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ON A QUESTION POSED BY D. LEVIATAN AND L. LORCH

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In this note we construct a pair of regular matrices T_1 and T_2 such that T_1 is stronger than T_2, and the T_2-transforms of all bounded sequences are such that the sets of limit-points are connected, while there is a bounded sequence such that the set of limit-points of its T_1-transform is not connected. This provides a negative answer to a question posed by Leviatan and Lorch [3, §6, (c)].

Let T_2 be any regular matrix having the property that for each bounded x the set of limit-points of T_2x is connected. (For example the method $(C, 1)$ has this property (see [1] or [2]).) Then by regularity, there exists a bounded sequence x, and two disjoint increasing sequences of indices, $\{m_r\}$ and $\{n_r\}$ with the property:

$$\lim_{r\to\infty}(T_2x)_{m_r} \neq \lim_{r\to\infty}(T_2x)_{n_r}$$

Let T_1 be the matrix obtained from T_2 by deletion of all rows except those of indices m_r and n_r. Evidently T_1 is stronger than T_2. The sequence T_1x is a subsequence of T_2x with only two limit-points and therefore disconnected.

Furthermore, we construct two matrices T_1 and T_2 having the above property and such that T_1 is strictly stronger than T_2. Let $T_2=(C, 1)$ and denote $\{p_r\}=\{m_r\}\cup\{n_r\}$. It is clear that we can obtain a subsequence $\{q_r\}$ including an infinite number of elements of each of $\{m_r\}$ and $\{n_r\}$ and such that $q_{r+1}>q_r\cdot g$ for some $g>1$ and all $r\geq 1$.

Corresponding to each of these indices, we choose $x_n=1$ in the first half of the block $q_r\leq n<q_{r+1}$ and $x_n=-1$ in the second half of the block. All remaining x_n are defined to be 0 (see [2], p. 392).

Clearly $\lim_{T_1}x=0$ and $\lim_{T_2}x$ does not exist; thus T_1 is strictly stronger than T_2.

References

1. H. G. Barone, *Limit points of sequences and their transforms by methods of summability*, Duke Math. J. **5** (1939), 740–752.

2. P. Erdös and G. Piranian, *Laconicity and redundancy of Toeplitz-matrices*, Math. Z. **83** (1964), 381–394.

3. D. Leviatan and L. Lorch, *On the connectedness of the limit-points of certain transforms of bounded sequences*, Canad. Math. Bull. (2) **14** (1971), 175–181.

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