ACCURACY ESTIMATES FOR THE DETERMINATION OF THE SOLAR SPACE-TIME METRIC BY HIPPARCOS

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ABSTRACT. Parallel to the determination of astrometric parameters for some 100,000 stars, the HIPPARCOS data may also be used to study the light deflection produced by the sun. Cowling (1983 thesis) has calculated the light deflection observable by HIPPARCOS for a general space-time metric, and we have included his list of fifteen metric coefficients as "global" parameters in simulated HIPPARCOS reductions. The estimated covariance matrices show that only 7-9 linear combinations of the Cowling parameters may be determined with some accuracy ($\sim 10^{-10}$). The value for the standard GR light deflection may be determined if the thermal effects on the satellite are small, but not to better than about 0.5%.

1. INTRODUCTION

ESA's astrometry satellite HIPPARCOS is due to be launched in 1988 for its 2.5 year mission. From a geostationary orbit, it will measure positions, parallaxes and proper motions for about 100,000 stars to a typical precision of about 2 milliarcseconds (mas). A number of papers describing the scientific goals of the project were presented at a colloquium in Strasbourg (Perryman and Guyenne, 1982).

A main feature of the HIPPARCOS instrument is the superposition of two fields of view about 58° apart on the sky. This "basic angle" may be kept constant at the milliarcsecond level, and it enables the construction of a reference system free of large-scale distortions. In principle, the satellite continously scans the sky in a predetermined pattern and measures the (one-dimensional) distances between the program stars. An intricate reduction process is then required to derive the astrometric parameters for each star.

The details of the reduction process are now being developed by two independent "consortia", FAST (headed by J. Kovalevsky) and NDAC (headed by E. Høg). In Lund, we have been studying the final stages of NDAC's reduction process, where intermediate quantities called "abscissae" are transformed into astrometric data. This is done in a

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least-squares adjustment where "global" parameters describing any large-scale distortion of the reference system may also be included. One obvious parameter to include is the general relativistic light deflection by the sun (amounting to about 4 mas even at 90° from the sun). It is also interesting to see how far the HIPPARCOS data may be used to solve for more general models of the solar space-time metric. Such questions can be answered by analysis of simulated data, and the results of such studies are reported here.

2. THE SIMULATED OBSERVATIONS

About 10,000 stars are created with random postitions, and with realisitc magnitudes, parallaxes and proper motions. The satellite is then assumed to scan over the sky according to the "nominal" scanning-law, where the rotation axis keeps an angle 43° to the sun while revolving around it 6.4 times each year. One then singles out "sets" of stars observed in a 12^{h} interval of time. All stars in a set lie in a rather narrow band along a great circle (RGC) in the sky, and their coordinates along the RGC are the abscissae. (In the real mission, a major reduction effort is spent deriving the abscissae from more elementary observations.) The simulation program simply makes one abscissa value for each star observed in a particular set, with magnitude-dependent observational errors. With 10,000 stars, each set contains about 180 observations, and 1825 sets (2.5 years of observation) are simulated.

3. THE LEAST-SQUARES ADJUSTMENT

All positions are known a priori within about 1-2 arcsec, and a linearized differential correction process is used to refine them. With five astrometric unknowns per star, the nominal normal equations are prohibitively large. It is possible, however, to eliminate (while correcting) the astrometric parameters, and the final normal equations contain only one unknown per set, plus the (small) number of global parameters. Each solution of the normal equations gives an estimate for the covariance matrix of the global parameters, and the task is to test different selections of globals and try to find a "best" one. In the search process, it was practical to use only 730-1095 sets, but a few full (1825 sets) solutions were also made.

4. THE SOLAR SPACE-TIME METRIC

Realistically, there seems to be no need to go beyond the PPN formalism (see e.g. Will and Nordtvedt, 1972) in order to describe the solar space-time metric. With observed constraints on the more "exotic" PPN parameters (cf. Will, 1979), the only one worth including as an HIPPARCOS global would be the deflection parameter γ. However, as pointed out by Schutz (1982), we may instead define a more general metric model and try to determine its parameters by HIPPARCOS. (Contrary to the PPN metric, we do not specify how it is created by the matter in the solar system but just fit it to the HIPPARCOS observations.) This idea was developed further by Cowling (1983), and the formulae in this section are due to him.

Cowling starts with the linear perturbation from flat space-time

$$ds^2 = (1 + h_{oo})dt^2 - (1 - h_{ii})[dr^2 + r^2(d\Theta^2 + \sin^2\Theta \, d\phi^2)] + \\ + 2\, h_{or} dr dt + 2\, h_{o\Theta} d\Theta dt + 2\, h_{o\phi} d\phi dt \quad (1)$$

with the $h_{\alpha\beta}$ expanded as

$$h_{oo} = A_{oo}^{(1)}/r + A_{oo}^{(2)}/r^2 + [B_{oo}^{(1)}/r + B_{oo}^{(2)}/r^2] \cos^2\Theta$$
$$h_{ii} = A_{ii}^{(1)}/r + A_{ii}^{(2)}/r^2 + [B_{ii}^{(1)}/r + B_{ii}^{(2)}/r^2] \cos^2\theta$$
$$h_{or} = A_{or}^{(1)}/r + A_{or}^{(2)}/r^2 + [B_{or}^{(1)}/r + B_{or}^{(2)}/r^2] \cos^2\Theta$$
$$h_{o\Theta} = [A_{o\Theta}^{(1)} + A_{o\Theta}^{(2)}/r] \sin\Theta \cos\Theta$$
$$h_{o\phi} = [A_{o\phi}^{(1)} + A_{o\phi}^{(2)}/r] \sin^2\Theta \quad (2)$$

These expansions are axially and reflectively symmetric, which seems a priori like a natural requirement. A non-axisymmetric metric is nevertheless possible in "preferred-frame" theories, and Cowling adds the following terms (now in rectangular coordinates)

$$h_{oo} = (C_{oo}x + D_{oo}y + E_{oo}z)/r^2$$
$$h_{ii} = (C_{ii}x + D_{ii}y + E_{ii}z)/r^2$$
$$h_{ox} = C_{ox}/r \qquad h_{oy} = C_{oy}/r \qquad h_{oz} = C_{oz}/r \quad (3)$$

Cowling then calculates (to first order in h) the resultant light--deflection from each of these therms, as observable by HIPPARCOS. In fact, he has transformed his data to a useful form where the effective deflection (in the abscissae) may be calculated from the three angles ψ, η and ϕ_o. Here, ψ is the main variable giving the position of a star relative to that of sun. The angle η is related to the "revolving scanning" of the satellite, making 6.4 revolutions per year. Finally, the non-axisymmetric terms need a reference direction ϕ_o, which is simply taken as the sun's longitude. (For simplicity, the plane of symmetry is taken to be the ecliptic instead of the solar equator, but the difference should be small).

The final result of Cowling's analysis is a set of fifteen parameters g_i and their differential coefficients $G^{(i)}$ such that the total deflection is

$$\Delta(\text{absc.}) = \Sigma\, g_i\, G^{(i)}\, (\psi, \eta, \phi_o) \quad (4)$$

The parameters are linear combinations of the A-E:s in (2) and (3), divided by powers of r_o ($\equiv$ 1 a.u.). The coefficients $G^{(i)}$ are written as Fourier series in ψ multiplied by trigonometric factors in η and

ϕ_o. The ψ-dependence is rather similar for all the parameters, however, which makes for troublesome correlations. See further Section 5. If General Relativity is correct, we expect nonzero values only for

$$\begin{aligned} g_1 (&\equiv -[A_{oo}^{(1)} + A_{ii}^{(1)}]/r_o) = 4GM_o/c^2 r_o = 3.95\ 10^{-8} \\ g_3 (&\equiv -[A_{oo}^{(2)} + A_{ii}^{(2)}]/r_o^2) = O(g_1^2) \leq 10^{-15} \\ g_9 (&\equiv -A_{o\phi}^{(2)}/r_o) = 2GJ_o/c^3 r_o^2 \sim 4\ 10^{-17} \text{ (rigidly rot. sun)} \end{aligned} \tag{5}$$

Even a much higher solar angular momentum (J_o) would not increase g_9 to detectable levels, cf. Richter and Matzner (1981).

5. THE CHOICE OF GLOBAL PARAMETERS FOR HIPPARCOS

Because the spin-axis of the satellite always keeps the same 43^o angle to the sun, thermal effects are expected to vary only with the angle ψ defined above. A standard set of global parameters are then f_1-f_{12}, with differential coefficients sin/cos $n\psi$. (f_1, with coefficient $\sin^2\psi$ is found to be inseparable from a parallax zero-point error and is better not included). At least g_1-g_7 are strongly correlated with the Fourier sine-terms, and there are also internal correlations within the Cowling group.

In order to find an "optimum" set of Cowling parameters, we have made some analyses of the eigenvectors/eigenvalues of the (global part of the) normal equations. One may show that these eigenvalues define a transformation to new, uncorrelated variables, with variances given by the inverse eigenvalues. The result of such an analysis is given in Table 1. For clarity, only coefficients above 0.1 are included, but

Table 1. Approximate transformation coefficients from the Cowling parameters g_i to uncorrelated "eigenvectors" of the normal equations.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}	g_{14}	g_{15}	ME
1								.66	.75							1.0
2	.19	.22	.44	.43	.49	-.48	.22					.10				2.9
3			.19	-.11						-.62	.72					4.8
4										.76	.64		.11			4.8
5	.25	.29	.58	-.34	-.30	.22				.17	-.23	.42				5.2
6	-.15	-.16	-.33	.13								.82			-.37	5.6
7								.72	-.62				.26	.16		8.2
8										-.10			.80	-.57		10
9								-.21	.29		-.11		.52	.80		10
10												.36			.93	12
11			.11	.26	.52	.75	-.29									33
12	.17			.67	-.51		-.50									46
13	.91		-.36	-.12	.15											185
14				.37	-.30	.39	.78									902
15		.90	-.41													1807

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this transformation still involves many g_i:s simultaneously. The interesting thing to note is the last column with teoretical mean errors (normalized to unity for the smallest one). Only a few parameters may be accurately determined, and it appears useless to include more than about ten.

It is of course possible to use the first rows of Table 1 as the definition of a set of "compound" Cowling parameters. In the real HIPPARCOS-reductions, the transfomration can only be found in retrospect, however, and we prefer to use the following set of compound variables with simpler coefficients:

$$\begin{aligned} h_1 &= g_9 + 0.88\ g_8 \\ h_2 &= (g_1 + 1.1\ g_2 + 2.3\ g_3) + 2.5(g_4 + g_5 - g_6) \\ h_3 &= (g_1 + 1.1\ g_2 + 2.3\ g_3) - 1.25(g_4 + g_5 - g_6) \\ h_4 &= g_8 - 0.88\ g_9 \\ h_i &= g_{i+5}, \quad i = 5,9 \end{aligned} \tag{6}$$

This transformation is completed in such a way that it may be inverted, and it is then easy to give the new differential coefficients in terms of Cowling's $G^{(i)}$. (The combinations of g_1 to g_3 and g_4 to g_6 in the parentheses in h_2 and h_3 reflects the fact that the corresponding G:s are roughly proportional to each other.) The main advantage of these h-parameters is their low degree of internal correlation (≤0.25). We may now define two standard sets of global parameters for HIPPARCOS. Set A includes the Fourier parameters f_2–f_{12} and the compound Cowling parameters h_1, h_4–h_9 (because h_2 and h_3 are strongly correlated with f_3 and f_5). Set B includes h_2 and h_3 but excludes then instead f_3 and f_5.

6. ESTIMATED COVARIANCES FOR THE GLOBAL PARAMETERS

From a number of test solutions with 730–1825 sets and 5–10,000 stars, the following results emerge. With σ^g as the mean error for the Fourier parameters f_2–f_{12}, we have approximately

$$\sigma^g = 1.6\ \sigma^a (N_{eff})^{-1/2} \tag{7}$$

where N_{eff} ($\simeq$ 27 N) is the is the "effective" number of observations of N stars. The mean abscissa error σ^a may be about 4.0 mas for 50,000 "good" stars, and we estimate $\sigma^g = 2.7\ 10^{-11}$ = 5.5 microarcseconds.

The interesting figures are now the mean errors relative to σ^g for the individual h_i. (For f_2–f_{12}, these ratios are close to unity). Table 2 shows that they are unfortunately rather large. The low value for h_1 is expected, because the $A_{o\phi}$-terms in the metric give a light-deflection with different sign on opposite sides of the sun. It should be noted, however, that the sensitivity of the HIPPARCOS observations is almost 6 orders of magnitude above the expected effect, see eq. 5.

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Table 2. Mean errors for the metric parameters relative to σ^g

h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9
0.66	(7.1)*	(8.1)*	5.6	2.6	2.6	4.8	5.0	5.0

* in set B (f_3 and f_5 excluded)

The general relativistic light-deflection (g_1) is included in h_2 and h_3, which may be determined only if f_3 and f_5 can be neglected. One has then

$$\begin{aligned} g_1 + 1.1g_2 + 2.3g_3 &= (h_2 + 2h_3)/3 \pm 5.9\ \sigma^g \\ g_4 + g_5 - g_6 &= (h_2 - h_3)/3 \pm 3.6\ \sigma^g \end{aligned} \qquad (8)$$

A non-zero result for $g_4 + g_5 - g_6$ most probably means that h_2 and h_3 are still dominated by instrumental effects. Only if $h_2 \simeq h_3$ (within the calculated mean error), we may use them to estimate the GR--deflection. In this favourable case, the error in g_1 (assuming g_2 and g_3 = zero) will still be about 0.03 mas (0.4 %).

7. CONCLUSIONS

For the general space-time metric defined by Cowling, the HIPPARCOS observations will determine about nine independent parameters. For one of them, the expected accuracy reaches 2 10^{-11}, for most of the others the mean errors are five times larger. Probably, a non-zero result will <u>not</u> be due to space-time effects, but will indicate instead some subtle instrument distortions (which may then be removed). <u>If</u> the instrumental effects are small, the GR light deflection may be confirmed to about 0.5 %.

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DISCUSSION

Grishchuk : is the Cowling metric a solution of some gravitational equation or is it a purely formal expression ?

Söderhjelm : it is a purely formal expression.

Grishchuk : is it possible that if we restrict ourselves only to metrics that are solutions of some equations, then the accuracy in γ would be better ?

Söderhjelm : may-be.

Alley : the accuracy in γ that can be expected from HIPPARCOS is worst than VLBI or radar estimates. Then why use HIPPARCOS ?

Söderhjelm : the main objective of HIPPARCOS is not relativity but astrometry. Some approximate estimations have suggested that perhaps HIPPARCOS could also improve estimates of relativistic parameters. We have shown in this paper that it is not the case.