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Theory of Global Disk Oscillations

Atsuo T. Okazaki
Faculty of Engineering, Hokkai-Gakuen University, Toyohira-ku, Sapporo 062-8605, Japan

Abstract. We discuss the characteristics of global oscillation modes in Be disks and review recent studies related to the disk oscillation model. Since the $m = 1$ modes are present only in near Keplerian disks and the mode confinement occurs only in the region in which the radial flow is subsonic, the model of global disk oscillation strongly prefers the viscous decretion disk scenario proposed by Lee et al. (1991), whereas it is incompatible with the wind-compressed disk scenario of Bjorkman & Cassinelli (1993), which predicts angular-momentum conserving disks with supersonic radial flow. Based on the viscous decretion disk scenario, we discuss transonic solutions of decretion and examine the effect of viscosity on the global one-armed modes.

1. Introduction

Extensive studies have revealed that a Be star has a two-component extended atmosphere, a polar wind region and a cool equatorial disk. The polar wind region consists of a low-density, fast outflow emitting UV radiation. The wind structure is well explained by the radiation-driven wind model, in which the radiative acceleration results from the scattering of the stellar radiation in an ensemble of spectral lines.

In contrast to the polar wind region, the equatorial disk consists of a high-density plasma with small radial velocity. The optical emission lines and the IR excess arise from the disk. Unfortunately, the nature of the disk, e.g., the origin, the structure and dynamics, and the evolution, is little understood, despite that large theoretical and observational efforts have been devoted to the study of Be stars.

Many Be stars exhibit long-term variations of the relative intensities of the violet and red components in double-peaked emission-line profiles. The period of the variation ranges from years to decades, which is 10^{3-4} times as long as the rotation period of the central stars. In addition, the variation is always accompanied by the profile shift: the profile as a whole shifts blueward (redward) when the red (violet) component is stronger (McLaughlin 1961; Hubert et al. 1987).

The bizarre phenomenon of the long-term V/R variation, which is a key to understanding of the structure and dynamics of Be disks, had been a longstanding enigma. In 1980's the promising candidate was the elongated disk model with apsidal motion (e.g., Hirata & Kogure 1984; Ballereau & Chauville 1989).

According to this model, the period of the long-term V/R variation is the period of the apsidal motion of a disk. The elongated disk model is capable of explaining the typical features of the long-term V/R variations. However, it was not a dynamical but, rather, a geometrical model, so it was quite difficult to understand why and how such low-frequency apsidal motion of a disk can continue for several decades.

A breakthrough came when Kato (1983) found that there exist global, $m = 1$ low-frequency oscillation modes in near Keplerian disks, where m is the azimuthal wave number. He estimated the period of the lowest order $m = 1$ mode to be about 10 yr in Be disks, which is in agreement with the observed periods of V/R variations. Applying Kato's theory, Okazaki (1991) constructed a global disk oscillation (GDO) model for Be stars and found that the characteristics of the global $m = 1$ eigenmodes are consistent with the observed properties of the long-term V/R variation. Since then, observational evidences supporting the GDO model have been accumulated (e.g., Hummel and Hanuschik 1997; Vakili et al. 1998).

In this paper we discuss the characteristics of global oscillation modes in Be disks and review recent studies related to the GDO model. For this purpose, we first consider possible modes of global oscillations in gaseous disks. We also consider the confinement of the global modes in Be disks. Based on the viscous decretion disk model proposed by Lee et al. (1991), we then discuss the steady disk structure and examine the effect of viscosity on the characteristics of GDOs.

2. Possible Global Oscillations in Cool Disks

Since the temperature of Be disks is of the order of 10^4 K, the pressure gradient force in the radial direction is much weaker than the gravitational force of the central star. Moreover, all observations imply that the radial flow is smaller than a few km s^{-1}, at least within ~ 10 stellar radii (Hanuschick 1994, 2000; Waters & Marlborough 1994). Therefore, the Be disk must be rotationally supported in the radial direction. The rotation velocity of the disk is then nearly Keplerian.

Let us consider an oscillation in the form of the normal mode which varies as $\exp[i(\omega t - m\phi)]$ with ω being the oscillation frequency. Then, the frequency of oscillation seen from an observer corotating with the disk matter at radius r is $\omega - m\Omega(r)$, where Ω is the angular frequency of disk rotation. If the oscillation is global, the variation of the pressure gradient force caused by this oscillation is necessarily much smaller than the variation of the Coriolis force. If seen from an observer corotating with the disk, the frequency of oscillation due to the Coriolis force is the epicyclic frequency $\kappa(r)$, which is given by $\kappa = [2\Omega(2\Omega + r\, d\Omega/dr)]^{1/2}$. Hence, in order for an oscillation to be global, the relation $[\omega - m\Omega(r)]^2 \sim \kappa^2(r)$ must be met over a wide region of the disk. Any oscillations breaking the condition must have very short wavelengths in the radial direction to compensate for the frequency difference by the pressure-restoring force.

We should note, however, that no oscillations satisfy the above condition in general, because two frequencies, $\omega - m\Omega$ and κ, depend on r separately. Hence, in general, no global persistent oscillations are present in cool disks in which the sound speed is much smaller than the rotation velocity. However, there is an exception. The epicyclic frequency, κ, in a near-Keplerian disk remains close to the angular frequency Ω at any radius. This indicates that only oscillations with $m = 1$ and $\omega \sim \Omega - \kappa$ ($\ll \Omega$) satisfy the above condition for global oscillations. Therefore, we conclude that possible global oscillations in near-Keplerian disks are very low-frequency ($\omega \ll \Omega$), $m = 1$ oscillations alone. Note that no global axisymmetric modes, such as radial pulsations, can exist in disks around Be stars.

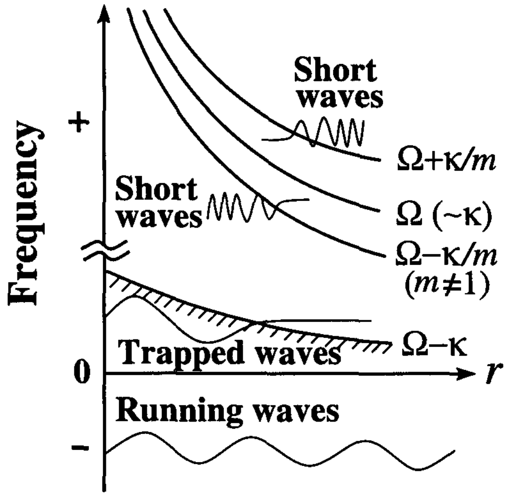

Figure 1. Propagation diagram for a near Keplerian disk. Waves can propagate in the region where $(\omega - m\Omega)^2 > \kappa^2$, but all oscillations with $m \neq 1$ have very short wavelengths. Only possible global waves are $m = 1$ waves that can propagate in the region where $\omega < \Omega - \kappa$ (hatched region). Prograde ($\omega > 0$) $m = 1$ waves can be confined to the inner part of the disk, whereas retrograde ($\omega < 0$) $m = 1$ waves are running waves and not confined to any part of the disk unless the disk is truncated, e.g., by the presence of a companion in a binary system.

Figure 1 shows the propagation diagram for a near Keplerian disk. Since the pressure always acts so as to increase the restoring force of oscillation, waves can propagate in the region where $(\omega - m\Omega)^2 > \kappa^2$ (see Kato 1983). Waves with $m \neq 1$, however, have very short wavelengths by the reason mentioned above, so they will dissipate in time. Only possible global waves are very, low-frequency $m = 1$ waves that can propagate in the region where $\omega < \Omega - \kappa$.

It is important to note that only prograde ($\omega > 0$) $m = 1$ waves can be confined to an inner disk, as shown in Figure 1. Since Be disks are not truncated except in binary systems, all observable $m = 1$ density waves should prograde, as pointed out by Papaloizou et al. (1992).

The presence of the prograde $m = 1$ density waves in Be disks has been confirmed observationally. Analysing the line-profile variability, Telting et al. (1994) showed the evidence for a prograde $m = 1$ density wave in the disk

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of β^1 Mon. Recently, Vakili et al. (1998) presented a direct interferometric evidence for a prograde $m = 1$ wave in the disk of ζ Tau.

Finally, we derive a condition for the confinement of global density waves in a disk with nonzero radial flow. The local dispersion relation for an oscillation which varies as $\exp[i(\omega t - k_r r - m\phi)]$ is written as

$$(\omega - m\Omega - k_r V_r)^2 - \kappa^2 = c_s^2 k_r^2, \tag{1}$$

where V_r is the radial velocity and c_s is the sound speed (e.g., Negueruela & Okazaki 2000). From equation (1), we note that waves can propagate in the region where $c_s^2[(\omega - m\Omega)^2 - \kappa^2] + V_r^2\kappa^2 > 0$ for which k_r is a real number, while they are evanescent in the region where $c_s^2[(\omega - m\Omega)^2 - \kappa^2] + V_r^2\kappa^2 < 0$. As shown in Figure 1, the confinement of any wave needs an evanescent region outside the propagation region. Since $(\omega - m\Omega)^2 - \kappa^2 \ll \kappa^2$ for global waves, the evanescent region is present only in disks with

$$\frac{|V_r|}{c_s} < \frac{|(\omega - m\Omega)^2 - \kappa^2|^{1/2}}{\kappa} \ll 1. \tag{2}$$

Therefore, global density waves can be confined to an inner disk only if the radial velocity is much smaller than the sound speed.

3. Global $m = 1$ Oscillations in Inviscid Disks

3.1. Unperturbed Disk Model

We take a geometrically thin, axisymmetric disk as an unperturbed equilibrium disk. For simplicity, we assume the disk to be isothermal. We use vertically integrated or averaged quantities to describe the disk. Neither the radial advective motion nor the viscous effect is taken into account in the unperturbed disk model discussed in this section. We will take them into account in later sections.

Following Papaloizou et al. (1992), we take into account the effect of rotation by including the quadrupole contribution to the potential around the rotationally-distorted central star. The external potential of the star of mass M and radius R is then approximated as

$$\psi \simeq -\frac{GM}{r}\left[1 + \frac{k_2 f^2}{3}\left(\frac{R}{r}\right)^2\right] \tag{3}$$

(Papaloizou et al. 1992), where k_2 is the apsidal motion constant and f is the rotation parameter defined by the ratio of the rotation velocity of the star to the Keplerian velocity at the stellar surface. According to Kogure & Hirata (1982), $0.2 \lesssim f \lesssim 0.6$ for B0-type Be stars and $0.5 \lesssim f \lesssim 1.0$ for B5-type Be stars. The value of k_2 for rapidly rotating stars is not well known. Theoretically, k_2 for non-rotating main-sequence stars has a maximum value of $\sim 10^{-2}$ at a mass between 7 and $10 M_\odot$ (Stothers 1974).

As mentioned previously, the radial flow in a Be disk is smaller than a few $\mathrm{km\,s^{-1}}$, at least within ~ 10 stellar radii. In such a flow with very small velocity gradient, the radiative force would arise not from the optically-thick strong lines but from an ensemble of optically-thin weak lines (Lamers 1986; Chen & Marlborough 1994). As the radiative force due to an ensemble of optically-thin weak lines, we adopt the parametric form proposed by Chen & Marlborough (1994):

$$F_{\rm rad} \simeq \frac{GM}{r^2}\eta\left(\frac{r}{R}\right)^{\epsilon}, \tag{4}$$

where we neglected the Eddington factor, which is as small as ~ 0.03 for a B0V star and ~ 0.003 for a B5V star.

Since the density profiles of Be disks are not well determined, we adopt a simple power-law form for the surface density Σ,

$$\Sigma(r) \propto \left(\frac{r}{R}\right)^{-n}, \tag{5}$$

where the index n is a constant.

3.2. Linear $m = 1$ Eigenmodes

A linear $m = 1$ perturbation which varies as $\exp[i(\omega - \phi)]$ is superposed on the unperturbed disk described above. We assume the perturbation to be isothermal, because in Be disks the thermal timescale is much shorter than the dynamical one. Since the eigenmode of interest is the mode confined to the inner part of the disk, we adopt $u_r = 0$ at some large radius as the outer boundary condition. As the inner boundary condition, we impose $u_r = 0$ at the star/disk interface.

We show the fundamental $m = 1$ mode in an inviscid disk with $n = 2$ in Figure 2a and the disk perturbed by the mode in Figure 2b. The period of the mode is 4.7 yr. Note that the perturbed disk becomes eccentric. Note also that the azimuthal component of the velocity perturbation anticorrelates with the density perturbation, except in an innermost narrow part of the disk. This property has been shown to cause the profile shift observed for the long-term V/R variations (Hummel & Hanuschik 1994, 1997; Hanuschik et al. 1995; Okazaki 1996; see also Hummel 2000). The global $m = 1$ oscillation model naturally explains both of the observed periodicities and the profile shift of the long-term V/R variations.

4. Viscous Decretion Disk Model

In this section, we consider an unperturbed disk model more realistic than in the previous section. Although there is no widely-accepted model as to how to form a near-Keplerian disk around a Be star, the viscous decretion disk model proposed by Lee et al. (1991) seems promising (see Porter 1999). In this model, the matter supplied from the equatorial surface of the star drifts outward by the viscous effect and forms the disk. Basic equations for viscous decretion disks are the same as those for viscous accretion disks, except that the sign of $\dot{M}$ (mass decretion/accretion rate) is opposite. Thus, viscous decretion is expected to produce a geometrically thin, near Keplerian disk around a Be star.

Adopting the Shakura-Sunyaev viscosity prescription, Okazaki (2000) showed that the equations which determine the velocity field in an isothermal decretion

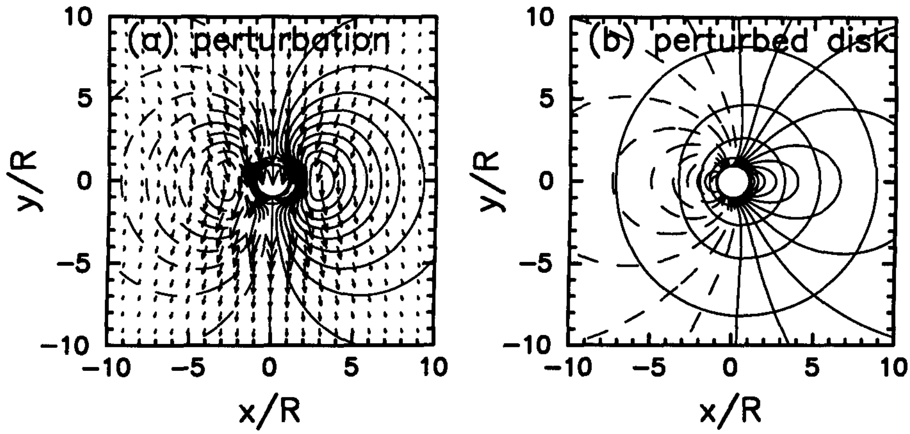

Figure 2. Fundamental $m = 1$ mode in a disk around a Be star: (a) the linear perturbation pattern and (b) the surface density and the line-of-sight velocity in the perturbed disk viewed equator-on. The period of the mode is 4.7 yr. Both disk and wave rotate counterclockwise. The central star is a B0V star with $M = 17.8M_{\odot}$, $R = 7.41R_{\odot}$, and $T_{\rm eff} = 2.80 \times 10^4$ (Allen 1973). We adopted the density profile with $n = 2$ and the disk temperature of $\frac{2}{3}T_{\rm eff}$. The rotation and radiative parameters adopted are $(k_2, f) = (6 \times 10^{-3}, 0.4)$ and $(\eta, \epsilon) = (5 \times 10^{-2}, 0.1)$, respectively. In panel (a), contours, which are separated by 0.1, denote the density perturbation, while arrows denote the perturbed velocity vectors normalized by the local angular velocity $r\Omega$. In panel (b) a nonlinear perturbation pattern similar to the linear one shown in panel (a) is assumed; for illustrative purposes, we normalized the amplitude of the perturbation so that the maximum value of the perturbed density is 50% of the unperturbed density. The density contours in panel (b) are separated by a factor of $10^{1/2}$, while the velocity contours are separated by $50\,{\rm km\,s^{-1}}$.

disk can be reduced to

$$\left(V_r - \frac{c_s^2}{V_r}\right)\frac{{\rm d}V_r}{{\rm d}r} = -\frac{GM}{r^2} + F_{\rm rad} + \frac{\ell^2}{r^3} + \frac{5}{2}\frac{c_s^2}{r} \tag{6}$$

and

$$\ell = \ell(R) + \alpha c_s^2\left[\frac{R}{V_r(R)} - \frac{r}{V_r}\right], \tag{7}$$

where α is the viscosity parameter, $\ell = rV_\phi$ is the specific angular momentum, and V_r and V_ϕ are the radial and azimuthal components of the vertically averaged velocity, respectively. Equations (6) and (7) show that the viscous decretion disk is a thermal wind in the equatorial region, in which the material is slowly accelerated outward by the pressure force.

Figure 3 shows some transonic solutions for viscous decretion disks. Neither the rotational distortion of the star nor the radiative force due to an ensemble

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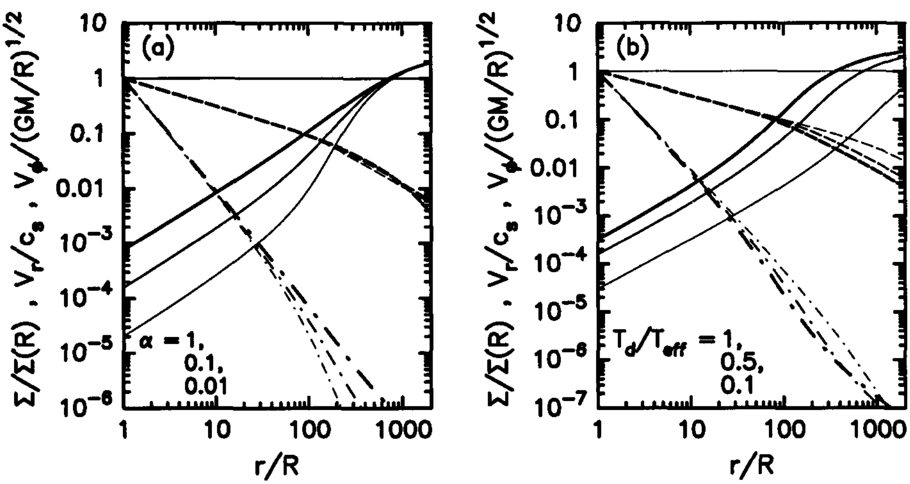

Figure 3. Structure of the viscous transonic decretion disks with different values of (a) α and (b) the disk temperature T_d/T_{eff}. Solid, dashed, and dash-dotted lines denote V_r/c_s, $V_\phi/(GM/R)^{1/2}$, and $\Sigma/\Sigma(R)$, respectively. The central star is a B0 main-sequence star. Neither the rotational deformation of the star nor the radiative force is taken into account. In panel (a) the disk temperature is fixed to be $T_d = \frac{1}{2}T_{eff}$. In panel (b) the value of α is fixed to be 0.1

of optically thin lines is taken into account, because their contribution to the steady disk structure is negligible as long as $\eta \ll 1$. In Figure 3, solid, dashed, and dash-dotted lines denote V_r/c_s, $V_\phi/(GM/R)^{1/2}$, and $\Sigma/\Sigma(R)$, respectively. For comparison purposes, we show transonic solutions for $\alpha = 1$ (thick lines), $\alpha = 0.1$ (lines with intermediate thickness), and $\alpha = 0.01$ (thin lines) in Figure 3a and those for $T_d/T_{eff} = 1$ (thick lines), 1/2 (lines with intermediate thickness), and 1/10 (thin lines) in Figure 3b. In Figure 3a, T_d/T_{eff} is fixed to be $\frac{1}{2}$, and in Figure 3b α is fixed to be 0.1. From Figure 3, we find that the sonic point is located far from the star and the outflow is highly subsonic for $r \ll 10^2 R$. This is because it is basically the pressure force which accelerates the flow up to a supersonic speed, and the pressure force does not work effectively in the region where it is much weaker than the effective gravity.

We also find that, in the subsonic part of the disk, V_r increases as r and Σ decreases as r^{-2}. In the inner subsonic region, V_ϕ decreases as $r^{-1/2}$ (Keplerian), while in the outer subsonic region and in the supersonic region it decreases as r^{-1} (angular momentum conserving).

The topology of the sonic point is nodal for $\alpha \gtrsim 0.95$, while it is a saddle-type for $\alpha \lesssim 0.9$ (Okazaki 2000). According to the theory of accretion, the sonic point in the former case is unstable, whereas in the latter case it is stable (e.g., Abramowicz and Kato 1989).

In this section, we have discussed the steady structure of disks formed by viscous decretion. Disks around some Be stars, however, are always transient. For example, μ Cen has exhibited outbursts followed by the formation of a transient disk (e.g., Hanuschick et al. 1993; see also Rivinius et al. 1998). Using a 3D Smoothed Particle Hydrodynamics approach, Kroll & Hanuschik (1997) successfully simulated the formation and decay of a near Keplerian disk by viscous effect during an outburst. Their results are in good agreement with the observations of μ Cen.

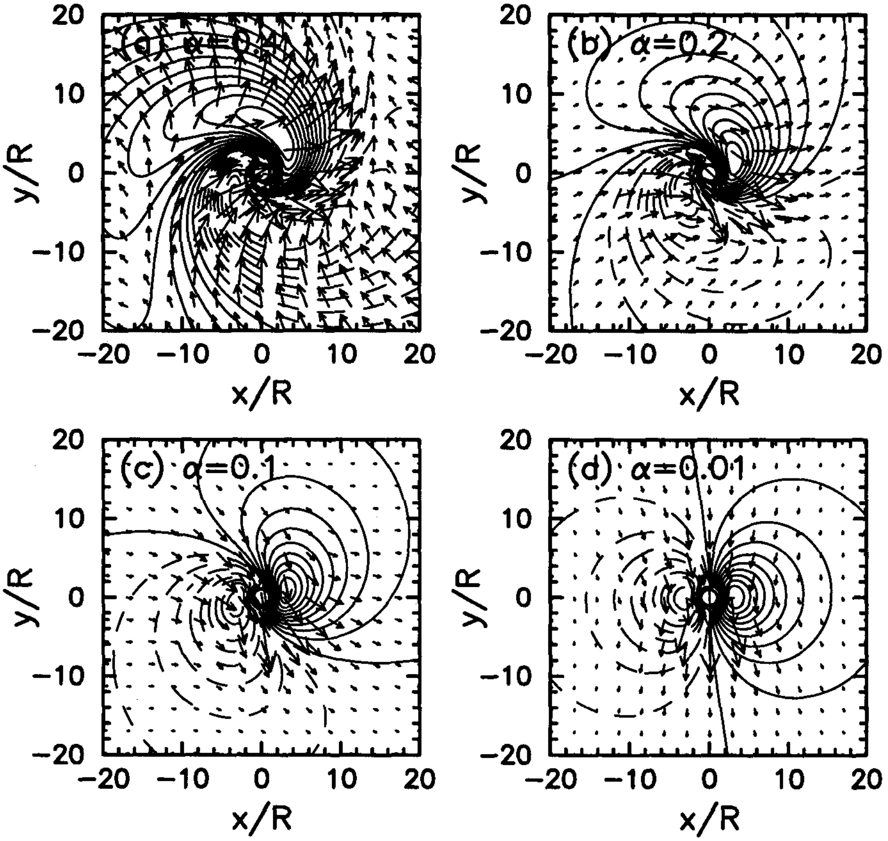

Figure 4. One-armed fundamental mode confined to the inner part of the transonic decretion disk for: (a) $\alpha = 0.4$, (b) $\alpha = 0.2$, (c) $\alpha = 0.1$, and (d) $\alpha = 0.01$. The central star is a B0 main-sequence star, the disk temperature is $T_{\rm d}/T_{\rm eff} = 2/3$, and the radiative parameters are $(\eta, \epsilon) = (5 \cdot 10^{-2}, 0.1)$. The effect of rapid rotation is neglected. In each panel, contours, which are separated by 0.1, denote the density perturbation, while arrows denote the perturbed velocity vectors normalized by the local angular velocity V_ϕ.

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5. Global Disk Oscillations in Viscous Decretion Disks

Kato et al. (1988) found that, in accretion disks, the pulsational (i.e., axisymmetric) modes, which are neutral when the disk is inviscid, become overstable when the viscous effect is taken into account. Using a similar analysis to nonaxisymmetric modes in decretion disks, one can easily show that the $m = 1$ modes, which are neutral in inviscid disks, become overstable when the viscous effect is taken into account (see Negueruela & Okazaki 2000 for details). The growth rate is given by

$$\mathrm{Im}\{\omega\} \simeq \alpha\Omega(k_r H)^2, \tag{8}$$

where k_r is the radial wave number and H is the scale-height of the disk. For the lowest order $m = 1$ mode, for which $k_r \sim \pi/r$ with r being the radius of the propagation region, the growth rate is $\sim \alpha\Omega(\pi H/r)^2$.

We have solved the equations for linear $m = 1$ perturbations in viscous transonic decretion disks. Some eigenmodes are given in Figure 4. They are for (a) $\alpha = 0.4$, (b) $\alpha = 0.2$, (c) $\alpha = 0.1$, and (d) $\alpha = 0.01$. From Figure 4, we observe that the characteristics of the modes are basically the same as those of the neutral modes discussed in section 3. Besides those characteristics, we note that the perturbation pattern of the overstable $m = 1$ modes is a leading, one-armed spiral. The spiral pattern becomes looser with increases in α. In other words, the propagation region of a one-armed spiral wave becomes larger with increasing α.

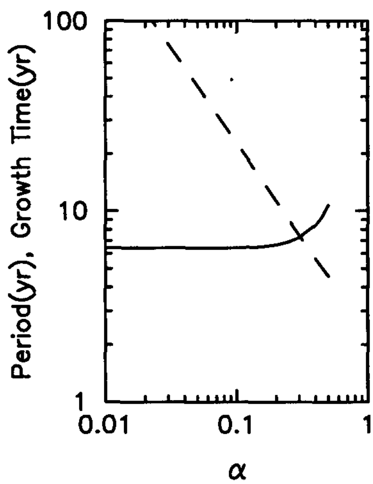

Figure 5. Oscillation period (solid line) and growth time (dashed line) of the one-armed fundamental mode confined to the inner part of the viscous transonic decretion disk. The stellar and disk parameters adopted are the same as those for Figure 4. Note that the period and the growth time become comparable for $\alpha \sim 0.3$.

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The period and growth time of the one-armed spiral mode is shown in Figure 5. We find that the growth time obtained numerically agrees well with the analytical one given above. Note that the timescale of the growth of the V/R variation is a measure of α. Thus, long-term V/R variation, in principle, enables us to probe the viscosity parameter in Be disks.

6. Disk Warping — Another Global Mode

In general, there can exist two types of $m = 1$ modes in near Keplerian disks. One is the so-called eccentric modes, in which the density perturbation and the horizontal component of the velocity perturbation are symmetric with respect to the equatorial plane. The modes we have discussed in the previous sections are eccentric modes.

The other type of $m = 1$ modes are z-antisymmetric modes, in which the density perturbation and the horizontal component of the velocity perturbation are antisymmetric with respect to the equatorial plane. The lowest order mode of this type is called a warping mode. If the disk has an optically thick part to the radiation from the central star, the warping mode can be excited (Pringle 1996). Porter (1998) showed that this is indeed the case in Be disks: the Be disks are optically thick in the IR and thus become unstable to warping.

Possible observational evidence for the warping mode in Be disks is the spectacular emission line variations in γ Cas and 59 Cyg (Hummel 1998). Both stars have shown two successive shell events associated with a synchronous quasi-cyclic variation of the line widths in all emission lines. Hummel (1998) interpreted the spectacular variation to be due to a tilted circumstellar disk with precessing nodal line, which is likely a warped inner disk (Porter 1998). The precession timescale of warped disks around early Be stars is about several hundreds of days (Porter 1998), which is roughly consistent with the observationally derived timescale of ~ 1000 days for the spectacular variation of γ Cas (Hummel 1998).

7. Summary

It is well established that a Be star has a two-component extended atmosphere, a polar wind region and a cool equatorial disk. In contrast to the polar wind region explained well by the radiation-driven wind model, the nature of the equatorial disk, e.g., the origin, the structure and dynamics, and the evolution, is little understood. Understanding even one of these aspects of the nature of Be disks is highly desirable.

In this paper, we have focused on the Be-disk structure and global disk oscillations (GDOs). Disks around Be stars are considered to be geometrically thin and near Keplerian. We have shown that the only possible global oscillations in such disks are very low-frequency, $m = 1$ oscillations. The $m = 1$ oscillations can be confined to an inner disk only if the radial flow in this part of the disk is very subsonic. Observationally, many Be stars exhibit long-term variations of the relative intensities of the violet and red components in double-peaked emission-line profiles. This long-term V/R variation has been considered to be a key to understanding of the structure and dynamics of Be disks. We have shown many features of the long-term V/R variation are naturally explained by even a simplest version of the GDO model, in which neither the radial advection nor the viscous effect is taken into account.

Next, we have considered the steady disk structure around Be stars, based on the viscous decretion disk scenario proposed by Lee et al. (1991). We have found that there is a transonic solution of decretion for any value of α, the viscosity parameter. The outflow, which is very subsonic near the star, is accelerated by pressure and becomes supersonic far from the star. The disk is near-Keplerian in the inner subsonic part, while it is angular-momentum conserving in the outer subsonic part and in the supersonic part. The transonic solution of viscous decretion is in good agreement with the observed properties of Be disks.

Then, we have examined the effects of viscosity on the GDO characteristics, solving equations for linear $m = 1$ perturbations on unperturbed transonic decretion disks. The characteristics of the $m = 1$ eigenmodes are basically the same as those in inviscid disks. Besides those characteristics, we have noted that the modes are overstable with the growth rate proportional to α. We have also noted that the perturbation pattern is a leading one-armed spiral, and the spiral pattern becomes looser with increasing α.

The GDO model based on the viscous decretion disk scenario agrees well with the observed disk structure and the characteristics of the long-term V/R variation. The current model, however, lacks knowledge about some important issues in order for the model to be a total theory of Be disks. The most important issue to tackle is the mass supply mechanism from the star. For many years, we have failed to find how material with the Keplerian angular momentum at the stellar surface can be supplied from the star. It is no doubt that finding it will become a great step toward understanding of the Be phenomenon. Another issue to explore is the evolution of the Be disks. Since the current model is based on the linear perturbation analysis, it can tell us little about how disks evolve. Nonlinear simulations of Be disks are therefore highly desirable.

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Discussion

P. Harmanec: You made a very clear presentation of what are the conditions under which V/R variations in Be star disks can occur. I believe this once more exposed the main unanswered problem we have here for decades: what is the ultimate cause of the Be phenomenon. Consider the case of ζ Tau, a B2e star in a 133 d binary. As I have shown in my review here, the radius of the Be star must be something like $5 - 6\,R_\odot$. Unless its mass is very anomalous, the Keplerian rotation near equator must be about $600\,\mathrm{km\,s^{-1}}$. The observed $v \sin i$ of $320\,\mathrm{km\,s^{-1}}$ must be close to the true equatorial velocity since the Hα photometry (Pavlovski et al. 1997, A&AS) shows that the star is observed nearly equator-on. Any scenario assuming a mass *outflow* from the star must, therefore, address the question where to take the energy needed to accelerate the material to about twice higher rotation rate! Note that a Keplerian disk is formed in a very natural way if the disk is formed instead by a gas stream inflowing from a secondary star in a binary system (either via Roche-lobe overflow or via strong stellar wind).

A. Okazaki: I agree with you that we have a great difficulty to construct a model to supply material from the central star, and it is a severe weakness in the current viscous decretion disk model. Although a Keplerian disk is naturally formed by accretion, there are many Be stars which show no signature of binarity or have compact companions. I prefer to consider mechanisms that work to both isolated and binary Be stars.

L. Balona: You have, quite naturally, not considered the effect of even a very weak magnetic field on the ionized disk. To what extent will your analysis be effective by the presence of such a field?

A. Okazaki: Since one-armed oscillation modes are very sensitive to the deviation of the disk rotation from the Keplerian rotation, the presence of a magnetic field can affect mode characteristics significantly. If Be stars have a global magnetic field > 100 Gauss, the analysis I have made above will break down.

S. Owocki: The viscous decretion disk is a nearly continuous diffusion process. Yet it seems many observers tend to favor a picture in which there is an inner "hole" in the disk. It seems that requires a more direct propulsion of material into a disk by a moment arm, e.g., by a magnetic field. I thus encourage observers to focus on the question of whether there really must be such an inner hole, to guide theorist on which kind of scenario to further explore.

J. Cassinelli: Is there a minimal disk mass required for the one arm instability, i.e., is self-gravity important? You say that the instability grows along with the growth on mass increase of the disk. So should we expect V/R effects to occur after some time after disk growth is initiated?

A. Okazaki: No, there isn't. The one-armed instability discussed above is driven by viscosity, and the self-gravity of Be disks is negligible. Since the growth time is roughly comparable with the drift timescale, V/R variation should become observable after some time after the disk formation process begins.

H. Henrichs: What can you say about the dependence of the V/R timescale on the spectral type of the central star?

A. Okazaki: The spectral dependence of the V/R timescale is complicated. There is no simple correlation between the V/R timescale and the spectral type of the star, because the timescale depends on many stellar and disk parameters. For example, higher temperature makes the V/R timescale longer, while stronger radiative force or faster rotation makes the timescale shorter. Moreover, the radiative force seems important in early Be stars, whereas the rapid rotation is much more important than the radiative force in late Be stars. As a result, the V/R variation seems to show no strong spectral dependence.

W. Hummel: You described how viscosity affects GDO. Can you comment on how GDO affects the viscosity and the transport of angular momentum?

A. Okazaki: Since the one-armed density wave in viscous decretion disks is a leading spiral, the angular momentum is transported from the wave to the disk matter. This means GDO acts to increase the effective viscosity.