Reflections of a Mathematician, by L.J. Mordell. Canadian Mathematical Congress, 1959. vii + 50 pages. $1.00.

The Canadian Mathematical Congress is to be congratulated on seizing the opportunity to publish this little book which so pleasantly reveals the personality of a Sadleirian Professor Emeritus. It should appeal to everyone who has any interest in mathematics, and particularly to any student who is beginning to do research.

In his attempt to answer the tantalizing question "What is mathematics?" the author refers to the borderline cases between pure mathematics and logic, where the distinction is as difficult to make as when a biologist wishes to decide whether some living organism is animal or vegetable. He recalls that his own interest in algebra arose from finding an old textbook in the 5-cent box of a second-hand bookstore in Philadelphia. He states that the indispensable qualities of a mathematician are "hope, faith, and curiosity, and the prime necessity is curiosity." Referring to Hardy's remark (A Mathematician's Apology, p. 10) that "mathematics is a young man's game", he gives some encouragement to older men by citing his own discovery, when over fifty, of the best possible estimate for the minimum numerical value of a binary cubic form. As an example of the important role played by the subconscious, he recalls that at one stage he had to set this problem aside because of what seemed a very formidable difficulty. But while spending a week-end in the Lake District he overcame the difficulty so easily that it was hard to realize why it had caused any trouble. Elsewhere he recalls that he rediscovered Eisenstein's formula for the class-number of definite ternary quadratic forms while walking along Oxford Street in May 1916.

After advising the student to face the fact that "every aspect of mathematics bristles with difficulties", he draws attention to the importance of clear presentation. "It is unwise of an author to take too much for granted. He should, on the other hand, avoid unnecessary or excessive detail." He quotes Polya's advice: "If you have two things to say, say them one at a time."

On the subject of the origin of problems, he remarks: "Signs of an active mathematician are that he can always find a problem and is usually occupied with one." As an instance of a good problem, he mentions an arithmetical counterpart of the four-colour map problem of topology. Every map on a sphere can be map-coloured with five colours, but nobody has

ever been able to prove that four will always suffice. Similarly, "every integer is the sum of at most five integer cubes, positive or negative, and there is an unproved conjecture that four cubes suffice."

He cites a very interesting example of the help that electronic computers can sometimes give to pure research not only in extending the borders of numerical knowledge (as when SWAC recognized $2^{2281} - 1$ as a prime number) but also in suggesting interesting theorems. He warns young mathematicians to remember that there may be no relation between the difficulty of a problem and its importance.

In the chapter on the element of luck, the author describes his own "lucky" discovery of the finite basis for the rational points on a plane cubic curve, and points out how much may depend on the people whom a young mathematician happens to meet. He refers to the striking case of Ramanujan "who was really a failed B.A." but became, with Hardy's timely encouragement, the most distinguished of all Indian mathematicians.

The author believes that mathematicians are more inclined than other scientists and scholars to praise their subject for its beauty, clarity, and precision. He describes some of the essential qualities of elegant mathematics. The problem must be interesting and significant. "The fundamental idea of the proof must be exceedingly simple, and just the right one. ...The application must seem inevitable and completely unexpected." The real mathematician "will find as much pleasure and satisfaction in his production as any artist, musician, painter, or sculptor does." Aesthetic judgment is required in choosing the right kind of problem to consider and the best way to present the solution. "But of course what really matters in the first place is to find any solution. Afterwards we may begin to look for the 'right solution'....Often the simpler the solution the more time and effort were required to find it."

Research workers are warned against undue haste in seeking publication. "Too often, papers appear which suffer from ... uninteresting generalization, ... or so much abstraction that only the bare bones of the subject are left, with very little indication of life in them."

On the subject of mathematical schools, the author describes the happy consequences for all concerned when a mathematician gathers around him a group of young researchers to whom he is willing to leave some of the detailed developments that follow from his own ideas.

He emphasizes the universality of mathematics, with its notation that is almost as free from barriers of language as is the notation of music. He shows by his own example the advan-

tage of travelling abroad to exchange ideas with mathematicians in many different countries.

He contrasts the achievements of the rare mathematical genius with those of the ordinary mathematician who must remain satisfied with a few slight contributions. He modestly places himself in the latter category. Looking back over his life, with its successes and its failures, he reflects that his efforts have been "worth while in every way.... Probably one is happiest when engaged in congenial work." He compares the efforts and rewards of mathematical research with those of mountaineering. After retirement he is still active and takes a keen interest in the work done by others --- unlike Sir Isaac Newton, who separated himself completely from the subjects that made him immortal.

An Appendix gives the solution (involving a double integral) of a delightful little problem about the time of waiting for a bus.

The reviewer hopes that the above outline will convey something of the spirit of this highly enjoyable booklet. It has been beautifully printed (by the Cambridge University Press) and is adorned with an excellent portrait of the author.

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Dynamics and Non-linear Mechanics, by E. Leimanis and N. Minorsky. Surveys in Applied Mathematics II. John Wiley and Sons, New York, 1958. 206 pages. $7.75.

This is volume 2 of a series Surveys in Applied Mathematics, and the contributions of the two authors are quite distinct.

Leimanis surveys, in 108 pages, recent advances in some classical problems of Dynamics, notably the motion of a rigid body free to rotate about a fixed point, and the problem of three bodies: both, of course, with the customary idealizations. In each of these cases the problem as originally posed was to solve a set of differential equations, and in due course it appeared that solution by elementary processes was impossible; so the problem becomes, more vaguely, to find out what we can about the solution, or to see whether specialization of the data makes it more tractable. In the rigid body problem let A, B, C be the principal moments of inertia at the fixed point O, (x_o, y_o, z_o) the coordinates of the mass-centre and (p, q, r) the angular velocity relative to principal axes in the body at O. The special cases A = B, $x_o = y_o = 0$ (the gyroscope) and $x_o = y_o = z_o = 0$ ('Poinsot motion') are easy and classical - 18th century. Then in 1888 Kowalevskaya startled the mathematical world by show-