NON-LINEAR COUPLING OF SPIRAL WAVES AND MODES : THE *m=1 - m=2* CONNECTION IN LOPSIDED GALAXIES

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Abstract : *It is suggested that the asymmetric structure of lopsided galaxies can be explained by non-linear coupling of* m=1 *and* m=2 *spirals. These structures cooperate to transfer energy and angular momentum outwards, much farther than they individually could.*

Many spiral galaxies present a "lopsided", *i.e.* very asymmetric, structure. They are two-armed spirals, often barred, with one arm much longer than the other. Velocity measurements show that the rotation center is shifted from the center of the bar, by a distance which may be comparable with the length of the bar. We discuss here the possibility that this structure is created by distinct, but non-linearly coupled, *m=1* and *m=2* spirals. Our hypothesis is that the *m=2* creates the 2-armed structure, while the *m=1* causes both the inner shift of the rotation center and the outer part of the longer arm. Their non-linear collaboration allows them to transfer energy and angular momentum much further out than either one individually could. We present models involving either a single *m=1* structure or two distinct ones, in a way which should be easily discriminated by detailed observations of galaxies.

We use the physics we have introduced in previous works[1,2], involving the non-linear collaboration of *m=2* modes with either an *m=0* or an *m=4* one, in order to explain the observed behaviour of particle simulations of galaxies with realistically peaked rotation curves. We repeat here the main arguments of these works : these simulations by Sellwood[3] showed that two long-lived *m=2* modes could coexist in a galaxy, although they could not be explained by linear theory since one of them touched its Inner Lindblad Resonance. We noticed that in fact the two *m=2* structures overlapped over a short radial interval, which coincided with both the corotation of the inner one and the Inner Lindblad Resonance of the outer one ; this coincidence was confirmed *a posteriori* in all the corresponding simulations, and will be important in what follows (we also note that Sellwood[4] in more recent simulations of disks with a "groove", observed the emission of spiral waves which deposited their energy at the Lindblad resonance, resulting in the emission of a new spiral with corotation at the ILR of the first one. We believe that this might be explained by the emission mechanism discussed below).

Then we considered the mechanism of non-linear wave or mode coupling. This occurs when two waves of frequency and mode numbers ω_1 and ω_2, m_1 and m_2 , coexist. They generate beat waves, at $\omega_3 = \omega_1 \pm \omega_2$ and $m_3 = m_1 \pm m_2$. If one of them corresponds to a wave which can propagate in the disk, it will reach a large amplitude and exchange energy with the "parent" waves, so that the evolutions of the three structures become strongly coupled. The strength of this process is proportional to the product of the amplitudes of the parent waves, so that it is usually considered negligible at realistically weak amplitudes. However we showed that the coincidence of the resonances allowed in this case non-linear coupling at relatively low amplitudes (furthermore one easily checks that if waves 1 and 2 are respectively at corotation and Lindblad resonance, wave 3 will also be at a Lindblad resonance). The reason lies in the presence, in the coupling coefficient, of an integral containing two small denominators corresponding to the two

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resonances. Or, from a different but equivalent point of view, if the two resonances come very close the particle orbits will become stochastic at correspondingly vanishing wave amplitudes. This allowed us to explain the numerical simulations, and also to look for galaxies showing indications of this behaviour. It might be the case of some SB(r) galaxies, showing a definite phase mismatch between the bar and the inner tips of the spiral (*e.g.* NGC 6300, as found by Buta[5]). Elmegreen[6] independently proposed, in detailed modelling of M51, that its structure could be best explained by two different $m=2$ spirals, with the same coincidence of resonances.

Before turning to lopsided galaxies, let us note that this coincidence and radial localization is not the generic case of wave coupling. Usually it rather takes place over an extended region, and the coupling coefficient is found from an integral over this region. For instance the equation describing the evolution of mode 3 involves the integral :

$$C \sim \int dr\, \phi_3^* \mathbf{N}\, \phi_1 \phi_2$$

where $\mathbf{N}$ is an operator defined in refs.(1-2), containing the double resonant denominator, and the star notes the complex conjugate. If we were in a homogeneous medium the radial Fourier components would separate, and the integral would vanish for all but the components verifying the selection rule $k_3=k_2+k_1$. This is not the case here. When the region where the waves overlap is localized the product behaves as a δ function. When it is extended the integral has to be computed, and in general the linear solutions can be used for the potentials, since the non-linear term is only a perturbation to the wave evolution equation. The most common case of non-linear coupling involves a wave and its harmonic, *e.g.* in the case of interest here one might have $m_1=m_2=1$, $m_3=2$, and $\omega_1 = \omega_2 = \omega_3 /2$. In this case the pattern frequencies are the same, the corotation resonances coincide, and the waves coexist over an extended radial range. Then one has to worry about the ILR of the $m=2$. Assume for instance that the $m=1$ is a mode, established in a resonant "cavity" between the galactic center and corotation. If the frequency is not high enough, the $m=2$ harmonic will have an ILR and thus never be reflected from the center, so that its amplitude will remain limited. The process in this case reduces to the emission of trailing $m=2$ waves inwards and outwards. If the $m=2$ has no ILR, it can be reflected at the center and return to corotation, where it will also be amplified, reaching high amplitudes which will further enhance the non-linear mechanism. One must note however that in general the mode condition (which selects discrete frequencies for linear modes) cannot be fullfilled both by the $m=1$ and $m=2$. This might lead to chaos in the temporal evolution of the structures.

In this first case (hereafter case I) the coupling can be strong, both because it affects a large radial range and because this range contains the corotation resonance. However this resonance is weaker, for tightly-wound waves, than the Lindblad resonances. For lopsided galaxies we can also consider a mechanism similar to the one discussed in our previous works, where the coupling is radially localized to a small interval containing the corotation of one of the waves and Lindblad resonances of the other two. In such cases the $m=1$'s have distinct pattern speeds, whose average is the pattern speed of the $m=2$. Then we have two possibilities : in case II coupling occurs at the corotation of the $m=2$, coinciding with the OLR of the inner $m=1$ and the ILR of the outer one. In case III coupling occurs at the OLR of both the $m=2$ and the inner $m=1$, and corotation of the outer one. Consideration of wave energy and angular momentum budgets allows us to predict different behaviours and aspects of galaxies involving these cases.

We assume here that the effect of this coupling is to prevent the waves from meeting their Lindblad resonances and being absorbed, so that the total wave energy and angular momentum are conserved. Thus, noting W_i and J_i the energy and angular momentum *given* by each wave in the non-linear process, we have $W_1 + W_2 + W_3 = 0$, $J_1 + J_2 + J_3 = 0$. On the other hand we know that W_i and J_i are related by $W_i = \omega_i J_i / m_i$. This gives $J_1 = J_2$, $W_1 = W_2$. In case II since mode 2 is emitted outwards from its ILR W_2 is positive, and thus also W_1. This means that, since mode 1 is at its OLR, a trailing $m=1$ arm arrives at corotation (mode 1) and another one (mode 2) leaves outwards ; since W_3 is negative some negative energy

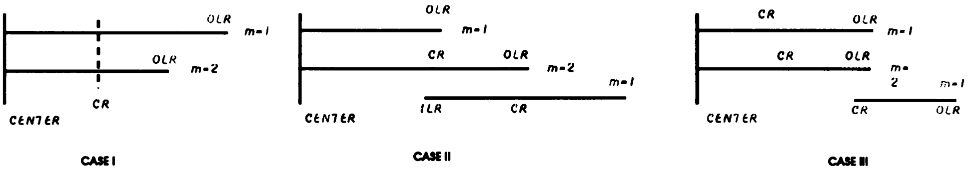

***Figure 1** : The radial extent and position of resonances for the modes in the three cases. CR, ILR and OLR denote the positions of respectively corotation and inner and outer Linblad resonances for each wave.*

of the *m=2* trailing wave arriving from the center is absorbed, corresponding to a decrease of the linear Swing amplification.

In case III for similar reasons W_2 and W_1 are negative, W_3 is positive, so that mode 3 is a trailing *m=2* wave arriving at its OLR, mode 2 is an *m=1* trailing wave leaving corotation outwards, and mode 1 is an *m=1* leading wave emitted inwards from its OLR.

In case I, because the pattern speeds are the same, we can reach no such conclusion. However modelisation of observed galaxies should rather easily allow to discriminate between these mechanisms, both from fitting the extent of the waves with measured rotation curves (allowing to identify the resonances), and from the presence or abscence of a leading *m=1* arm emitted inwards. To complete this discussion, we can also note :

i) That case I fortuitously corresponds to the *ad-hoc* model used by Marcellin and Athanassoula[7], in modelling the velocity fields of NGC 1313, with a bar rotating as a solid body around a fixed point (*i.e.* the rotation frequencies of the bar around its center and around the fixed point coincide).

ii) That in cases II and III, since the modes have different pattern speeds, the *m=2* and *m=1* arms should have an arbitrary relative phase angle. However numerical simulations of Sparke and Sellwood[8] have shown that, even when this is the case for the potential perturbations, it may be much harder to distinguish from the stellar densities which have a more fuzzy aspect. This is very similar to the observation of ref. 7 that in NGC 1313 the outer part of the longer arm (*i.e.* the outer *m=1* arm in our analysis) is nearly detached from the inner part (the *m=2*).

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