THE DETERMINATION OF ABSOLUTE PHASE OF A LONG BASELINE INTERFEROMETER AND ITS APPLICATION TO THE PRECISE MEASUREMENT OF THE CONSTANT OF NUTATION

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Long Baseline Interferometry (L.B.I.) refers to a dramatic technical breakthrough in astronomy accomplished almost exactly ten years ago in the spring of 1967 by a group of workers at the National Research Council of Canada and some major Canadian Universities. The Canadian effort was duplicated within a month by an independent American effort and it was realized soon afterward that the new technique of L.B.I. held enormous potential for geodynamical as well as astronomical studies.

If exploited to its maximum capability L.B.I. promised to determine the earth's orientation in space to $\pm 10^{-3}$ " arc and to measure intercontinental terrestrial distances to $\pm$ 1 cm. These astounding capabilities required for their successful execution a solution to the problem of "fringe identification" or in standard optical parlance the "determination of the absolute order of the interference". The difficulties posed by this problem are chiefly responsible for the failure over the past ten years of L.B.I. to achieve anywhere near its ultimate capabilities as a tool of geodynamical research. A number of recent developments in the field of L.B.I. indicate that a solution to the difficult "fringe identification problem" is at hand and that we stand today poised on the brink of an exciting era in which a variety of outstanding geodynamical problems will be attacked with unprecendent experimental accuracy by L.B.I. techniques.

The modern long baseline interferometer consists of two fully steerable parabolic antennae at positions $\vec{r}_1$, $\vec{r}_2$ receiving radiation from a source whose direction is indicated by the unit vector $\hat{s}$. The received radiation is amplified, translated in frequency to a video band by mixing with a harmonic signal derived from a local oscillator phase locked to a high quality frequency standard and, after further amplification, is recorded on standard video tape. The frequency standard is also used to drive a clock to provide timing information on the video tape which is used to obtain correlation between the two tapes.

It can be shown [1] that the phase $\Phi(t)$ of a long baseline interferometer is given by

E. P. Fedorov, M. L. Smith and P. L. Bender (eds.), Nutation and the Earth's Rotation, 129–138.

$$\Phi(t) = \int_{0}^{t+\tau/2} \omega_1(t)dt - \int_{0}^{t-\tau/2} \omega_2(t)dt + \lambda_1(t+\tau/2) - \lambda_2(t-\tau/2) - \omega_0 \cdot [\tau(t) - \Delta\tau_I - \tau_g(t)]$$

where

(a) $\omega_1(t)$ $\omega_2(t)$ are the nominal local oscillator frequencies at each antenna site.

(b) $\lambda_1(t)$, $\lambda_2(t)$ are the local oscillator "phase errors" at each antenna site due to the integrated effects of the departures of the actual local oscillator frequencies from their nominal frequencies.

(c) ω_0 is the observing frequency of the interferometer.

(d) $\tau(t)$ is the correlation delay - the amount by which one video tape record is physically delayed relative to the other during correlation.

(e) $\Delta\tau_I$ is the differential instrumental delay - the difference, from one antenna to the other, between the signal propagation delays from the focus of the antenna, through the receivers, to the write heads of the video tape recorders.

(f) $\tau_g(t) = 1/c\ \hat{s} \cdot [\vec{r}_2(t-\tau/2) - \vec{r}_1(t+\tau/2)]$ is the "geometric delay". This is the signal propagation delay from one antenna located $\vec{r}_2$ to the other antenna located at $\vec{r}_1$ when due account is taken of the motions of the antenna relative to an inertial frame during the transit of the signal.

The expression for $\Phi(t)$ appears to be acausal involving events occurring both before <u>and</u> after the time t. This is of course the case since the interferometer described does not operate in "real time".

Denoting

$$\Delta\lambda(t) = \lambda_1(t+\tau/2) - \lambda_2(t-\tau/2)$$

$$\Delta\tau(t) = \tau(t) - \Delta\tau_I - \tau_g(t)$$

we can write the expression for the phase $\Phi(t)$ as

$$\Phi(t) = \int_{0}^{t+\tau/2} \omega_1(t)dt - \int_{0}^{t-\tau/2} \omega_2(t)dt + \Delta\lambda(t) - \omega_0 \cdot \Delta\tau(t)\ .$$

Now, the values of the two integrals are essentially known exactly as is the value ω_0. It is the quantities $\Delta\lambda(t)$ and $\Delta\tau(t)$ which have traditionally been uncertain by many hundreds or thousands of turns of phase. It is these two quantities which it is now possible to determine within an accuracy of less than 2π and hence determine the "absolute phase" of the interferometer.

(I) <u>Bandwidth Synthesis</u>

Consider first the question of $\Delta\tau(t)$. This quantity is typically uncertain by an amount equal to the "delay resolution" of the interferometer which is in turn given by the reciprocal of the recorded bandwidth. For a recorded bandwidth of 5 MHz $\Delta\tau$ is about 200 nanoseconds. At an observing frequency of 10 GHz (3 cm. wavelength) this results in a phase uncertainty of $\omega_0 \cdot \Delta\tau$ of the order of 2000 turns of phase. The solution to the problem of a precise measurement of $\Delta\tau$, precise enough to resolve all 2π ambiguities, is provided by the technique of "bandwidth synthesis".

Bandwidth synthesis as a technique for identifying fringes in a long baseline interferometer was proposed by T. Gold [2] in 1967 and later discussed in greater detail by A.E.E. Rogers [3] in 1970. Bandwidth synthesis takes advantage of the fact that while a long baseline interferometer does a poor job of measuring absolute phase it can measure relative phase very well. It is in reality the radio interferometer analog of the technique in classical optics known as the "method of exact fractions" used to calibrate Fabrey-Perot etalons and also used to measure the meter in units of the wavelength of visible radiation.

We see from the expression for interferometer phase that

$$\Delta\tau(t) = - d\Phi(t)/d\omega_o$$

and so interferometer phase can be expressed implicitly

$$\Phi(t) = \int_o^{t+\tau/2} \omega_1(t)dt - \int_o^{t-\tau/2} \omega_2(t)dt + \Delta\lambda(t) + \omega_o \cdot d\Phi(t)/d\omega_o$$

The technique of bandwidth synthesis then is to scan the observing frequency ω_o across a wide band of frequencies measuring the change of interferometer phase with frequency, a measurement requiring only relative phase changes, in order to establish the value of the derivative $d\Phi/d\omega_o$. By sequentially switching the observing frequency across the entire bandwidth of the front end receiver (150 MHz) it is possible to determine $\Delta\tau$ to a very high accuracy.

Consider the case of a typical modern long baseline interferometer: System Tmperature $T_S \sim 100°K$, Antenna Temperature $T_A \sim 1° K$, Recorded Bandwidth $B \sim 5$ MHz, Correlation Integration Interval $2\Delta T \sim 0.4$ seconds, Frequency Stability $\delta\omega_o/\omega_o \sim 1 \times 10^{-12}$.

It can be shown [3] that the instrumental sensitivity to phase measurements $\delta\Phi$ is $\delta\Phi = \pm T_S/T_A\sqrt{2\Delta TB}$ rad. which for the above values of the parameters gives a phase sensitivity of $\delta\Phi \sim \pm 0.07$ rad. The uncertainty in $\Delta\tau$, $\delta[\Delta\tau]$, equals the uncertainty in the derivative, $\delta[\Delta\Phi/\Delta\omega_o]$, and is approximately equal to $\delta[\Delta\tau] = \delta[\Delta\Phi/\Delta\omega] \cong \delta[\Delta\Phi]/\Delta\omega_o$ or $\delta[\Delta\tau] = 47$ ps. This is sufficient to resolve all 2π ambiguities in the interferometer phase due to inadequate delay resolution for observing frequencies of 10 GHz or less (3 cm. wavelength or longer).

(II) Satellite Phase Synchronization

Consider secondly the question of $\Delta\lambda(t)$. This quantity is typically uncertain by an amount which grows at the rate of hundreds of turns of phase per day and results from the independent "cycling" of the two on-site frequency standards. Each time one local oscillator gains a turn on the other a "fringe" is generated and the phase of the correlation function is incremented by 2π. Recently Yen has developed a simple, elegant technique using geostationary satellite telemetry to synchronize the phase of remote frequency standards.

Yen's method presupposes a geostationary communications satellite visible from both ends of the interferometer baseline. Let $d(t)$ be the propagation phase delay between the antenna at $\vec{r}_1$ and the satellite at time t and let $d(t) + \delta(t)$ be the same quantity for the antenna $\vec{r}_2$. The total transit time for a signal to pass from $\vec{r}_1$ to $\vec{r}_2$ via the satellite is $T(t) = 2d(t) + \delta(t)$. Let ϕ_{11}, ϕ_{22} be the phases, before transmission to the satellite, of two SHF c.w. signals phase locked at each antenna site to the on-site frequency standard. Denote by ϕ_{12}, ϕ_{21} the phases of the SHF c.w. signals transmitted via the satellite from $\vec{r}_1$ to $\vec{r}_2$ and from $\vec{r}_2$ to $\vec{r}_1$ respectively. Clearly $\phi_{21}(t) = \phi_{22}(t-T(t))$ and $\phi_{12}(t) = \phi_{11}(t-T(t))$.

Most communications satellites use an onboard translation oscillator and mixer to convert the frequency of the up-link signal to the frequency of the down-link signal. If the same translation oscillator, whose phase is ϕ_{ss}, serves to convert the SHF c.w. signals arriving at the satellite from both antennae then $\phi_{21}(t) = \phi_{22}(t-T(t)) - \phi_{ss}(t-d(t))$ and $\phi_{12}(t) = \phi_{11}(t-T(t)) - \phi_{ss}(t-d(t)-\delta(t))$. If the satellite transmits a c.w. beacon, phase ϕ_{bs}, which is phase locked to the translation oscillator then $\phi_{bs}(t) = 1/k_s \cdot \phi_{ss}(t)$ where k_s is a known constant. If ϕ_{b1} ϕ_{b2} are the received beacon phases at $\vec{r}_1$ and $\vec{r}_2$ respectively then $\phi_{b1}(t) = 1/k_s \cdot \phi_{ss}(t-d(t))$ and $\phi_{b2}(t) = 1/k_s \cdot \phi_{ss}(t-d(t) -\delta(t))$. From this it can be shown that $\phi_{21}(t) = \phi_{22}(t-T(t)) - k_s \cdot \phi_{b1}(t)$ and $\phi_{12}(t) = \phi_{11}(t-T(t)) - k_s \cdot \phi_{b2}(t)$.

The phase of each on-site frequency standard can be written $\phi_{11}(t) = \omega_1 t + \lambda_1(t)$, $\phi_{22}(t) = \omega_2 t + \lambda_2(t)$ where $\omega_1 \omega_2$ are the known nominal frequencies of each standard and $\lambda_1(t)$ $\lambda_2(t)$ are the unknown phase errors. At each antenna site the following observable phase differences can be measured: $\Delta_1(t) = \phi_{21}(t) + k_s \cdot \phi_{b1}(t) - \omega_2/\omega_1 \cdot \phi_{11}(t)$ and $\Delta_2(t+\delta(t)) = \phi_{12}(t+\delta(t)) + k_s \cdot \phi_{b2}(t+\delta(t)) - \omega_1/\omega_2 \cdot \phi_{22}(t+\delta(t))$. From the above one can show that $\Delta_1(t) = -\omega_2 \cdot \tau(t) + \lambda_2(t-\tau(t)) - \omega_2/\omega_1 \cdot \lambda_1(t)$ and $\Delta_2(t+\delta(t)) = -\omega_1 \cdot \tau(t) + \lambda_1(t+\delta(t)-\tau(t)) - \omega_1/\omega_2 \cdot \lambda_2(t+\delta(t))$. The difference $D(t) = \Delta_1(t) - \omega_2/\omega_1 \, \Delta_2(t+\delta(t))$ reduces to $D(t) = [\lambda_2(t-\tau(t)) + \lambda_2(t+\delta(t))] - \omega_2/\omega_1 [\lambda_1(t) + \lambda_1(t+\delta(t) - \tau(t))]$.

For a geostationary satellite $\tau(t) \sim 250$ m sec. and for a 2000 km difference in path length between each end of the baseline and the satellite $\delta(t) \sim 6$ m sec. The position of the geostationary satellite is sufficiently stable that $|d\tau(t)/dt| \sim |d\delta(t)/dt| < 5 \times 10^{-9}$. If the frequency standard at each antenna site have stabilities of the order of 10^{-12} then linear interpolation will show that $1/2\, D(t) = \lambda_2(t-d(t)) - \omega_2/\omega_1 \, \lambda_1(t-d(t))$ with errors of the order of 5×10^{-3} rad. For identical frequency standards at each antenna site $\omega_2/\omega_1 \simeq 1$ to parts in 10^{12} and so $1/2\, D(t) = \Delta\lambda(t-d(t))$. For high quality frequency standards $\Delta\lambda(t)$ is sufficiently slowly varying and $d(t)$ sufficiently well known for this technique to allow an a posteriori determination of $\Delta\lambda(t)$ accurate to 10^{-2} rad or better at the observing frequency of the interferometer.

(III) The Use of "Phase Stable" L.B.I. to Study Nutation

It would appear from the preceding analysis that it is possible with state of the art technology to operate a long baseline interferometer in an "a posteriori" phase stable mode. Consider the characteristics of such an instrument assuming an observing frequency of 10 GHz and a baseline length of 5000 km. The minimum fringe spacing on the celestial sphere is 1"2 x 10^{-3} arc and will remain fixed in space relative to the baseline, at least in an a posteriori sense, with a long term stability of a high quality frequency standard. Tidal displacements of the antennae relative to some geocentric average terrestrial frame will cause displacements of the fringe positions relative to the same geocentric average terrestrial frame of the order of ± 10" x 10^{-3} arc. The use of on-site gravimeters could provide corrections for the antenna tidal displacements to ± 1 cm. and allow an a posteriori correction to the fringe position relative to some average terrestrial frame to ± 3" x 10^{-4} arc.

In addition it can be shown [4] that, allowing for antenna zenith angles as large as 70°, random fluctuations in the fringe positions with a time scale of 15-30 minutes due to unmodelled atmospheric effects could be less than 4.5π rad or ± 2"7 x 10^{-3} arc r.m.s. The use of bandwidth synthesis and satellite phase synchronization would allow the continued determination of "absolute interferometer phase", i.e. the identification of each fringe, for extended periods of time. The combined corrupting effects of the earth's atmosphere and earth tidal deformation on the positioning of fringes relative to some average terrestrial frame could be as small as ± 3"0 x 10^{-3} arc.

We shall consider how a long baseline interferometer might be used to measure the nutation. For our purposes we shall regard the nutation as being the process whereby the direction of the earth's rotation axis is displaced periodically relative to an inertial frame. In L.B.I. observations of the nutation the inertial frame will be defined implicitly by the assigned coordinates of a variety of extragalactic compact radio sources. By definition, the direction of the earth's rotation axis is specified by the declinations of objects on the celestial sphere and so the study of the earth's nutation by L.B.I. methods is reduced to the problem of observing changes in the declination of compact extra galactic radio sources. It is possible to use a long baseline interferometer in the manner of a polar telescope to make high precision differential declination measurements of circumpolar or semi-circumpolar radio sources. (Circumpolar radio sources are visible from both ends of the baseline for 24 sidereal hours while semi-circumpolar sources are so visible for at least 12 sidereal hours).

The measurement method basically consists of determining the interferometer phase difference between upper and lower culminations of the radio source. Upper and lower culmination are defined as the points in time at which the radio source transits the meridian of the interferometer baseline and at which times the source is moving parallel to the fringe pattern. If t_u and t_ℓ are the times of upper and lower culmination respectively then $\Delta\Phi_u^\ell$ is given by $\Delta\Phi_u^\ell = \Phi(t_\ell) - \Phi(t_u)$.

The interferometer phase $\Phi(t)$ can be written as

$$\Phi(t) = \int_0^t [\omega_1(t)-\omega_2(t))]\ dt + [\omega_1(t)+\omega_2(t)]/2\cdot\tau(t) + \lambda_1(t+\tau/2) - \lambda_2(t-\tau/2) - \omega_o\cdot\tau(t) + \omega_o\cdot\Delta\tau_I + \omega_o\cdot\tau_g(t)$$

with errors of the order of $\pm 10^{-4}$ turns of phase. The quantity $\omega_1(t)-\omega_2(t)$ is the difference between the nominal frequencies of the two local oscillators and is set to the a priori value of the differential Doppler shift between the antennae, $\omega_d(t)$, $\omega_1(t) - \omega_2(t) = \omega_d(t)$. The observing frequency ω_o is defined by $\omega_o = [\omega_1 + \omega_2]/2$ and so

$$\Delta\Phi_u^\ell = \int_{t_u}^{t_\ell} \omega_d(t)dt + \Delta\lambda(t_\ell) - \Delta\lambda(t_u) + \omega_o\cdot[\Delta\tau_I(t_\ell)-\Delta\tau_I(t_u)]+\omega_o\cdot[\tau_g(t_\ell)-\tau_g(t_u)]$$

where
$$\Delta\lambda(t_\ell) = \lambda_1(t_\ell + \tau_\ell/2) - \lambda_2(t_\ell - \tau_\ell/2)$$
$$\Delta\lambda(t_u) = \lambda_1(t_u + \tau_u/2) - \lambda_2(t_u - \tau_u/2)$$

and where
$$\tau_u = \tau(t_u)\ , \qquad \tau_\ell = \tau(t_\ell)\ .$$

This can be rewritten as

$$\omega_o\cdot[\tau_g(t_\ell)-\tau_g(t_u)] = \Delta\Phi_u^\ell - \int_{t_u}^{t_\ell}\omega_d(t)dt - \Delta\lambda(t_\ell)+\Delta\lambda(t_u)-\omega_o\cdot[\Delta\tau_I(t_\ell)-\Delta\tau_I(t_u)]$$

On the RHS the quantity $\Delta\Phi_u^\ell$ can be determined with an accuracy of roughly $\pm$ 0.07 rad by simply counting the accumulated interferometer phase as the source moves from upper to lower culmination. The integral over the frequency $\omega_d(t)$ is known to a high precision. This follows from the fact that $\omega_d(t)$ is only of the order of 2×10^4 Hz or less and its nominal value is known with an accuracy comparable to the setability of the hydrogen maser frequency which can be taken to be of the order of one part in 10^{12}. Hence the uncertainty in $\omega_d(t)$ is of the order of 2×10^{-8} Hz and its integral over 12 hours, 4.5×10^4 seconds, has an uncertainty of only $\pm 10^{-3}$ turns of phase or $\pm 6 \times 10^{-3}$ rad. The accumulated differential local oscillator phase errors at upper and lower culmination $\Delta\lambda(t_u)$, $\Delta\lambda(t_\ell)$ can be determined by the method proposed by Yen to roughly $\pm$ 0.01 rad. It appears possible to determine the RHS, excluding the terms involving instrumental delays $\Delta\tau_I$, to an accuracy of at least $\pm$ 0.1 rad.

The largest source of error in this measurement will arise out of uncertainties in the value of the term involving instrumental delays. If we were to require the uncertainty in this term to be no larger than the uncertainty in the rest of the RHS namely $\pm$ 0.1 rad, for an observing frequency of 10 GHz we would require knowledge of $\Delta\tau_I(t_\ell)-\Delta\tau_I(t_u)$ accurate to $\pm 1/2\pi \times 10^{-11}$ sec. Now $\Delta\tau_I(t_u)$ and $\Delta\tau_I(t_\ell)$ can only be established by monitoring the instrumental delay at each antenna separately.

Fortunately $\Delta\tau_I(t_\ell)-\Delta\tau_I(t_u) = [\tau_{I1}(t_\ell)-\tau_{I1}(t_u)]-[\tau_{I2}(t_\ell)-\tau_{I2}(t_u)]$ where τ_{I1}, τ_{I2} are the instrumental delays at each antenna site, and so we see that it is only necessary to make an accurate determination at each antenna site of the change in instrumental delay over the interval t_u to t_ℓ. This can be done by monitoring the relative phase at the output of the receiver system of a number of C.W. signals of known frequency injected into the input of the receiver system. Such phase calibration methods of monitoring instrumental delay variations can be made with an accuracy of $\pm$ 0.2 nsec [5].

With the variations in differential instrumental delay known to roughly $\pm$ 0.2 nsec the term $\omega_0 \cdot [\Delta\tau_I(t_\ell)-\Delta\tau_I(t_\omega)]$ is uncertain by as much as $\pm$ 2 rad or $\pm$ $2/2\pi$ fringes for an observing frequency of 10 GHz. This constitutes the major source of measurement error in the technique. We see that the number N where $N = \omega_0/2\pi \cdot [\tau_g(t_\ell)-\tau_g(t_u)]$ corresponds to the total number (not necessarily integral) of interferometer fringes crossed by the source as it moves from upper to lower culmination and can be determined observationally to roughly $\pm$ $2/2\pi$ fringes. In this sense N is a measure, in units of the variable interferometer fringe spacing, of the angular distance between the positions of the radio source at upper and lower culmination.

Although it is possible to use the number N and knowledge of the interferometer fringe spacing to make an absolute determination of the polar distance Δ and hence the declination δ of the source this cannot be done to a very high accuracy. The accuracy in the fringe spacing is limited by the accuracy in the baseline length which is typically no better than a few parts in 10^7. Thus though the fringe spacing for the interferometer being considered here has a minimum value of $1''.2 \times 10^{-3}$ arc this number is uncertain at the level of 10^{-10} "arc. In attempting to interpret N in terms of Δ this systematic error will accumulate and will ultimately produce an error in Δ of the order of 10^{-10} N "arc. If N is small as it will be for sources near the pole this is of little consequence. However for this systematic error in the measurement of Δ to be less than 10^{-3} "arc we require N to be less than 10^7 fringes. At a fringe spacing of the order of $1''.2 \times 10^{-3}$ arc this means the source must be within about 1.8° of the celestial pole. Obviously as a method for making high accuracy declination determinations for sources at large distances from the celestial pole this procedure fails not because of errors in the determination of N but because of the inability to convert N reliably to an accurate value of Δ.

Fortunately to study the earth's nutation it is necessary only to study changes in the declination of a source which can be established to an accuracy comparable to the interferometer fringe spacing by observing changes in the numerical value of N. A difference ΔN between the fringe counts $N(t_1)$ and $N(t_2)$ at epochs t_1 and t_2 will indicate a change in the angular diameter of the parallel of declination of the source. This would imply that the source had moved relative to the celestial pole and a continuing sequence of such observations would facilitate a study of earth nutation to a high accuracy, comparable with the fringe spacing of the interferometer.

The expected errors implicit in this procedure to study nutation require examination. Firstly each measurement $N(t_1)$, $N(t_2)$ is corrupted by the differential precession and nutation which occurred during the interval of approximatley 12 sidereal hours required to complete the fringe count. This amounts to roughly 70" x 10^{-3} arc and must be corrected for on the basis of the present theory of precession and nutation. If the present theory of precession and nutation has a precision of four significant figures then the differential precession and nutation occurring over this 12 hour interval can be corrected for and the measurement reduced to a single epoch with an accuracy of roughly $\pm$ 1" x 10^{-4} arc.

Secondly the measurements must be corrected for the effects of polar motion. In general it is necessary to consider both the effects of polar motion occurring during the fringe count and as well as the effects of polar motion occurring in the interval between the separate determinations $N(t_1)$ and $N(t_2)$. In the former case, it must be recognized that there are today no observations made of the short term (minutes to hours) variations in the pole position. However if the motion of the pole position is "reasonably smooth", i.e. barring the possibility of large impulsive torques on the mantle which shift the pole position discontinuously, one can place a reasonable upper limit on the rate of polar motion of 6" x 10^{-8} arc sec^{-1}. If the polar motion were to persist at this rate for 12 hours as the source moved from upper to lower culmination it would alter the observed value of Δ by 2".6 x 10^{-3} arc. However judging from the similarities in the observed rates of polar motion between the smoothed ILS-IPMS data and the smoothed BIH data it is likely that corrections for the effects of the rotation of the baseline and the resulting rotation of the fringe pattern on the sky due to polar motion over a 12 hour interval could be made a posteriori with a precision of $\pm$ 30% or perhaps much better when the rate is large. Hence the observed value of N could be corrected for the effects of polar motion with an accuracy of roughly $\pm$ 5" x 10^{-4} arc.

In the latter case it should be pointed out that the method of measurement being proposed is to first order insensitive to the effects of polar motion occurring in the interval between separate determinations $N(t_1)$ and $N(t_2)$. This follows from the fact that rotations of the baseline relative to inertial space resulting from polar motion drag the fringe pattern across the sky and, to first order, cause the same number of fringes to enter and leave the interior of the small circle of the source's parallel of declination, thus preserving, to first order, the value of N. This property of fringe compensation to preserve the value of N is satisfied exactly for an East-West baseline. So for an East-West baseline the measurement method is insensitive to polar motion occurring in the interval between separate determinations $N(t_1)$ and $N(t_2)$.

For baselines which are not East-West this exact fringe compensation for polar motion does not occur. It can be shown that the fringe miscompensation δN, that is the difference between fringes entering and leaving the interior of the small circle of the sources parallel of declination due to polar motion, is given by $\delta N = 2\, b/\lambda \sin \delta_b \cos \delta\ \delta\theta$ where:

(i) λ is the wavelength being observed by the interferometer. In the case under consideration λ = 3 cm.

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(ii) b is the total length of the interferometer baseline. In the case under consideration b = 5000 km.

(iii) δ_b is the declination of the vector baseline and δ is the declination of the source.

(iv) $\delta\theta$ is the amount of polar motion which has occurred in the stated interval parallel to the meridian of the vector baseline. The measurement is insensitive to polar motion orthogonal to the meridian of the vector baseline.

Now for reasonably low declination sources cos $\delta \sim 1/2$ and this formula reduces to $\delta N \sim 1.6 \times 10^8 \sin \delta_b\ \delta\theta$ fringes. The angle $\delta\theta$ has a maximum value of about 0".3 arc or 1.5×10^{-6} rad. However polar motion determinations by the BIH and others provide values of $\delta\theta$ which are accurate to $\pm$ 0".02 arc, $\pm 10^{-7}$ rad, or thereabouts. Therefore it is possible to correct the determinations $N(t_1)$, $N(t_2)$ for fringe miscompensation due to polar motion occurring in the interval between the determinations with an accuracy of $\pm\ \delta N$ where $\delta N \sim 16 \sin \delta_b$ fringes or or an accuracy of $\pm 19.2 \times 10^{-3} \sin \delta_b$ arc. For this measurement error due to polar motion to be less than $\pm$ 1 fringe or $\pm$ 1".2 $\times 10^{-3}$ arc the declination of the baseline must be less than 3 1/2° or so.

The fringe count will also be corrupted by unmodelled atmospheric phase delays and unmodelled tidal displacements of the antennae. By far the largest of these two corrupting effects is the atmosphere with an r.m.s. phase error of typically $\pm$ 4.5 rad or $\pm$ 2".7 $\times 10^{-3}$ arc on single observations for zenith angles of 70° [4]. This r.m.s. atmospheric phase error at one antenna will combine with another of similar magnitude at the other antenna to produce a net corrupting effect on interferometer phase which is $\sqrt{2}$ times as large or roughly $\pm$ 4" $\times 10^{-3}$ arc. The tidal errors are of the order of $\pm$ 3" $\times 10^{-4}$ arc. While the tidal errors are likely to be systematic the atmospheric corruption to the fringe count is likely to be random with (it is hoped) a mean value of zero and a time scale of fluctuations of the order of 15-30 minutes. This means that the corrupting effects of the atmosphere on the fringe count can be average by curve fitting or some equivalent means over the 12 hour interval required to make the measurement. This will reduce the corrupting effects of the atmosphere by a factor of $1/\sqrt{N}$ where $N \sim 25$. In this way it would seem possible to reduce this effect to something of the order of $\pm$ 8" $\times 10^{-4}$ arc.

These results are summarized in Table I. It can be readily seen that the measurement errors are strongly dominated by error #3 due to fringe miscompensation resulting from polar motion between fringe counts. Fortunately this source of error can be made to vanish exactly if observations are taken on an East-West baseline. Assuming this to be the case we shall neglect this source of error entirely. The remaining sources of error listed in Table I will combine to give a net residual error after corrections of the order of $\pm$ 1" $\times 10^{-3}$ arc if they are random quantities. This residual error after corrections must be combined with the measurement error on the fringe count of $\pm 1/\pi$ fringe or $\pm$ 4" $\times 10^{-4}$ arc arising out of uncertainties in the variations in the

TABLE I: Showing error budget for residual errors following corrections to the fringe count for a variety of geophysical effects.

1.	Residual error in fringe count after correcting for differential precession and nutation during measurement	$\pm$ 1" x 10^{-4} arc
2.	Residual error in fringe count after correcting for differential polar motion during measurement	$\pm$ 5" x 10^{-4} arc
3.	Residual error in ΔN (differential fringe count) after correcting for polar motion between separate determinations of N	$\pm$ 19".2 x 10^{-3} sin δ_b arc
4.	Residual error in fringe count resulting from unmodelled tidal displacements of antennae	$\pm$ 3" x 10^{-4} arc
5.	Residual error in fringe count resulting from unmodelled atmospheric phase delays	$\pm$ 8" x 10^{-4} arc

differential instrumental delay of the interferometer. Considering these results it seems that it would be possible with an interferometer of this design to study variations in the orientation of the earth's rotation axis in inertial space with an accuracy of the order of $\pm$ 2" x 10^{-3} arc.

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