483	13 *t*	·0003	·0004
487	3 *b*	·0613	·0614
490	11 *t*	·0088	·0086
491	7 *t*	(*Against* No. 27) $-\frac{n^2-1}{12}\cdot\frac{du_0}{dx}$	$+\frac{n^2-1}{12}\cdot\frac{du_0}{dx}$

ERRATA IN THE TABLES.

Page	Age	Column	Error	Correction
498	0	Δ Colog p_x	0·96353	$\bar{1}$·96353
„	13	„	·00000	0·00000
505	61	R_x	73314·37	77314·37
547	90	A_x	·84512	·85412

THE LEGAL STAMP DUTY ON RE-ASSURANCE POLICIES, EFFECTED BY WAY OF GUARANTEE ON A COPY OF THE ORIGINAL POLICY.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—Up to the present it has been, I believe, the general practice to stamp re-assurances in the same way as direct policies, the result being that the Government receive double stamp duty on the amount re-assured.

This society has recently obtained an adjudication of the proper stamp which should be placed on re-assurances, effected by way of guarantee on a copy of the original policy, and it would appear that a sixpenny stamp is legally sufficient for this purpose, whatever be the amount of the policy. It would also appear that the copy policy does not require to be authenticated with a shilling stamp.

The means adopted of getting the duty assessed were as follows:

The original policy, kindly lent us by the re-assuring office, was lodged at Somerset House, together with our guarantee endorsed upon a copy of it. We were then required to stamp our guarantee with a sixpenny stamp, and supply the authorities with a copy of the original policy, endorsed with a copy of our guarantee, and after some delay our guarantee was returned to us marked—

2,926. Commissioners of Inland Revenue

13 Jun, 9. (26) Adjudged duly stamped. (6)

(89)

I did not understand that any special form or style of guarantee would be insisted on, the view taken by Somerset House being, it seemed, that a re-assurance in this form was of the nature of an indemnity.

I thought of raising the question as to the stamp duty on an ordinary policy granted by one office to another and bearing a re-assurance endorsement, but did not do so at the moment, thinking