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BOOK REVIEWS

KHUKHRO, E. I. *Nilpotent groups and their automorphisms* (de Gruyter Expositions in Mathematics Vol. 8, de Gruyter, Berlin, New York 1993) 252pp., 3 11 013672 4, about £90.

This is an excellent coverage, by a leading expert in the field, of the use of Lie ring methods in the theory of (mostly!) finite p-groups. The book falls into two sections. The first is on Linear Methods, and is a very good textbook which could stand alone as such. The exposition starts *ab initio*, defining groups, rings, modules, isolators, Lie rings associated with lower central series and the like. Here are proved such classical results as the fact that the associated Lie rings of groups of prime exponent p satisfy the $(p-1)$-st Engel condition, and the nilpotency of soluble Engel Lie rings.

The second section is devoted to the subject indicated by the title. Chapter 4 contains proofs of the theorem of Higman, Kostrikin and Kreknin on regular automorphisms, with extensions to almost regular automorphisms. These results are applied in Chapter 5 to give a proof of Higman's theorem establishing a bound for the nilpotency class of groups admitting a regular automorphism of prime order. Then comes the beautiful result that every finite p-group admitting an automorphism of order p with exactly p^m fixed points contains a subgroup of (p,m)-bounded index and p-bounded class. Some highlights from the highly technical Chapter 6 (*Nilpotency in varieties of groups with operators*) are as follows. Let M_p be the variety of operator groups consisting of all groups with a splitting automorphism ϕ of prime order p, which means that the operator identities $x^{\phi^p}=x$, $xx^{\phi}\ldots x^{\phi^{p-1}}=1$ hold. Corollary 6.4.2 states that soluble groups in M_p are nilpotent, while 6.4.5 establishes the positive solution for the Restricted Burnside Problem in M_p: the locally nilpotent groups in M_p form a subvariety, that is, for each d the nilpotency classes of d-generator nilpotent groups in M_p are (d,p)-bounded. Chapter 7 contains another proof of this latter result. For finite p-groups, the study of splitting automorphisms of prime order is equivalent to the study of groups different from their Hughes subgroups. Highlights here are as follows. Another version of the RBP in M_p is that every d-generator finite p-group admitting a partition contains a subgroup of index p which is nilpotent of (d,p)-bounded class not more than $f(d-1,p)$, where f is the function implicit in the above result confirming RBP for M_p. There are many other interesting Hughes-type results in this chapter, indeed too many to mention here. Finally, in Chapter 8, the focus is on nilpotent p-groups admitting automorphisms of general p-power order having fixed-point set of given order, and includes the latest results of Shalev and the author.

The exposition throughout is clear, despite the technical nature of the subject; the author's pleasant style has been enhanced by J. C. Lennox's assistance with the pitfalls of English. Each chapter contains a final section of *Comments*, giving insights, overviews and reflections on proof. Altogether, this is a book to be recommended for its wealth of detail, covered extremely sketchily in this review, of a difficult area of nilpotent group theory.

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HÖRMANDER, L. *Notions of convexity* (Progress in Mathematics Vol. 127, Birkhäuser, Basel, Berlin, Boston 1994) 424 pp., 3 7643 3799 0, £38.

Lars Hörmander is the author of two well-known books: *The analysis of linear partial*