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b-GENERALIZED DERIVATIONS OF SEMIPRIME RINGS HAVING NILPOTENT VALUES

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Abstract

Let R be a semiprime ring with extended centroid C and with maximal right ring of quotients $Q_{mr}(R)$. Let $d: R \to Q_{mr}(R)$ be an additive map and $b \in Q_{mr}(R)$. An additive map $\delta: R \to Q_{mr}(R)$ is called a (left) b-generalized derivation with associated map d if $\delta(xy) = \delta(x)y + bxd(y)$ for all $x, y \in R$. This gives a unified viewpoint of derivations, generalized derivations and generalized σ-derivations with an X-inner automorphism σ. We give a complete characterization of b-generalized derivations of R having nilpotent values of bounded index. This extends several known results in the literature.

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1. Results

Throughout the paper, unless specially stated, R is always a semiprime ring with Martindale symmetric ring of quotients $Q_s(R)$. We let $Q_{mr}(R)$ (respectively $Q_{ml}(R)$) denote the maximal right (respectively left) ring of quotients of R. It is known that $R \subseteq Q_s(R) \subseteq Q_{mr}(R)$. The overrings $Q_s(R)$ and $Q_{mr}(R)$ of R are semiprime rings with the same center C, which is a regular self-injective ring. The ring C is called the extended centroid of R. Also, R is a prime ring if and only if C is a field. We refer the reader to the book [1] for details.

An additive map $d: R \to R$ is called a *derivation* if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. For $b \in R$, we let $\mathrm{ad}(b)$ denote the map $x \mapsto [b, x] := bx - xb$ for $x \in R$. Clearly, $\mathrm{ad}(b)$ is a derivation of R, which is called the *inner derivation* of R induced

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by the element b. It is known that any *derivation* d of R can be uniquely extended to a derivation of $Q_{mr}(R)$. A derivation $d\colon R \to R$ is called X-*inner* if its extension to $Q_{mr}(R)$ is inner. In this case, it is easy to check that $d = \mathrm{ad}(q)$ for some $q \in Q_s(R)$. An additive map $\delta\colon R \to R$ is called a *generalized derivation* if there exists a derivation d of R such that $\delta(xy) = \delta(x)y + xd(y)$ for all $x, y \in R$ (see [2, 14, 18]). The derivation d is uniquely determined by δ, and is called the *associated derivation* of δ.

Let σ be an automorphism of R. An additive map $\delta\colon R \to R$ is called a (right) σ-*derivation* if $\delta(xy) = x\delta(y) + \delta(x)\sigma(y)$ for $x, y \in R$. Basic examples of σ-derivations are derivations and $\sigma - 1$. Given $b \in R$, the map $x \mapsto xb - b\sigma(x)$ for $x \in R$ obviously defines a σ-derivation, which is called the *inner* σ-*derivation* induced by b. It is clear that any σ-derivation of R can be uniquely extended to a σ-derivation of $Q_{mr}(R)$. In [21], Lee and Liu gave a common generalization of both generalized derivations and σ-derivations. An additive map $g\colon R \to R$ is called a right *generalized* σ-*derivation* if there exists an additive map $\delta\colon R \to R$ such that $g(xy) = xg(y) + \delta(x)\sigma(y)$ for all $x, y \in R$. It is clear that δ is uniquely determined by the map g. The additive map δ is called the *associated map* of g. Our present study is motivated by the following results.

Let $d\colon R \to R$ be a derivation, $\delta\colon R \to R$ a generalized derivation, $g\colon R \to R$ a right generalized σ-derivation, and n a fixed positive integer. Also, the rings R in (4)–(6) are prime.

(1) Suppose that $d(x)^n = 0$ for all $x \in R$. Then $d = 0$ (see [10, 12, 13]).

(2) Let λ be a left ideal of R. Suppose that $d(x)^n = 0$ for all $x \in \lambda$. Then $\lambda d(\lambda) = 0$ (see [16, Theorem 6]).

(3) Suppose that $\delta(x)^n = 0$ for all $x \in R$. Then $\delta = 0$ (see [18, Theorem 5]).

(4) Suppose that $\delta(x)^n = 0$ for all $x \in \rho$, a right ideal of R. Then there exist $b, c \in Q_{mr}(R)$ and $\beta \in C$ such that $\delta(x) = bx - xc$ for all $x \in R$ and $(b - \beta)\rho = 0 = (c - \beta)\rho$ (see [18, Theorem 6]).

(5) Suppose that $g(x)^n = 0$ for all $x \in R$. Then $g = 0$ (see [21, Theorem 2.7]).

(6) Let $a, b, q \in Q_{mr}(R)$. Suppose that $(a\delta(qx) - bx)^n = 0$ for all $x \in R$. Then either $a\delta(q) - b = 0 = aq$ or there exist $a_0, b_0 \in Q_{mr}(R)$ and $\mu \in C$ such that $\delta(x) = a_0x + xb_0$ for $x \in R$ and $aa_0q - b = -b_0aq = \mu aq$.

Let us consider a special case of (5). Suppose that the extension of σ to $Q_{ml}(R)$ is inner; that is, there exists a unit $u \in Q_{ml}(R)$ such that $\sigma(x) = uxu^{-1}$ for $x \in R$. Let δ be the associated map of g. Then $g(xy) = xg(y) + d(x)yu^{-1}$ for all $x, y \in R$, where $d(x) := \delta(x)u$ for $x \in R$. Notice that $d\colon R \to Q_{ml}(R)$. See [3, 4] for the Lie ideal case.

In (6), let $d\colon R \to R$ be the associated derivation of δ; that is, $\delta(xy) = \delta(x)y + xd(y)$ for $x, y \in R$. We let $\widetilde{\delta}(x) := a\delta(qx) - bx$ for $x \in R$. Then $\widetilde{\delta}(x) = aqd(x) + (a\delta(q) - b)x$ for $x \in R$. A direct computation shows that $\widetilde{\delta}(xy) = \widetilde{\delta}(x)y + (aq)xd(y)$ for $x, y \in R$. Since d can be uniquely extended to $Q_{mr}(R)$, so can $\widetilde{\delta}$. In view of [17, Theorem 3] (or see Fact 1.5 below), R and $Q_{mr}(R)$ satisfy the same differential identities. Thus, $\widetilde{\delta}(x)^n = 0$ for all $x \in Q_{mr}(R)$.

Motivated by the results (1)–(6) above, we give the following definition.

Definition 1.1. (1) Let $d\colon R \to Q_{mr}(R)$ be an additive map and $b \in Q_{mr}(R)$. An additive map $\delta\colon R \to Q_{mr}(R)$ is called a (left) *b-generalized derivation* with *associated map d* if $\delta(xy) = \delta(x)y + bxd(y)$ for all $x, y \in R$.

(2) Let $d\colon R \to Q_{ml}(R)$ be an additive map and $b \in Q_{ml}(R)$. An additive map $\delta\colon R \to Q_{ml}(R)$ is called a right b-generalized derivation with associated map d if $\delta(xy) = x\delta(y) + d(x)yb$ for all $x, y \in R$.

Clearly, a generalized derivation is a 1-generalized derivation and a right generalized σ-derivation is a right u^{-1}-generalized derivation if $\sigma(x) = uxu^{-1}$ for $x \in R$, where u is a unit in $Q_{ml}(R)$. For $a, b, c \in Q_{mr}(R)$, the map $x \mapsto ax + bxc$ for $x \in R$ is a left b-generalized derivation. Analogously, for $a, b, c \in Q_{ml}(R)$, the map $x \mapsto xa + bxc$ for $x \in R$ is a right c-generalized derivation. We note that left or right b-generalized derivations appear canonically in [7, Theorems 1.1 and 1.3]. The goal of the paper is to give a complete characterization of b-generalized derivations having nilpotent values of bounded index. By symmetry, it suffices to deal with one of left and right b-generalized derivations. For simplicity of notation, a b-generalized derivation always means a left b-generalized derivation.

To state the main theorem of the paper, we have to recall some basic properties of idempotents of C. We write $\mathbf{B}$ for the set of all idempotents of C. The set $\mathbf{B}$ forms a Boolean algebra with respect to the operations $e \dotplus h := e + h - 2eh$ and $e \cdot h := eh$ for all $e, h \in \mathbf{B}$. It is complete with respect to the partial order $e \le h$ (defined by $eh = e$) in the sense that any subset S of $\mathbf{B}$ has a supremum $\bigvee S$ and an infimum $\bigwedge S$. Given a subset S of $Q_{mr}(R)$, we define $\mathrm{E}[S]$ to be the infimum of $e \in \mathbf{B}$ such that $ex = x$ for all $x \in S$. If $S = \{b\}$, we write $\mathrm{E}[b]$ instead of $\mathrm{E}[S]$ for simplicity. Note that, for $a, b \in Q_{mr}(R)$, $aRb = 0$ if and only if $\mathrm{E}[a]\mathrm{E}[b] = 0$. By the characterization, it is easy to see that if a b-generalized derivation δ has associated maps d and d', then $\mathrm{E}[b]d(x) = \mathrm{E}[b]d'(x)$ for all $x \in R$. We refer the reader to the book [1] for details.

We are now in a position to state the main theorems of the paper.

Theorem 1.2. *Let R be a semiprime ring, $b \in Q_{mr}(R)$, and let $\delta\colon R \to Q_{mr}(R)$ be a b-generalized derivation with associated map d. Suppose that $\delta(x)^n = 0$ for all $x \in R$, where n is a positive integer. Then there exists $q \in Q_{mr}(R)$ such that $E[b]d(x) = [q, x]$ for $x \in R$, $\delta(x) = -bxq$ for $x \in R$, and $qb = 0$.*

By symmetry, we also have the following result whose proof parallels that of Theorem 1.2.

Theorem 1.3. *Let R be a semiprime ring, $b \in Q_{ml}(R)$, and let $\delta\colon R \to Q_{ml}(R)$ be a right b-generalized derivation with associated map d. Suppose that $\delta(x)^n = 0$ for all $x \in R$, where n is a positive integer. Then there exists $q \in Q_{ml}(R)$ such that $E[b]d(x) = [q, x]$ for $x \in R$, $\delta(x) = qxb$ for $x \in R$, and $bq = 0$.*

Let I be an ideal of R. By the semiprimeness of R, the left annihilator of I in R coincides with the right annihilator of I in R. The ideal I is called *dense* if its left annihilator in R is zero. We write $C\{X_1, X_2, \ldots\}$ for the free algebra over C in noncommutative indeterminates $X_1, X_2, \ldots$ and $Q_{mr}(R) *_C C\{X_1, X_2, \ldots\}$ for the free product of the C-algebras $Q_{mr}(R)$ and $C\{X_1, X_2, \ldots\}$. Let $f(X_i) \in Q_{mr}(R) *_C C\{X_1, X_2, \ldots\}$ and T be a subring of $Q_{mr}(R)$. We say that f is a GPI (that is, a *generalized polynomial identity*) of T if $f(x_i) = 0$ for all $x_i \in T$. By a *derivation word* Δ, we mean that Δ is of the form $d_1 d_2 \cdots d_s$, where each d_i is either a derivation of $Q_{mr}(R)$ or the identity map of $Q_{mr}(R)$. By a *differential polynomial* $f(X_i^{\Delta_j})$, we mean that all Δ_j are derivation words and $f(Z_{ij})$ is a generalized polynomial in noncommutative indeterminates Z_{ij}. The differential polynomial $f(X_i^{\Delta_j})$ is called a *differential identity* of T if $f(x_i^{\Delta_j}) = 0$ for all $x_i \in T$. We will use the following facts in the proofs below.

Fact 1.4. Let I be a dense ideal of R. Then I and $Q_{mr}(R)$ satisfy the same GPIs with coefficients in $Q_{mr}(R)$ (see [1, Theorem 6.4.1] for a semiprime ring R and [6, Theorem 2] for a prime ring R).

Fact 1.5. Let I be a dense ideal of R. Then I and $Q_{mr}(R)$ satisfy the same differential identities with coefficients in $Q_{mr}(R)$ (see [17, Theorem 3]).

Fact 1.6. Let ρ be a right ideal of R and $a \in Q_{mr}(R)$. Suppose that $(ax)^n = 0$ for all $x \in \rho$. Then $a\rho = 0$ (see Fact 1.4 and [11, Lemma 1.1]).

Fact 1.7. Let $\phi\colon I \to Q_{mr}(R)$ be a right R-module map, where I is a dense ideal of R. Then there exists $a \in Q_{mr}(R)$ such that $\phi(x) = ax$ for all $x \in I$ (see [19, Lemma 2.1] with the same proof by replacing 'a nonzero ideal in a prime ring' with 'a dense ideal in a semiprime ring').

Fact 1.8. Let $d\colon R \to Q_{mr}(R)$ be a derivation. Then d can be uniquely extended to a derivation from $Q_{mr}(R)$ to itself (see, for instance, [17, Lemma 2]).

2. The prime case

We begin with the following key result.

Proposition 2.1. *Let R be a prime ring, $a, b, c \in R$, and n a positive integer. Suppose that $(ax + bxc)^n = 0$ for all $x \in R$. Then there exists $\beta \in C$ such that $a = \beta b$ and $(c + \beta)b = 0$.*

A prime ring R is called a GPI-*ring* if it satisfies a nontrivial (that is, nonzero) generalized polynomial with coefficients in $Q_{mr}(R)$. The prime ring R is called *centrally closed* if $R = RC$. In particular, the prime ring $Q_{mr}(R)$ is centrally closed. The following lemma is a special case of [24, Theorem 1]. Since the proof below is neat and self-contained, we give its proof here for the convenience of the reader. We also remark that Chang proved the following lemma with the extra assumption that b is invertible in R (see [5, Lemma 2.1]).

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Lemma 2.2. *Let R be a prime ring, $a, b, c \in R$, and n a positive integer. Suppose that $(b(ax + xc))^n = 0$ for all $x \in R$. Then there exists $\beta \in C$ such that $b(a - \beta) = 0$ and $(c + \beta)b = 0$.*

Proof. Suppose first that R is not a GPI-ring. This implies that $(b(aX + Xc))^n$ is a trivial generalized polynomial. In particular, ba and b are dependent over C. That is, $b(a - \beta) = 0$ for some $\beta \in C$. Thus,

$$\begin{aligned} 0 &= (c+\beta)(b(ax+xc))^n bx \\ &= (c+\beta)(b((a-\beta)x + x(c+\beta)))^n bx = ((c+\beta)bx)^{n+1} \end{aligned} \tag{2.1}$$

for all $x \in R$. In view of Fact 1.6, $(c + \beta)b = 0$.

Suppose next that R is a GPI-ring. It follows from Fact 1.4 that

$$(b(ax + xc))^n = 0 \tag{2.2}$$

for all $x \in RC$. Let F denote the algebraic closure of C if C is an infinite field and let $F = C$ if C is a finite field. Then (2.2) holds for all $x \in \widetilde{R}$ (see [22, Lemma 2.3]), where $\widetilde{R} := RC \otimes_C F$. In view of [8, Theorem 3.5], $\widetilde{R}$ is a centrally closed prime F-algebra. By [23, Theorem 3], $\widetilde{R}$ is a primitive ring with a minimal idempotent e such that $e\widetilde{R}e = Fe$. Hence, there exists a left vector space V over F such that $\widetilde{R}$ acts densely on ${}_F V$.

Given $v \in V$, we claim that $v(ba)$ and vb are dependent over F. Suppose not; then there exists $x \in \widetilde{R}$ such that $v(ba)x = v$ and $vbx = 0$. Then $0 = v(b(ax + xc))^n = v$, which is a contradiction. This proves the claim.

If $\dim_F Vb \geq 2$, it is routine to prove that there exists $\widetilde{\beta} \in C$ such that $ba = \widetilde{\beta} b$; that is, $b(a - \widetilde{\beta}) = 0$. Thus, by (2.1) we have $(c + \widetilde{\beta})b = 0$. Suppose next that $\dim_F Vb = 1$. Choose $v_0 \in V$ such that $Vb = Fv_0 b$. Write $v_0 ba = \widetilde{\gamma} v_0 b$ for some $\widetilde{\gamma} \in F$. Let $v \in V$. Then $vb = \widetilde{\alpha} v_0 b$ for some $\widetilde{\alpha} \in F$. Then $vba = \widetilde{\alpha} v_0 ba = \widetilde{\alpha}\widetilde{\gamma} v_0 b = \widetilde{\gamma} vb$.

In either case, there exists $\widetilde{\beta} \in F$ such that $ba = \widetilde{\beta} b$. Choose a basis $\mu_0, \mu_1, \ldots$ for F over C, where $\mu_0 = 1$, and write $\widetilde{\beta} = \beta\mu_0 + \beta_1\mu_1 + \cdots$, where $\beta, \beta_1, \ldots \in C$. Then $ba = \beta a$. That is, $b(a - \beta) = 0$. It follows from (2.1) that $(c + \beta)b = 0$. □

Proof of Proposition 2.1. It follows from Fact 1.4 that

$$(ax + bxc)^n = 0 \tag{2.3}$$

for all $x \in Q_{mr}(R)$. We claim that $a \in bQ_{mr}(R)$. Clearly, we may assume $a \neq 0$.

Suppose that R is not a GPI-ring. Then a and b are dependent over C. In particular, $a \in bQ_{mr}(R)$, as asserted. Suppose next that R is a GPI-ring. In this case, $Q_{mr}(R)$ is also a prime GPI-ring (see Fact 1.4). Since $Q_{mr}(R)$ is a centrally closed prime ring, it follows from [23, Theorem 3] that $Q_{mr}(R)$ is a primitive ring with nonzero socle. Write $H := \mathrm{soc}(Q_{mr}(R))$, the socle of $Q_{mr}(R)$. Note that H is a regular ring (see [9]); that is, for any $w \in H$, $wzw = w$ for some $z \in H$. For $z \in H$, we write $\ell_H(z)$ for the left annihilator of z in H; that is, $\ell_H(z) = \{x \in H \mid xz = 0\}$.

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We first consider the case that $a, b \in H$. Let $w \in \ell_H(b)$. By (2.3),

$$0 = w(a(xw) + b(xw)c)^n ax = (wax)^{n+1}$$

for all $x \in Q_{mr}(R)$. In view of Fact 1.6, $wa = 0$. That is, $w \in \ell_H(a)$. Up to now, we have proved that $\ell_H(b) \subseteq \ell_H(a)$

Since $a, b \in H$, there exist $u, v \in H$ such that $aua = a$ and $bvb = b$. Set $f := au$ and $g := bv$. Then f, g are idempotents. Then $\ell_H(g) \subseteq \ell_H(f)$; that is, $H(1-g) \subseteq H(1-f)$. So $(1-g)f = 0$. Then $a = fa = gfa = bvfa \in bH$, as asserted.

For the general case, let $w \in H$. We see that $(awx + bwxc)^n = 0$ for all $x \in Q_{mr}(R)$. Since $aw, bw \in H$, the first case implies that $aw \in bwH$. Write $aw = bwt$ for some $t \in H$, depending on w. Replacing x by wx in (2.3),

$$(bw(tx + xc))^n = (a(wx) + b(wx)c)^n = 0$$

for all $x \in Q_{mr}(R)$. By Lemma 2.2, there exists $\beta_w \in C$, depending on w, such that $bw(t - \beta_w) = 0$. That is, $aw = \beta_w bw$ for $w \in H$.

Fix an idempotent $e_0 \in H$ such that $ae_0 \neq 0$. Then $ae_0 = \beta be_0$ for some $\beta \in C$. Let f be an idempotent of H. Then $af = \beta_f bf$ for some $\beta_f \in C$. We claim that $\beta_f = \beta$ if $af \neq 0$. Indeed, there exists $h = h^2 \in H$ such that $e_0 H + fH = hH$ and $ah = \beta_h bh$ for some $\beta_h \in C$. Note that $he_0 = e_0$ and $hf = f$. Thus,

$$ae_0 = ahe_0 = \beta_h bhe_0 = \beta_h be_0,$$

implying that $\beta_h = \beta$. Similarly, $\beta_h = \beta_f$ and so $\beta = \beta_f$. Thus, $(a - \beta b)f = 0$ if $af \neq 0$.

Let $f = f^2 \in H$ with $af = 0$. We claim that $bf = 0$. By Litoff's theorem [9], there exists an idempotent $h \in H$ such that $e_0, f \in hHh$. If $ah = 0$ then $ae_0 = ahe_0 = 0$, which is a contradiction. Thus, neither ah nor $a(h - f)$ is zero. Note that $h - f$ is an idempotent. Then

$$ah = \beta bh \quad \text{and} \quad a(h - f) = \beta b(h - f).$$

This implies that $\beta bf = 0$, so $bf = 0$ follows. Up to now, we have proved that $(a - \beta b)f = 0$ for any idempotent $f \in H$ with $af = 0$.

In either case, $(a - \beta b)f = 0$ for any idempotent $f \in H$. Since H is a regular ring, $(a - \beta b)H = 0$ and so $a = \beta b$. Rewrite (2.3) as $(bx(c + \beta))^n = 0$ for all $x \in Q_{mr}(R)$. So $(x(c + \beta)b)^{n+1} = 0$ for all $x \in Q_{mr}(R)$. By Fact 1.6, $(c + \beta)b = 0$ follows. □

The following characterizes b-generalized derivations of semiprime rings.

Theorem 2.3. *Let R be a semiprime ring, $b \in Q_{mr}(R)$, and let $\delta: R \to Q_{mr}(R)$ be a b-generalized derivation with associated map d. Then* $\mathrm{E}[b]d : R \to Q_{mr}(R)$ *is a derivation and there exists $\widetilde{b} \in Q_{mr}(R)$ such that $\delta(x) = bd(x) + \widetilde{b}x$ for all $x \in R$.*

Proof. Expanding $\delta((xy)z)$ and $\delta(x(yz))$ respectively, we see that

$$bx(d(yz) - yd(z) - d(y)z) = 0$$

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for all $x, y, z \in R$. The semiprimeness of R implies that $\mathrm{E}[b]d(yz) = y\mathrm{E}[b]d(z) + \mathrm{E}[b]d(y)z$ for all $y, z \in R$; that is, $\mathrm{E}[b]d\colon R \to Q_{mr}(R)$ is a derivation. Let $\mu\colon R \to Q_{mr}(R)$ be the map defined by $\mu(x) = bd(x)$ for $x \in R$. Then

$$\mu(xy) = b\mathrm{E}[b]d(xy) = b\mathrm{E}[b]d(x)y + bx\mathrm{E}[b]d(y)$$
$$= bd(x)y + bxd(y) = \mu(x)y + bxd(y)$$

for all $x, y \in R$. Thus, we have $(\delta - \mu)(xy) = (\delta - \mu)(x)y$ for all $x, y \in R$. In view of Fact 1.7, there exists $\widetilde{b} \in Q_{mr}(R)$ such that $\delta(x) = bd(x) + \widetilde{b}x$ for all $x \in R$. □

Theorem 2.4. *Let R be a prime ring, $b \in Q_{mr}(R)$, and let $\delta\colon R \to Q_{mr}(R)$ be a nonzero b-generalized derivation with associated map d. Suppose that $\delta(x)^n = 0$ for all $x \in R$, where n is a positive integer. Then there exists $q \in Q_{mr}(R)$ such that $d = \mathrm{ad}(q)$, $\delta(x) = -bxq$ for $x \in R$, and $qb = 0$.*

Proof. In view of Theorem 2.3, there exists $\widetilde{b} \in Q_{mr}(R)$ such that $\delta(x) = bd(x) + \widetilde{b}x$ for all $x \in R$. By assumption,

$$(bd(x) + \widetilde{b}x)^n = 0 \tag{2.4}$$

for all $x \in R$. By Fact 1.8, d can be uniquely extended to a derivation from $Q_{mr}(R)$ to itself, also denoted by d. In view of Fact 1.5, (2.4) holds for all $x \in Q_{mr}(R)$. Note that $Q_{mr}(Q_{mr}(R)) = Q_{mr}(R)$.

Suppose first that d is X-outer. In view of [15, Theorem 2], $(by + \widetilde{b}x)^n = 0$ for all $x, y \in Q_{mr}(R)$. Then $b = 0 = \widetilde{b}$ (see Fact 1.6). This implies that $\delta = 0$, which is a contradiction. Thus, d is X-inner. Then there exists $q' \in Q_{mr}(R)$ such that $d(x) = [q', x]$ for $x \in R$. Since R and $Q_{mr}(R)$ satisfy the same GPIs (see Fact 1.4), we rewrite (2.4) as

$$((bq' + \widetilde{b})x - bxq')^n = 0$$

for all $x \in Q_{mr}(R)$. In view of Proposition 2.1, there exists $\mu \in C$ such that $bq' + \widetilde{b} = \mu b$ and $(q' - \mu)b = 0$. Let $q := q' - \mu$. Then $d = \mathrm{ad}(q)$, $bq = -\widetilde{b}$ and $qb = 0$. Therefore,

$$\delta(x) = bd(x) + \widetilde{b}x = b(qx - xq) - bqx = -bxq \quad \text{for } x \in R,$$

as asserted. □

3. Proof of Theorem 1.2

Let R be a semiprime ring with extended centroid C. We call $\{e_\nu \mid \nu \in \Lambda\} \subseteq \mathbf{B}$ an *orthogonal subset* if $e_\nu e_\mu = 0$ for $\nu \neq \mu$ and a *dense subset* of $\mathbf{B}$ if $\sum_{\nu\in\Lambda} e_\nu C$ is an essential ideal of C. The ring $Q_{mr}(R)$ is *orthogonally complete* in the following sense: Given any dense orthogonal subset $\{e_\nu \mid \nu \in \Lambda\}$ of $\mathbf{B}$, $Q_{mr}(R)$ is ring-isomorphic to the direct product $\prod_{\nu\in\Lambda} Q_{mr}(R)e_\nu$ via the map

$$x \mapsto \langle xe_\nu\rangle \in \prod_{\nu\in\Lambda} Q_{mr}(R)e_\nu \quad \text{for } x \in Q_{mr}(R).$$

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Therefore, given any subset $\{a_\nu \in Q_{mr}(R) \mid \nu \in \Lambda\}$, there exists a unique $a \in Q_{mr}(R)$ such that $a \mapsto \langle a_\nu e_\nu \rangle$. The element a is written as $\sum_{\nu\in\Lambda}^{\perp} a_\nu e_\nu$ and is characterized by the property that $ae_\nu = a_\nu e_\nu$ for all $\nu \in \Lambda$. A subset T of $Q_{mr}(R)$ is called orthogonally complete if $0 \in T$ and $\sum_{\nu\in\Lambda}^{\perp} a_\nu e_\nu \in T$ for any dense orthogonal subset $\{e_\nu \mid \nu \in \Lambda\}$ of $\mathbf{B}$ and any subset $\{a_\nu \mid \nu \in \Lambda\} \subseteq T$. Denote by $\mathrm{Spec}(\mathbf{B})$ the set of all maximal ideals of the complete Boolean algebra $\mathbf{B}$. Let T be a subset of $Q_{mr}(R)$. The intersection of all orthogonally complete subsets of $Q_{mr}(R)$ containing T is called the *orthogonal completion* of T and is denoted by $\widehat{T}$. In view of [1, Proposition 3.1.14 and Corollary 3.1.15], $\widehat{R}$ is a subring of $Q_{mr}(R)$ and

$$\widehat{R} = \Big\{\sum\nolimits_{\alpha\in\Lambda}^{\perp} x_\alpha e_\alpha \mid \{e_\alpha \mid \alpha \in \Lambda\} \text{ is a dense orthogonal subset of } \mathbf{B} \text{ and } x_\alpha \in R \text{ for all } \alpha \in \Lambda\Big\}.$$

Moreover, $\widehat{R} \cap \mathbf{m}Q_{mr}(R)$ is a prime ideal of $\widehat{R}$ for all $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$ (see [1, Theorem 3.2.15]).

Proposition 3.1. *A derivation $d\colon Q_{mr}(R) \to Q_{mr}(R)$ is X-inner if and only if $\overline{d}\colon Q_{mr}(R)/\mathbf{m}Q_{mr}(R) \to Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$ is X-inner for any $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$.*

The proof of Proposition 3.1 is the same as that of [20, Proposition 2.2]. Let $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$. It is known that $\mathbf{m}Q_{mr}(R)$ is a prime ideal of $Q_{mr}(R)$. We use the notations: $\overline{Q_{mr}(R)} = Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$, $\overline{C} = C + \mathbf{m}Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$, and $\overline{\widehat{R}} = \widehat{R} + \mathbf{m}Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$. Then both $\overline{Q_{mr}(R)}$ and $\overline{\widehat{R}}$ are prime rings having the same extended centroid $\overline{C}$ (see [1]). Keeping these notations we have the following.

Lemma 3.2. *Let $v, x \in Q_{mr}(R)$. Suppose that $\overline{x} \in \overline{C}\overline{v}$ for any $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$, where $\overline{z} := z + \mathbf{m}Q_{mr}(R)$ for $z \in Q_{mr}(R)$. Then $x \in Cv$.*

Proof. Consider the set $\Sigma = \{e \in \mathbf{B} \mid ex \in Cv\}$. We see that if $e \le f \in \Sigma$ then $e \in \Sigma$. Also, if $e, f \in \Sigma$ are orthogonal then clearly $e \dotplus f \in \Sigma$. This means that Σ is an ideal of the complete Boolean algebra $\mathbf{B}$. If $1 \in \Sigma$ then $x \in Cv$, as asserted. Suppose on the contrary that $1 \notin \Sigma$. By Zorn's lemma, there exists $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$ such that $\Sigma \subseteq \mathbf{m}$. We work in $Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$. Since $\overline{x} \in \overline{C}\overline{v}$, there exists $a \in Cv$ such that $\overline{x} = \overline{a}$. Therefore, $ex = ea$ for some $e \in \mathbf{B} \setminus \mathbf{m}$. Note that $ea \in Cv$, implying $e \in \Sigma$. This is a contradiction. □

The next theorem extends Proposition 2.1 to the semiprime case.

Theorem 3.3. *Let R be a semiprime ring, $a, b, c \in R$, and n a positive integer. Suppose that $(ax + bxc)^n = 0$ for all $x \in R$. Then there exists $\beta \in C$ such that $a = \beta b$ and $(c + \beta)b = 0$.*

Proof. By Fact 1.4, $(ax + bxc)^n = 0$ for all $x \in Q_{mr}(R)$. Let $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$. Working in $Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$, we see that $(\overline{a}\,\overline{x} + \overline{b}\,\overline{x}\,\overline{c})^n = 0$ for all $\overline{x} \in Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$. In view of Proposition 2.1, $\overline{a} \in \overline{C}\,\overline{b}$. Since $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$ is arbitrary, it follows from Lemma 3.2

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that $a \in Cb$. Write $a = \beta b$ for some $\beta \in C$. Then $(bx(c+\beta))^n = 0$ for all $x \in R$. By Fact 1.6, $(c+\beta)b = 0$ follows, as asserted. □

Lemma 3.4. *Theorem 1.2 holds if* $\mathrm{E}[b] = 1$.

Proof. Since $\mathrm{E}[b] = 1$, it follows from Theorem 2.3 that $d\colon R \to Q_{mr}(R)$ is a derivation. By Fact 1.8, d can be uniquely extended to a derivation $\widetilde{d}\colon Q_{mr}(R) \to Q_{mr}(R)$. Clearly,

$$\widetilde{d}\Big(\sum\nolimits^{\perp}_{\nu\in\Lambda} x_\nu e_\nu\Big) = \sum\nolimits^{\perp}_{\nu\in\Lambda} d(x_\nu)e_\nu,$$

where $x_\nu \in R$. We claim that δ can be also uniquely extended to a b-generalized derivation of $\widehat{R}$, say $\widetilde{\delta}$, with associated map $\widetilde{d}\colon \widehat{R} \to Q_{mr}(R)$, by defining

$$\widetilde{\delta}\Big(\sum\nolimits^{\perp}_{\nu\in\Lambda} x_\nu e_\nu\Big) = \sum\nolimits^{\perp}_{\nu\in\Lambda} \delta(x_\nu)e_\nu,$$

where $x_\nu \in R$. Indeed, let $\sum^{\perp}_{\nu\in\Lambda} x_\nu e_\nu = 0$, where $x_\nu \in R$. Then $x_\nu e_\nu = 0$ for any ν. Fix an x_ν. Choose a dense ideal I of R such that $x_\nu I \cup e_\nu I \subseteq R$. Note that $d(ye_\nu) = \widetilde{d}(ye_\nu) = \widetilde{d}(y)e_\nu = d(y)e_\nu$ for $y \in I$ since $\widetilde{d}$ is a derivation. Thus,

$$0 = \delta(x_\nu(ye_\nu))^n = (\delta(x_\nu)ye_\nu + bx_\nu d(ye_\nu))^n = (\delta(x_\nu)ye_\nu)^n,$$

implying that $(\delta(x_\nu)e_\nu y)^n = 0$ for all $y \in I$. Fact 1.4 implies that $(\delta(x_\nu)e_\nu y)^n = 0$ for all $y \in Q_{mr}(R)$. By Fact 1.6, $\delta(x_\nu)e_\nu = 0$. So $\sum^{\perp}_{\nu\in\Lambda} \delta(x_\nu)e_\nu = 0$. This proves that $\widetilde{\delta}$ is well defined. It is routine to check that $\widetilde{\delta}$ is an additive map.

We claim that $\widetilde{\delta}\colon \widehat{R} \to Q_{mr}(R)$ is a b-generalized derivation with associated map $\widetilde{d}$. Indeed, let $\widetilde{x}, \widetilde{y} \in \widehat{R}$. Write

$$\widetilde{x} = \sum\nolimits^{\perp}_{\nu\in\Lambda} x_\nu e_\nu \quad \text{and} \quad \widetilde{y} = \sum\nolimits^{\perp}_{\nu\in\Lambda} y_\nu e_\nu,$$

where $x_\nu, y_\nu \in R$. Then $\widetilde{xy} = \sum^{\perp}_{\nu\in\Lambda} (x_\nu y_\nu)e_\nu$ and

$$\begin{aligned}
\widetilde{\delta}(\widetilde{x}\,\widetilde{y}) &= \sum\nolimits^{\perp}_{\nu\in\Lambda} \delta(x_\nu y_\nu)e_\nu \\
&= \sum\nolimits^{\perp}_{\nu\in\Lambda} (\delta(x_\nu)y_\nu + bx_\nu d(y_\nu))e_\nu \\
&= \Big(\sum\nolimits^{\perp}_{\nu\in\Lambda} \delta(x_\nu)e_\nu\Big)\Big(\sum\nolimits^{\perp}_{\nu\in\Lambda} y_\nu e_\nu\Big) + b\Big(\sum\nolimits^{\perp}_{\nu\in\Lambda} x_\nu e_\nu\Big)\Big(\sum\nolimits^{\perp}_{\nu\in\Lambda} d(y_\nu)e_\nu\Big) \\
&= \widetilde{\delta}(\widetilde{x})\widetilde{y} + b\widetilde{x}\widetilde{d}(\widetilde{y}),
\end{aligned}$$

as asserted.

Let $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$. Clearly, $\widetilde{d}(\mathbf{m}\widehat{R}) \subseteq \mathbf{m}Q_{mr}(R)$ since $\widetilde{d}$ is a derivation. We claim that $\widetilde{\delta}(\mathbf{m}\widehat{R}) \subseteq \mathbf{m}Q_{mr}(R)$. Let $x \in \mathbf{m}\widehat{R}$. Then $xe = 0$ for some $e \in \mathbf{B} \setminus \mathbf{m}$. Applying the same argument as in the first paragraph, we see that $\widetilde{\delta}(x)e = 0$. Thus $\widetilde{\delta}(x) \in \mathbf{m}Q_{mr}(R)$. This proves our claim.

Thus, $\widetilde{\delta}$ and $\widetilde{d}$ canonically induce the maps $\widetilde{\delta}_{\mathbf{m}}\colon \widehat{R}/\mathbf{m}\widehat{R} \to Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$ and $\widetilde{d}_{\mathbf{m}}\colon \widehat{R}/\mathbf{m}\widehat{R} \to Q_{mr}(R)/\mathbf{m}Q_{mr}(R)$, where

$$\widetilde{\delta}_{\mathbf{m}}(\overline{\widetilde{x}}) := \overline{\widetilde{\delta}(\widetilde{x})} \quad \text{and} \quad \widetilde{d}_{\mathbf{m}}(\overline{\widetilde{x}}) := \overline{\widetilde{d}(\widetilde{x})}$$

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for $\overline{\widetilde{x}} = \widetilde{x} + \mathbf{m}\widehat{R}$, where $\widetilde{x} \in \widehat{R}$. Note that $Q_{mr}(R)/\mathbf{m}Q_{mr}(R) \subseteq Q_{mr}(\widehat{R}/\mathbf{m}\widehat{R})$. It is clear that $\widetilde{\delta}_{\mathbf{m}}$ is a $\overline{b}$-generalized derivation with associated map $\widetilde{d}_{\mathbf{m}}$. Note that $\overline{b} \neq \overline{0}$ since $\mathrm{E}[b] = 1$.

We work in the prime ring $\widehat{R}/\mathbf{m}\widehat{R}$ with extended centroid $\overline{C}(:= C + \mathbf{m}\widehat{R}/\mathbf{m}\widehat{R})$. Let $\overline{\widetilde{x}} = \widetilde{x} + \mathbf{m}\widehat{R} \in \widehat{R}/\mathbf{m}\widehat{R}$, where $\widetilde{x} \in \widehat{R}$. Write $\widetilde{x} = \sum^{\perp}_{\nu\in\Lambda} x_\nu e_\nu$, where $x_\nu \in R$. Then $\widetilde{\delta}(\widetilde{x}) = \sum^{\perp}_{\nu\in\Lambda} \delta(x_\nu)e_\nu$ and

$$\widetilde{\delta}_{\mathbf{m}}(\overline{\widetilde{x}})^n = \overline{\widetilde{\delta}(\widetilde{x})}^{\,n} = \overline{\Big(\sum\nolimits^{\perp}_{\nu\in\Lambda}\delta(x_\nu)e_\nu\Big)^n} = \overline{\sum\nolimits^{\perp}_{\nu\in\Lambda}\delta(x_\nu)^n e_\nu} = \overline{0}.$$

In view of Theorem 2.4, the derivation $\widetilde{d}_{\mathbf{m}}$ is X-inner. It follows from Proposition 3.1 that $\widetilde{d}$ is X-inner. Thus, $\widetilde{d} = \mathrm{ad}(q')$ for some $q' \in Q_{mr}(R)$. Moreover, in view of Theorem 2.4, for any $\mathbf{m} \in \mathrm{Spec}(\mathbf{B})$ we have $\overline{q'}\,\overline{b} = \overline{q'b} \in \overline{C}\,\overline{b}$. By Lemma 3.2, $q'b = \beta b$ for some $\beta \in C$. Set $q := q' - \beta$. Then $d = \mathrm{ad}(q)$ and $qb = 0$.

Let $x, y \in R$. Then

$$\delta(xy) = \delta(x)y + bxd(y) = \delta(x)y + bx(qy - yq),$$

implying that

$$\delta(xy) + bxyq = (\delta(x) + bxq)y.$$

By Fact 1.7, there exists $w \in Q_{mr}(R)$ such that $\delta(x) = -bxq + wx$ for all $x \in R$. Thus, $(wx - bxq)^n = 0$ for all $x \in R$ and hence for all $x \in Q_{mr}(R)$ (see Fact 1.4). In view of Theorem 3.3, there exists $\mu \in C$ such that $w = \mu b$ and $(q - \mu)b = 0$. Thus, by the fact that $qb = 0$, we see that $\mu = 0$ and $w = 0$. That is, $\delta(x) = -bxq$ for all $x \in R$, as asserted. □

Proof of Theorem 1.2. Let $e := \mathrm{E}[b]$, $\delta_1(x) := e\delta(x)$ and $d_1(x) := ed(x)$ for $x \in R$. Then $(1 - e)\delta(xy) = (1 - e)\delta(x)y$ for all $x, y \in R$. By Fact 1.7, there exists $w \in Q_{mr}(R)$ such that $(1 - e)\delta(x) = wx$ for all $x \in R$. But $(wx)^n = 0$ for all $x \in R$. This implies that $w = 0$; that is, $(1 - e)\delta(x) = 0$ for all $x \in R$.

Note that $\delta_1 : R \to Q_{mr}(R)$, $d_1 : R \to Q_{mr}(R)$, and $\delta_1(xy) = \delta_1(x)y + bxd_1(y)$ for all $x, y \in R$. Applying the same argument given in the proof of Lemma 3.4, d_1 is a derivation and can be uniquely extended to a derivation $\widetilde{d}_1 : \widehat{R} \to Q_{mr}(R)$ by defining

$$\widetilde{d}_1\Big(\sum\nolimits^{\perp}_{\nu\in\Lambda} x_\nu e_\nu\Big) = \sum\nolimits^{\perp}_{\nu\in\Lambda}(ed(x_\nu))e_\nu, \quad \text{where } x_\nu \in R.$$

On the other hand, δ_1 can be extended to a map $\widetilde{\delta}_1 : \widehat{R} \to Q_{mr}(R)$ by defining

$$\widetilde{\delta}_1\Big(\sum\nolimits^{\perp}_{\nu\in\Lambda} x_\nu e_\nu\Big) = \sum\nolimits^{\perp}_{\nu\in\Lambda}(e\delta(x_\nu))e_\nu, \quad \text{where } x_\nu \in R.$$

Note that $\widetilde{d}_1(e\widehat{R}) \subseteq eQ_{mr}(R)$ and $\widetilde{\delta}_1(e\widehat{R}) \subseteq eQ_{mr}(R)$. Working on $eQ_{mr}(R)$,

$$\widetilde{\delta}_1(xy) = \widetilde{\delta}_1(x)y + bx\widetilde{d}_1(y)$$

for all $x, y \in e\widehat{R}$. Note that $Q_{mr}(e\widehat{R}) = eQ_{mr}(R)$ and that $(\widetilde{\delta}_1(x))^n = 0$ for all $x \in e\widehat{R}$. Since $\mathrm{E}[b] = e$ and the extended centroid of $e\widehat{R}$ is equal to eC, it follows from Lemma 3.4 that

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there exists $q \in eQ_{mr}(R)$ such that $ed(x) = [q, x]$ for $x \in e\widehat{R}$, $e\delta(x) = -bxq$ for $x \in e\widehat{R}$, and $qb = 0$.

Choose a dense ideal I of R such that $(1 - e)I \subseteq R$. Let $x, y, z \in I$. Then

$$\begin{aligned}\delta(x(1-e)y) &= \delta(x)(1-e)y + bxd((1-e)y)\\ &= bxed((1-e)y) = bx(ed(y) - ed(e)y - ed(y)) = 0,\end{aligned}$$

since $\delta(x)(1 - e) = 0$ and ed is a derivation on $Q_{mr}(R)$. So $\delta((1 - e)I^2) = 0$. Let $x \in I^2$. Then

$$\delta(x) = e\delta(x) = e\delta(ex + (1 - e)x) = e\delta(ex) = -b(ex)q = -bxq.$$

Up to now, we have proved that $\delta(x) = -bxq$ for $x \in I^2$. Let $y \in R$ and $x \in I^2$. We notice that $ed(x) = ed(ex) = e[q, ex] = [q, x]$. Then $yx \in I^2$ and

$$-byxq = \delta(yx) = \delta(y)x + byd(x) = \delta(y)x + byed(x) = \delta(y)x + by[q, x],$$

implying that $(\delta(y) + byq)x = 0$. That is, $(\delta(y) + byq)I^2 = 0$ and so $\delta(y) = -byq$, as asserted. □

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