whence $\epsilon^{-nz} = \frac{dv}{dz}$;

giving,

$$\int 10^{-10^z}\epsilon^{-nz}dz = -10^{-10^z}\frac{\epsilon^{-nz}}{n} - \frac{(\log_e 10)^2}{n}\int 10^{-10^z}\epsilon^{-(n-\log_e 10)z}.dz.$$

I am, Sir,
Your very obedient servant,

London, 12 *June* 1873. W. M. MAKEHAM.

ERRATUM.—In the paper "On the Integral of Gompertz's Function," above referred to, there is a misprint. For the second formula on p. 309,

$$\log\frac{1}{g^{q^x}\epsilon^{-(a+\delta)x}}\int_x^\infty g^{q^x}\epsilon^{-(a+\delta)}.dx$$

should be read,

$$\log\frac{1}{g^{q^x}\epsilon^{-(a+\delta)x}}\int_x^\infty g^{q^x}\epsilon^{-(a+\delta)x}.dx.$$

ON THE RELATION BETWEEN THE NET PREMIUM AND THE RATE OF INTEREST.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the current volume of the *Journal*, p. 227, I gave a demonstration in reference to the relation between the value of a policy and the rate of interest according to which it is calculated, and in the present letter I propose to examine in a similar way the relation between the net premium and the rate of interest.

We have $\mathrm{P}_x = \frac{1}{1+a_x} - (1-v)$,

$$\therefore \quad \frac{d\mathrm{P}}{dv} = \frac{-\frac{da}{dv}}{(1+a)^2} + 1 \text{ (omitting the subscript } x\text{)},$$

$$= \frac{(1+a)^2 - \frac{da}{dv}}{(1+a)^2}.$$

Thus, since P_x increases or decreases, when v increases, according as $\frac{d\mathrm{P}}{dv}$ is positive or negative, we have only to examine whether

$$(1+a)^2 > \text{ or } < \frac{da}{dv}.$$

Now, $(1+a)^2 = (1+a_x) + p_x v(1+a_x) + {}_2p_x v^2(1+a_x) + \ldots.$

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and as shown in my former letter,

$$v\frac{da}{dv}=a_x+p_xva_{x+1}+{}_2p_xv^2a_{x+2}+\ldots.$$

or
$$\frac{da}{dv}=\frac{a_x}{v}+\frac{p_xva_{x+1}}{v}+\frac{{}_2p_xv^2a_{x+2}}{v}+\ldots.$$

If, now, a_{x+1}, a_{x+2}, ... are none of them greater than a_x,

then
$$\frac{da}{dv}<\frac{a_x}{v}+\frac{p_xva_x}{v}+\frac{{}_2p_xv^2a_x}{v}+\ldots.$$

Hence, under the same condition, we shall certainly have

$$(1+a)^2>\frac{da}{dv},$$

if
$$(1+a_x)+p_xv(1+a_x)+{}_2p_xv^2(1+a_x)+\ldots.$$
$$>\frac{a_x}{v}+\frac{p_xva_x}{v}+\frac{{}_2p_xv^2a_x}{v}+\ldots.$$

that is, if
$$1+a_x>\frac{a_x}{v},$$

or
$$i\cdot\frac{1}{i}+a_x>a_x+ia_x,$$

or
$$\frac{1}{i}>a_x;$$

that is, if the value of a perpetuity of 1 is greater than the value of a life annuity of 1, the rate of interest being the same in both cases.

In other words, since the value of the perpetuity is necessarily the greater, $\frac{dP}{dv}$ is positive; therefore P_x, the net premium, increases as the rate of interest decreases, provided that a_x is not less than a_{x+1}, a_{x+2}, ...

I am, Sir,
Your obedient servant,
W. SUTTON.

18 *Lincoln's Inn Fields,*
1 *March* 1873.

ON THE FORMULA FOR THE MARKET VALUE OF A COMPLETE ANNUITY.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—The usefulness of the expression for the value of a life annuity in terms of d and p, first proposed by the late Griffith Davies, is obvious, whether from a theoretical or practical point of view. From the theoretical, in that it shows the elements of which the value consists; and from the practical, in that it is of universal application, equally valid whether p and d be based on the same rate of interest or not, or when p is a purely arbitrary quantity.

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