8.4 RADIATION FROM PULSARS

I. LERCHE*

Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Ill., U.S.A.

Abstract. We suggest that an oscillating interface, carrying a steady current between a rotating, radiating, magnetic dipole and an external plasma is the source of pulsar radiation produced over a broad band of high frequencies (10^8–10^{18} Hz). The efficiency of the radiation mechanism is so high that the oscillation energy goes mainly into radiation in each cycle of the oscillation.

Recently Pacini (1968) and Gunn and Ostriker (1969) have shown that a rotating neutron star, with a magnetic dipole axis oriented perpendicular to the spin axis, provides a reasonable and plausible explanation of the basic 'clock' mechanism for pulsars. Pacini has pointed out that the radiation pressure due to the oscillating field pushes any material outward, thereby creating a cavity dominated by the radiation, and surrounded by plasma on the outside. The radiation sweeps all material out of the cavity so that the radiation in the cavity is describable to a first approximation by a vacuum field. Gunn and Ostriker demonstrated that a charged particle placed in such a vacuum field is rapidly accelerated to relativistic energies as it is swept outward by the radiation pressure.

This paper is concerned with the interesting physical consequences for radiation output from the interface between the external plasma and the radiation field assuming the particles at the interface are relativistic, so that they can respond to the basic pulsing low frequency ($\sim$30 Hz) electromagnetic radiation at a velocity close to c.

The boundary of the cavity terminates the radiation field, with external plasma pressure balancing the radiation pressure. At the interface the stress of the vacuum dipole radiation field is transferred to the particles in the plasma by the Lorentz forces of a surface current. The dipole radiation pattern rotates with the neutron star at an angular frequency Ω. Thus the radiation pressure on the interface oscillates about its mean value with a frequency Ω, and so the position of the interface oscillates about its mean value with a frequency Ω. Thus the steady current in the interface, which transfers the stress of the dipole field to the plasma outside the cavity, oscillates perpendicularly to its direction along the interface with an angular frequency Ω. *Such an oscillating current sheet radiates energy.*

In order to illustrate the manner in which high frequency radiation is produced in such a system, consider the particular situation of an infinite, plane, current sheet in vacuum carrying a steady current of density j_0 per unit length which points in the x-direction. This particular example contains most of the essential physics of the conversion mechanism. The problem can also be considered from the point of view of a finite thickness, partially coherent, interface and/or an independent particle picture. Since all three points of view yield essentially the same results (apart from

* Alfred P. Sloan Foundation Fellow.

Davies and Smith (eds.), The Crab Nebula, 449–454.

numerical factors of order unity) we have concentrated here on the point of view which most succinctly illustrates the essential physics of the basic problem.

Take the current sheet to move sinusoidally in the y direction at an angular frequency Ω and with amplitude y_0.

Then Maxwell's equations are

$$c^2\nabla^2\mathbf{A} - \frac{\partial^2\mathbf{A}}{\partial t^2} = 4\pi c j_0 \hat{x}\,\delta[y - y_0 \sin\Omega t], \tag{1}$$

which gives the electric field, $\mathbf{E} = -c^{-1}\,\partial\mathbf{A}/\partial t$, as

$$\mathbf{E} = \hat{x}\pi j_0 c^{-1} \sum_{n=1}^{\infty} J_n[n\Omega y_0/c] \exp[in\Omega(y - ct)/c], \quad y > y_0. \tag{2}$$

[For details concerning this and other points in the paper we refer the reader to the original papers (Lerche, 1970a, b, c).] Then the time averaged Poynting vector is

$$\begin{aligned}\mathbf{P} &= \hat{y}(\pi j_0^2/8c) \sum_{n=1}^{\infty} J_n[n\Omega y_0/c]^2, \quad y > y_0; \\ &\equiv \hat{y}(\pi j_0^2/16c)\,[(1 - \Omega^2 y_0^2/c^2)^{-1/2} - 1].\end{aligned} \tag{3}$$

Now the relativistic particles move nearly at c. So simple arguments based on the forces involved indicate that y_0 could be as large as c multiplied by half a period of the oscillation, $\pi c/\Omega$. But if y_0 were as large as $\pi c/\Omega$, then the radiation output would be infinite. So energy considerations require that y_0 be less than c/Ω. With radiation damping reducing y_0 to something less than c/Ω we write $\Omega y_0 = c(1 - \frac{1}{2}\varepsilon^2)$. [We are unable to solve the dynamical equations of motion for a relativistically oscillating interface, including radiation reaction. We would, of course, be extremely interested to see calculations related to this problem.]

For $\varepsilon \ll 1$ the total power radiated is

$$P = \frac{\pi j_0^2}{16c\varepsilon} \quad \text{erg cm}^{-2}\,\text{sec}^{-1}. \tag{4}$$

We expect that ε adjusts itself so that the power emitted, P, equals the power available, $P_0 \equiv cB(r_c)^2/16\pi$, where r_c is the distance of the interface from the neutron star. A very crude estimate of r_c can be obtained as follows. The balance of particle pressure with radiation pressure at r_c gives $nmc^2\mathscr{E} = B_*^2 R_*^6 \Omega^4/(8\pi c^4 r_c^2)$, where B_* is the surface field of the neutron star, R_* its radius, $mc^2\mathscr{E}$ the mean relativistic particle energy and n the number density of particles at r_c. If we also take the displacement current to be $O(nec)$ at r_c then we find $n(r_c) \approx \Omega^2\mathscr{E}me^{-2} \simeq 10^2\ \text{cm}^{-3}$, and $r_c R_*^{-1} \simeq \simeq eB_* R_*^3\Omega/(8mc^2\mathscr{E}) \approx 10^5$. We have assumed, with Gunn and Ostriker, that $B_* \simeq 10^{12}$ G, $\Omega \simeq 10^2\ \text{sec}^{-1}$, $R_* \simeq 10^6$ cm, $\mathscr{E} \simeq 10^6$. Also the wave zone of the dipole field starts at about $r \simeq 10^2\ R_*$ for the numbers given. Thus the plasma-radiation interface is far out in the wave zone of the dipole field. And the interface is probably $O(c/\Omega)$ thick, which is a cyclotron radius at r_c.

From the balance of Lorentz force with dipole stress $j_0 B(r_c) = cB(r_0)^2/8\pi$ and $P \lesssim P_0$, we have $\varepsilon \gtrsim \frac{1}{64}$ for an infinitely thin current sheet. The differential spectrum of the radiation is, for $\omega \gg \Omega$,

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} = \frac{\pi j_0^2}{8c\Omega} J_{\omega/\Omega}\left[\frac{\omega}{\Omega}\left(\frac{\Omega y_0}{c}\right)\right]^2, \tag{5}$$

or, using the asymptotic expansion of the Bessel function for $\omega \gg \Omega$ and $\Omega y_0/c \approx 1$, we have

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} \simeq \frac{\Gamma(\frac{1}{3})^2 j_0^2}{16 \cdot 2^{1/3} \cdot 3^{1/6} \pi c \Omega^{1/3} \omega^{2/3}} \quad \text{erg cm}^{-2}\,\text{sec}^{-1}\,(\mathrm{d}\omega)^{-1}, \tag{6}$$

for $\omega \lesssim 3\Omega\varepsilon^{-3} \equiv \omega_{\max}$; and where $\omega = n\Omega$, $n \gg 1$.

Thus the differential power spectrum varies as $\omega^{-2/3}$. The larger ε^{-1} becomes the higher is the upper limit to the emission frequency. There is no radiation for $\omega > \omega_{\max}$. But for an infinitely thin current sheet, encompassing the total current j_0, the emission mechanism is too efficient. With $\varepsilon = \frac{1}{64}$ ($\omega_{\max} \simeq 7000\,\Omega$) it would be difficult to obtain X-ray emission since 10 keV radiation ($\omega_{\max} \gtrsim 10^{19}$ sec^{-1}) requires $\varepsilon^{-1} \sim 10^6$.

However it should be remembered that the current sheet is $O(c/\Omega)$ thick, with a distribution of current across the interface which gives partial incoherence. Since we do not have a detailed description of the interface structure and dynamics, including radiation reaction, the problem of partial, or complete, incoherence across a finite thickness interface remains unsolved. About all we can do is to introduce a phenomological constant, μ, into the formula for emitted power and write $P = \mu\pi j_0^2/16c\varepsilon$, and $\mathrm{d}P/\mathrm{d}\omega = \mu \times [\mathrm{d}P/\mathrm{d}\omega$ from Equation (5)]. Here μ, which is small compared to unity, represents the incoherent nature of the finite thickness interface for emission at frequencies well in excess of Ω. And now $\mu = 64\varepsilon$ for balance of power emitted with power available. The effect of the incoherent nature (for $\omega \gg \Omega$) of the finite thickness interface is to increase the spectral range of the emission (by allowing ε^{-1} to be of the order of 10^6 for 10 keV pulsed X-ray emission, for example) but to decrease the power at a given frequency by μ. Thus increasing the spectral range by a factor 10^{14} (ε^{-1} going from 64 to 10^6) decreases the power at a given frequency by a factor 2×10^4.

For example, a finite thickness interface with the current per unit area, $\mathbf{j}$, oscillating as

$$\mathbf{j}(y, t) = \hat{x} \int_{-\infty}^{\infty} J(Y_0)\, \delta\{y - Y_0 - y_0 \sin[\Omega t + \varphi(Y_0)]\}\, \mathrm{d}Y_0,$$

with $y_0\Omega = c(1 - \frac{1}{2}\varepsilon^2)$ and $\varphi(Y_0)$ a *random* phase, also gives $\mathrm{d}P \propto \omega^{-2/3}\,\mathrm{d}\omega$. The power output is proportional to $\int_{-\infty}^{\infty} J(Y_0)^2\,\mathrm{d}Y_0$, with $\int_{-\infty}^{\infty} J(Y_0)\,\mathrm{d}Y_0 = j_0$.

This is to be contrasted with the emission from an infinitely thin current sheet, carrying the current per unit length j_0, for which the power output is proportional to j_0^2. The $\omega^{-2/3}$ dependence of the differential power output is a property of the $\sin(\Omega t)$

motion. The *level* of power at a fixed frequency is a property of the structure of the current distribution in the interface. Only if the motion differs from $\sin(\Omega t)$ does the shape of the power spectrum change. For example if each point in the interface moves with $y = Y_0 + y_0(t)$ with $d^2 y_0/dt^2 \propto \delta$-function during each period and $y_0(t) = y_0(t + 2\pi/\Omega)$ then $dP \propto \omega^{-2}\, d\omega$, assuming random phase between neighboring points.

It has not escaped our attention that a finite thickness interface can explain several other features of pulsar emission. Elsewhere (Lerche, 1970b, c) we show that such an interface can be used to explain pulse structure, finite pulse width (transit time delay of radiation through the interface), polarization 'swing' (rotation of Lorentz current through the interface) through a pulse, progressive 'march' of individual features through many pulses, and highly directional emission. Since this paper is concerned with the basic physics of the emission process rather than detailed models of the interface structure, we do not believe it is appropriate to enter into a discussion of these points here. Instead we refer the interested reader to the original papers (Lerche, 1970a, b, c.)

Extensive observations of pulsars at many frequencies have been made. [In order to keep the number of references under control we shall refer generically to the 'Progress Report on Pulsars' by Maran and Cameron (1969) and the 'Review of Theories of Pulsars' by Chiu (1970). The interested reader is referred to both for the chronology of pulsar observations and theories. The Maran-Cameron report was up to date as of March, 1969; the Chiu report was up to date as of May, 1969.] Very preliminary results of infra-red observations of NP 0532 have also been reported (Neugebauer *et al.*, 1969). They are not included here in view of their preliminary nature.

In order to apply our results to actual pulsars we choose to discuss pulsar NP 0532 as an illustrative case.

In the case of NP 0532 the basic oscillation frequency is $\Omega = 2\pi\ 30\ \mathrm{sec}^{-1}$ corresponding to a pulsing period of $\frac{1}{30}$ sec. The time averaged emission spectrum has been seen with a bandwidth extending from the radio (408 Mc/sec – 1720 Mc/sec) through optical (3400 Å–8300 Å) through X-ray (1.5 keV–10 keV). The intensity levels seen in these bands are

$$P(\text{X-ray}) : P(\text{optical}) : P(\text{radio}) \simeq 7000 : 100 : 1,$$

and the differential power spectrum, $dP/d\omega$, is proportional to $\omega^{-\alpha}$ with $\alpha = 0.9 \pm 0.2$ in the radio region, $\alpha = 0.5 \pm 0.2$ in the optical region, $\alpha = 1 \pm 0.2$ in the X-ray region (Maran and Cameron, 1969; Oke, 1969, Gorenstein *et al.*, 1968, Bradt *et al.*, 1969).

The overall shape of the theoretical spectrum (Equation 6) from radio to X-ray agrees with the observations since the theory predicts $\alpha = 0.7$. Upon integration Equation (6) the theory also predicts that, in the bandwidths given above, the power levels should be in the ratios

$$P(\text{X-ray}) : P(\text{optical}) : P(\text{radio}) = 1600 : 80 : 1.$$

which, within a factor 4 or so, are in accord with the experimentally determined values.

Also, if we *assume* $\omega_{max} = 10$ keV/$\hbar$, then $\varepsilon^{-1} \simeq 10^6$. The upper cut-off frequency is very sensitive to the choice of ε and, conversely, ε is not a very sensitive function of the upper cut-off frequency. No optical emission has yet been seen from any pulsar other than NP 0532. These null observations then suggest that ω_{max} for such pulsars is less than about 10^{15} sec^{-1}.

The motion and radiation described above encompass the essential physics of the conversion mechanism.

We wish to stress that, within the framework of the Pacini and Gunn-Ostriker rotating magnetic neutron star picture, consideration of the radiation-plasma interface is of prime importance. For the underlying neutron star only provides the basic 'clock'; the high frequency radiation observed originates in, and at, the interface through the motion and dynamics of the surface current produced by the Lorentz forces transferring the dipole stress to the external plasma.

It is this fact we wish to emphasize rather than the particular model of an infinitely thin current sheet oscillating sinusoidally in vacuum, and the detailed illustration of this situation as applied to NP 0532. The agreement of this simple model with the observations is clearly encouraging, but is obviously less than a full resolution of the interface dynamics, motion and radiation.

However we point out that the essential physics of the interface motion is captured in the simple illustrative situation described above, and while more complex models of the motion, structure and dynamics of the interface may alter the 'fine grain' details of the radiation output, they will not change the basic physics involved.

In short, we believe that the physical mechanism outlined in this paper, and discussed in detail elsewhere (Lerche, 1970a, b, c), operates for pulsars, but that our numerical values for power output, etc., are probably good to a factor 10 or so only. The spectral index of 0.7 is probably good to a factor of order unity – the precise value depending on a detailed model for the interface structure and motion.

Acknowledgements

I am grateful to Professor S. Chandrasekhar for his encouragement and help in matters concerning the realization of this paper. I also wish to thank Professor E. N. Parker for many stimulating discussions concerning the basic physical mechanism. This work was supported by the United States Air Force, Office of Aerospace Research under contract F-19628-69-C-0041.

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Discussion

R. N. Manchester: Is the transition at which the radiation occurs the same as the one discussed by Rees earlier – that is, the boundary of the nebula.

I. Lerche: No, my interface is not at the distance of Rees'. If I remember correctly Rees' interface is at the outside edge of the nebula, whereas mine is at 10^{11} cm from the pulsar.

M. J. Rees: I do not at all understand what is supposed to provide the huge external pressures which prevents your current layer from being pushed out at least to the edge of the Nebula.

I. Lerche: Either the excess plasma pressure, if it is non-uniform, or an imbedded magnetic field in the plasma will hold the interface at its mean position.

F. D. Drake: In order to produce the detailed pulse shapes which are stable over years, how does your model maintain a fixed complex small scale current structure in the emitting region?

I. Lerche: I don't know! All I know is that the total current per unit length must be of order $cB/4\pi$ always. Why it should be distributed with a fixed small scale structure throughout the interface I do not know.