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**On 107.27: Graham Jameson writes:** the author's reasoning requires the additional assumption that  $f_j > 0$  for  $1 \leq j \leq S$ . This condition appears in the working at the top of page 346, but is not included in the initial statement of the result. It is clearly essential for the proof: if  $f' = 0$ , then one cannot conclude that  $u_t < M$ . In fact, a trivial example shows that the theorem can fail without this assumption: take  $f_1 = 0$  and  $f_2 = 1$ , so that  $u_n = u_{n-2}$ , hence  $u_n$  equals 1 for even  $n$  and 0 for odd  $n$ . The author's method can be compared with [1], where the same result is established in non-probabilistic language.

There are important applications of the renewal theorem, notably in population dynamics, in which the above assumption does not hold. The condition that is needed for the conclusion to hold is actually as follows [2, p. 330]: if  $K(f)$  is the set of  $j$  for which  $f_j > 0$ , then the greatest common divisor of the members of  $K(f)$  must be 1. This more general version requires considerably more work: the proof in [2] is not easy.

#### References

1. H. Flanders, Averaging sequences again, *Math. Gaz.* **80** (March 1996), pp. 219-222.
2. W. Feller, *An Introduction to Probability Theory and its Applications*, Wiley (1971).

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