bracket are verified. Next this bracket is derived via explicit reduction from a Lagrangian particle formulation of fluid dynamics. The Euler–Poisson bracket leads naturally to the conservation of vorticity in terms of Casimir functionals. The chapter ends with an application of Noether's Theorem. Unfortunately, the author decided not to mention the concept of particle relabelling, which is at the very heart of the Lagrangian to Euler reduction process. The next chapter provides an extensive discussion of stability results for steady Euler flows. The stability theory of steady flows is complicated by the fact that stationary flows do not, in general, satisfy the first order necessary conditions for an energy minimum. Thus the classical stability methods break down. V. I. Arnold suggested the construction of an invariant pseudo-energy functional. For parallel shear flows Arnold's linear stability theorems reduce to Fjortoft's results. Furthermore, Arnold established sufficient conditions which would establish nonlinear stability. The author presents Arnold's stability results as well as important recent developments, such as Andrew's Theorem, from a general variational point of view and its associated Hamiltonian formulation.

An interesting generalization of the two-dimensional vorticity equation is provided by the Charney–Hasegawa–Mima (CHM) equation, which arises from the shallow-water equations for rotating fluids in the limit of small Rossby numbers. The CHM equation has dispersive linear wave solutions, called Rossby waves, and has also nonlinear steadily travelling dipole vortex solutions. These solutions play an important role in large scale evolution of the planetary atmosphere. The Hamiltonian structure of the CHM equations and its derivation are discussed in Chapter 5. A large portion of that chapter is then devoted to the stability of steady solutions. An important new feature is the existence of steadily travelling waves. The discussion of their stability leads to important modifications in the previously presented framework; these are also discussed in Chapter 5. The final chapter is concerned with the Hamiltonian structure and the associated stability theory for the celebrated Korteweg–de Vries (KdV) equations.

The book is presented in a refreshingly non-technical style with plenty of details and exercises provided. The reader should be familiar with basic fluid dynamics, classical mechanics, variational calculus, and stability theory. The text can be recommended for advanced undergraduate students and graduate students in applied mathematics and physical sciences. All in all, this is a well-written introduction to Hamiltonian fluid dynamics and basic stability results.

S. REICH

Balser, W. *Formal power series and linear systems of meromorphic ordinary differential equations* (Universitext, Springer, 2000), xviii+299 pp., 0 387 98690 1 (hardcover), £32.50.

Divergent series occur in a variety of situations. A typical example (considered in the Introduction of the book under review) is of the (meromorphic) ODE,

$$t^2 f' = f - t, \tag{1}$$

which admits the formal asymptotic expansion,

$$f(t) \sim \sum_{n=0}^{\infty} n! t^{n+1},$$

which of course has zero radius of convergence. Attempts to sum divergent series go back at least to Euler. Some such attempts are completely formal and resemble typical student standards of rigour. For example, consider the series,

$$1 - 2 + 4 - 8 + 16 - \cdots , \tag{2}$$

the partial sums S_n of which satisfy $S_n = (1 - (-2)^n)/3$. If we want to associate a 'sum' S with this series, we see that it has to satisfy $2S = 1 - S$, i.e. $S = 1/3$, which should be inspected

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in the light of behaviour of the partial sums. Considerations of 'computations' of such form led Abel in 1826 to write famously to Holmboë: 'Les séries divergentes sont, en général, quelque chose de bien fatale, et c'est une honte qu'on ose y fonder aucune démonstration.'

This would encourage one to avoid divergent series as an analytic tool. However, Abel continues to say that even though proofs based on divergent series are non-rigorous, 'Pour la grande partie, les résultats sont justes, … c'est là une chose bien étrange.' Hence he concludes, 'Je m'occupe à en chercher la raison, problème très intéressant.' The mathematical physics and applied mathematics wholeheartedly agree with his conclusion. M. V. Berry, one of the foremost proponents of the use of divergent series, writes [**2**] that 'an asymptotic series … is a compact encoding of a function, and its divergence should be regarded not as a deficiency, but a source of information about the function.'

A considerable amount of effort has been expended in trying to devise summation methods for divergent series that are *consistent*: if the method associates a 'sum' $\mathcal{S}$ with a formal series f, we want $\mathcal{S}(f)$ to be the usual sum if f is a convergent series. Of course we also want $\mathcal{S}(\alpha f + \beta g) = \alpha\mathcal{S}(f) + \beta\mathcal{S}(g)$, etc. There are many summation procedures that satisfy these requirements; those associated with the names of Césaro and Peano are well known.

However, without doubt the best known, and most frequently used, especially in physics (for example, see [**3**] for a review), summation method is that studied by Borel. Briefly, it works as follows: Let $f(t)$ be a formal series in t, $\sum_{n=0}^{\infty} a_n t^n$, where $|a_n| \leqslant n!$. Consider the function $g(x)$ defined by

$$g(x) = \sum_{n=0}^{\infty} \frac{a_n}{n!} x^n,$$

where the series definitely converges for $|x| < 1$, and associate with $f(t)$ the expression

$$\tilde{f}(t) = \int_0^{\infty} \mathrm{e}^{-x} g(xt)\,\mathrm{d}x = \frac{1}{t}\int_0^{\infty} \mathrm{e}^{-x/t} g(x)\,\mathrm{d}x.$$

If $\tilde{f}(t)$ is defined for all t, we call $f(t)$ *Borel summable*.

Note that the Borel summation method can be used to sum the series of (2), but not the series associated with (1) (check what is happening at $t = 1$). This brings us to Balser's book, the main purport of which is to describe a summation theory suitable for formal power series associated with meromorphic ODEs. This is basically the *multisummability* theory developed originally in a different form by Ecalle, the rough idea of which is to iterate the Borel summation procedure. Since the amount of notation and definitions needed to describe multisummability is prohibitively large, we do not attempt it here.

This is not a particularly good book. It has the serious drawback of not differentiating clearly between important and less important topics. For example, multisummability is never rigorously defined (a rough definition of a multisummable formal power series is on p. 173, though the concept is already used on p. 165); a crucial summability result is, without being flagged, tucked away as part (d) of Theorem 42. I think it is fair to say that, though it covers a lot of material (theory of systems of meromorphic ODEs, Stokes's phenomenon, Ecalle's acceleration operators), it does not give a coherent overview of what is a rapidly expanding area with applications in many branches of natural sciences, all the more so due to the development of computer algebra systems. Though the author professes to address a wide audience, I feel that the target audience are other summability experts, to whom the author presents a different approach to Ecalle's theory. The exercises in the book vary widely in the degree of difficulty; the index is not particularly useful, and the style is uneven and sometimes flippant. I do not believe Balser has answered the question he poses on p. ix, i.e. what formal series are good for, as there are no examples of applications of summability procedures and of the use of sums as 'sources of information.'

The bibliography of the book, though it inexplicably omits any reference to the work of Berry (e.g. [**1**]) and his students, is very helpful. Most of the work on multisummability, resurgence (never defined in the book), etc., is in French, and fully represented in the volume under review. In fact, it is scandalous that the path-breaking work of Ecalle is not available in English. It is to be hoped that perusal of Balser's book, flawed though it is, will stimulate further activity in this 'problème très intéressant', which might lead to the translation of Ecalle's work (as well as that of Ramis, or the monograph on resurgence by Candelpergher, Nosmas, and Pham, all referenced in Balser's book), or to the writing of a more reader-friendly book of an applied character.

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