New Cosmological Data and the Values of the Fundamental Parameters
IAU Symposium, Vol. 201, 2005
Anthony Lasenby and Althea Wilkinson, eds.

Cosmic Acceleration: A Natural Remedy For Horizon and Flatness Problems

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Abstract. By assuming that the recently found cosmic acceleration is a genuine and ever present effect, we show that, the resultant modified Friedman model is free from the horizon and flatness problems; and there is no need to invoke the additional "inflationary" initial phase as a separate ingredient.

1. Introduction

Over the past few years, studies of luminosity-distance studies involving type supernova I, have suggested that the universe is undergoing accelerated expansion (AE) (Bachall et al. 1999). Such AE is possible either due to the presence of a cosmological constant (Λ) or "quintessence". The simplest way to broadly represent the AE would be to write $S(t) = \alpha t^n$, where α is a constant and $n > 1$. In the past era of supposed decelerated expansion ($n < 1$), one would encounter the so-called horizon and flatness puzzles. However, we shall show below that once we accept the fact the universe is undergoing AE rather than DE and assume that it was so in the past too, there is no need to postulate an *ad hoc* inflationary phase at least for the purpose of solving the horizon or flatness problems. Recall that the range of the causal horizon is defined as

$$d_H = S(t) \int_0^t \frac{dt'}{S(t')} \tag{1}$$

We can see that

$$d_H = \frac{S(t)}{k} \int_0^t t'^{-n} dt' = \frac{S(t)}{k} \frac{t'^{(1-n)}}{(1-n)} \Big|_0^t \tag{2}$$

For $n > 1$, the above integral blows up and there is no horizon problem.

In the presence of the cosmological term, the relationship between present density u, Hubble constant H, and scale size S is

$$H^2 + \frac{k}{S^2} = \frac{8\pi G u}{3} + \frac{\Lambda c^2}{3} \tag{3}$$

where k is the curvature constant. Dividing both sides by H^2, after some manipulation, we see that

$$\Omega - 1 = \frac{k}{H^2 S^2} - \frac{\Lambda c^2}{3H^2} \tag{4}$$

Here note that

$$\dot{S} = n\alpha t^{\beta}; \qquad \beta = n - 1 > 0; \qquad H = \frac{\dot{S}}{S} = n/t \tag{5}$$

Using the foregoing relationships, we find from Eq. (4) that for the present epoch $t = t_1$ and for the past epoch $t = t_2$, we have

$$\frac{\Omega_1 - 1}{\Omega_2 - 1} = \frac{k\alpha^{-2}t_1^{-2\beta} - \frac{\Lambda_1 t_1^2 c^2}{3}}{k\alpha^{-2}t_2^{-2\beta} - \frac{\Lambda_2 t_2^2 c^2}{3}} \tag{6}$$

As $t_2 \to 0$, the second term in the denominator of the above equation $\to 0$, and we obtain

$$\frac{\Omega_1 - 1}{\Omega_2 - 1} = \left(\frac{t_1}{t_2}\right)^{-2\beta} - \left(\frac{\Lambda_1}{\Lambda_2}\right) t_1^2 t_2^{2\beta} \tag{7}$$

Again as $t_2 \to 0$, both the terms on the RHS of the foregoing equation $\to 0$ if Λ varies less sharply than t^{-2} and $\Omega_1 - 1 = \mathcal{O}(0)(\Omega_2 - 1)$. So in the present universe, we should actually have $\Omega_1 = \Omega_{matter} + \Omega_\Lambda \to 1$. Then there is no flatness problem, and on the other hand, the universe should be completely flat at any appreciable value of t. In fact very recent observations indeed show that this is precisely the case (Bernandis et al. 2000).

2. Discussion and Conclusion

The age of the universe in the present case could be finite. Because, by using Eq. (5) we find that $t = \int dS/\dot{S} = S^{1/n}/\alpha$. In contrast there could be more involved models incorporating AE for which one may have $t = \infty$ for $S \neq 0$ (Fakir 2000). In other words there could be singularity free models for which horizon and flatness problems naturally do not arise. However, we found that even in a more orthodox model of AE with a finite age of the universe, horizon and flatness problems cease to exist and, thus, there is no need to postulate, in an *ad hoc* manner the occurrence of a brief spell of inflation. It can be shown that even if the horizon and flatness problems disappear with the idea of inflation in the present epoch, in principle, they may reappear, in future if $k \neq 0$ and the universe were undergoing DE. On the other hand, such problems do not recur for the present simple model incorporating AE. And although, the singularity and a hot early epoch lie in a finite past, in the present simple case, note that, the universe begins with a "whimper" ($\dot{S} = 0$) rather than a "bang"($\dot{S} = \infty$). Note that it is a different question as to how the AE is taking place and for which there could be hundreds of answers depending on the models and assumptions.

References

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