Correspondence

DEAR EDITOR,

This letter comes from a retired Civil Engineer following up recent articles [1, 2] written in response to a request for information on the mathematics of bowls [3].

The articles proved the constancy of the delivery angle for a given bowl on a given green (angle θ in [1, Figure 2]). What has to be realised however is that on a given speed of green each of the fourteen bowls (7 Medium and 7 Heavy Weight) in general use before the introduction of narrow bias bowls, behaves differently. Having the idea that bowls would be the easiest of the ball games to analyse, I requested assistance from a neighbour, Alan Harper, a physicist with the CSIRO in Sydney, who built the swing arm bowler for Brearley and Bolt's experiments. He put me onto Daish [4] whose book appeared to discuss all ball games except bowls. However, Daish postulated that a ball will skid in becoming rotational when struck dead centre by a club (putted golf ball, billiard ball). A lawn bowl pushed forward when delivered will also behave in this way. If V is the initial velocity of delivery, it is easily proved that when the bowl stops skidding and becomes fully rotational, its velocity of translation is $5/7V$ and the distance, d, it skids is $0.25V = 4$ feet when $V = 16$ feet/sec. The $5/7V$ value is constant in that if the green has less friction the bowl travels further in reaching the $5/7V$ value. On a slightly damp green the skid of a correctly delivered bowl has been seen as a faint streak on the grass and heard as a faint hiss, which has been confirmed with other bowlers.

There is more to Bowls, as a game than pure mathematics; it calls for an application of mathematics, physics and geometry. A knowledge of geometry of the body is essential if a bowl is to be effectively delivered. Numerous articles have been written on this aspect by Ezra Wyeth in *Bowls Alive in N.S.W.* Wyeth is an Australian who used to spend six months each year in America.

Yours sincerely,

JOHN E. McGLYNN

3 Lewis Street, Balgowlah Heights, Sydney, 2093, N.S.W., Australia

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DEAR EDITOR,

An abnormal distribution of intelligence

A recent popular book of puzzles produced by a leading Social Club for the intelligent advertises that 'membership is accepted from all persons with an IQ of 148 or above. This represents 2% of the population.' Anyone who has taught any statistics will recognize that this doesn't seem right. IQ tests have long been standardized to have a mean of 100 and a standard deviation of 15 or 16 and the scores are a good example of the normal distribution.

If the standard deviation is 16, then a score of 148 is three standard deviations from the mean and a table of the normal distribution shows that the probability of being this far (or farther) from the mean is .00135 or .135% so the stated probability is off by a factor of about 15! The 2% point of the normal is 2.0625 and multiplying this by 16 gives 33, so the appropriate IQ value should be 133.

Taking the standard deviation as 15, the probability of achieving 148 or higher is .00069 or .069% which is off by a factor of about 29. The 2% point is then actually at an IQ of about 131.

All in all, this did not seem very intelligent to me and I wrote to the *Guardian* which had repeated the assertion in an article on Tuesday 2 June 1997. They published my comment along the lines above, followed by a reply by Allan White of the Univ. of Birmingahm explaining that the Mensa test has a standard deviation of 22.5. I wrote again to the *Guardian* (not published) as follows

> Dear Sir,
>
> I am pleased that Allan White has cleared up the statistical confusion regarding Mensa tests. The following was cut from my original letter: 'Mensa's statistics would make sense if they took a standard deviation of about 23 points, or if they shift the average IQ to about 115, but these are so far from the accepted standard that one should not call them IQ tests.' Using a scale that measures 22.5 where people expect 15 leads to inflated scores, a bit like using a two-foot rule and calling it a yardstick, i.e. their scaling is a foot short of a yard. Doing this on a market stall would bring out the Trading Standards Officers. Mensa is welcome to use any test and scaling they want, but it is deceptive to call these IQ scores.
>
> Sincerely,
>
> David Singmaster.

I wonder if other readers of the *Gazette* have been similarly puzzled by mentions of extremely high IQ scores.

Yours sincerely,

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DEAR EDITOR,

Some new inverse cotangent identities for π

We would like to report on our progress in this field following on from the work described in [1, 2 and 3]. We have obtained numerous new identities (available from the authors) in which the integral cotangent values have at least 5, 6, 7, 8 and 9 digits, respectively. Using Lehmer's measure [1] as our criterion, our best results in these categories (with dates of discovery) are the following;

Best 5-digit cotangent identity:

{1} = 19162{40515} + 12000{51412} + 9000{219602} + 11407{734557} + 26463{1039784} – 6271{6826318} – 2988{7626068} – 15764{9639557} + 183{21072618} + 8419{2539791558}

(12 May 97) Measure: 1.63086

Best 6-digit cotangent identity:

{1} = 36462{390112} + 135908{485298} + 274509(683982} – 39581{1984933} + 178477{2478328} – 114569{3449051} – 146571{18975991} + 61914{22709274} – 69044{24208144} – 89431{201229582} – 43938{2189376182}

(First discovered by Arndt, 1993: see remarks below) Measure: 1.63050

Best 7-digit cotangent identity:

{1} = 439650{1065376} + 562730{1610057} – 182273{6682866} + 279594{6826318} – 274772{7626068} – 20463{9639557} + 849753{21072618} + 318002{72079977} + 183379{103224943} + 165192{130357318} – 248691{2539791558} – 45243{18221678207} + 224134{30446482737}

(19 Aug 97) Measure: 1.71934

Another 7-digit cotangent identity:

2{1} = 893758{1049433} + 655711{1264557} + 310971{1706203} + 503625{1984933} – 192064{2478328} – 229138{3449051} – 875929{18975991} – 616556{21638297} – 187143{22709274} – 171857{24208144} – 251786{201229582} – 432616{2189376182}

(4 Apr 97) Measure: 1.73115

Best 8-digit cotangent identity:

{1} = 2196033{12477035} + 7498561{18975991} – 560323{22709274} – 863651{24208144} + 7645132{42267682} + 1796827{44279097} + 1860735{63199427} + 2003244{201229582} + 3646482{306903943} + 4944419{1258140850} – 1981094{1624720807} – 1106947{2189376182} – 5587995{17249711432} – 1534215{52254287493} – 1573796{579766497643}

(11 Sep 97) Measure: 1.77957

Best 9-digit cotangent identity:

2 {1} = 82367111{201229582} + 69151993{231373438} + 147962666{262992072} + 98495517{284862638} – 40395090{318942476} + 8487197{1258140850} – 4706801{1504779193} – 103925268{1624720807} + 171256136{1848369102} + 117453109{2189376182} + 2612971{2539791558} – 37214354{3712239557} + 120244869{41734246913} + 51797709{52254287493} + 39817224{66492889557} – 24376984{73276714818} – 127113931{579766497643} + 70137324{69971515635443}

(5 Sep 97) Measure: 1.87698

(Note: $\{n\} = \text{arccot}\, n$; $\{1\} = \pi/4$)

We believe that, apart from the 7-digit identity with measure 1.73115 (whose interest lies in the fact that it contains only 12 terms), these are the best identities in these categories so far discovered by anyone; they supersede the corresponding 'bests' listed in [2]. All were discovered by ourselves, using methods outlined in [3], based on Todd's reduction process. An additional advantage of the database system described in [3] is that a very much faster algorithm for carrying out Todd's process can be used, since only a limited, known, set of potential prime factors is involved; this permits the use of double-length floating-point arithmetic, removing the 31-bit ceiling on cotangent values previously imposed.

We need to be cautious in claiming priority for these identities. Hwang Chien-lih independently discovered the above 6-digit identity with measure 1.63050 on 12th May, 1997. At the beginning of June he came across that same identity in an Internet file which appeared to contain the successful results of attempts to enhance Gauss's remarkable 4-digit identity (measure 1.9568 – see [1]) by involving the additional primes 73, 89, 97, 101 and 109. According to its discoverer, Joerg Arndt [4], the 1.63050 identity first came to light in the autumn of 1993.

We are also extremely grateful to Joerg Arndt for pointing out a misattribution in [1]. The identifier $\{1\} = 44\{57\} + 7\{239\} - 12\{682\} + 24\{12943\}$ (measure: 1.58604) was originally published by Carl Størmer [5, p. 85] in 1896, and it is therefore incorrect to refer to it as 'Wrench's identity'.

Yours sincerely,

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Editor's Note: This will be the last report on this topic for the time being.

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DEAR EDITOR,

In a recent edition of the *Gazette* and again in his book *New Mathematical Diversions,* Martin Gardner repeats the claim that Lewis Carroll invented the game of Doublets (WARM to COLD, APE to MAN, etc.) at Christmas in 1877, calling the game WORD-LINKS.

However, when researching my biography of George Boole, I examined Boole's notebooks which are held in the Library of the Royal Society in London. These notebooks contain several word-links in Boole's hand, so, as Boole died in 1864, it seems that the origins of this fascinating game go back further than 1877. Actually, from other evidence relating to his marriage, I would place Boole's notebooks as after 1855.

I am not suggesting that Boole invented this game and it is also possible that Carroll rediscovered it independently in good faith.

Yours sincerely,

DES MACHALE
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Problem corner

Solutions are invited to the following problems. They should be addressed to Graham Hoare at Dr Challoner's Grammar School, Chesham Road, Amersham, Bucks HP6 5HA, and should arrive not later than 10 March 1998, please.

81.I (J. Wolstenholme, 1862; spotted by Tony Crilly)

Prove that if p is a prime, $p \geqslant 5$, then $\binom{2p-1}{p-1} - 1$ is divisible by p^3.

81.J (Cyril F. Parry)

Triangle ABC, with angles $\frac{4\pi}{7}, \frac{2\pi}{7}, \frac{\pi}{7}$ and sides a, b, c, is inscribed in a circle of unit radius. If $u_n = a^{2n} + b^{2n} + c^{2n}$ for integral n, show that

(i) $u_n = \frac{7}{2}\binom{2n}{n} = \dfrac{7.2n!}{2.n!\,n!}$ for $1 \leqslant n \leqslant 6$, and

(ii) find the corresponding combinatorial function of n for u_n when (a) $n \geqslant 7$ and (b) $n \leqslant 0$.

81.K (Snookered: Tantalising Triangular Sums by Oyler)

If a number of balls can be arranged to form a triangular array as in, for instance, the red balls at the start of a game of snooker then that number is a member of the set of triangular numbers.

Each quartet (W, X, Y, Z), say, consists of either three consecutive or three singly alternate (take one miss one etc.) or three doubly alternate take

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