7

The decay kinematics of the massless relativistic string

7.1 Introduction

In this chapter we consider the situation when a $q\bar{q}$-state is produced with a large amount of energy at a single space-time point. It will be called the original pair and we assume that q and $\bar{q}$ interact through a constant attractive force, κ. The pair will then form a yoyo-hadron state as described in the previous chapter and immediately start to separate.

The state composed of the two particles and the force field, if it contains a larger mass than that of the stable hadrons, will decay into smaller-mass particles. Such a decay process is of course of a quantum mechanical nature.

Although we will at this point use semi-classical arguments, we will later show that the resulting formulas fit into both a quantum mechanical tunnelling process and a statistical mechanics scenario.

The major assumption will be that *a string state may decay by the production of new pairs of $q\bar{q}$-particles along the force field*. Using the earlier interpretation that a q or $\bar{q}$ corresponds to the endpoint of a string, the production process corresponds to creating new endpoints, i.e. to breaking up the original string into smaller pieces.

The q- and $\bar{q}$-particles will be treated as massless during the discussion. This assumption is necessary in a semi-classical framework for the conservation of energy-momentum. A massless pair produced at a single space-time point does not take any energy from the field. A massive pair (mass μ) will, however, in classical physics need a field region $\delta x = 2\mu/\kappa$. We will later consider the quantum mechanical modifications which are necessary in order to treat the production of massive pairs.

The production point of a new pair is called a *vertex*. Figure 7.1 shows the development in space-time of parts of a $q\bar{q}$-state, with some of the vertices produced.

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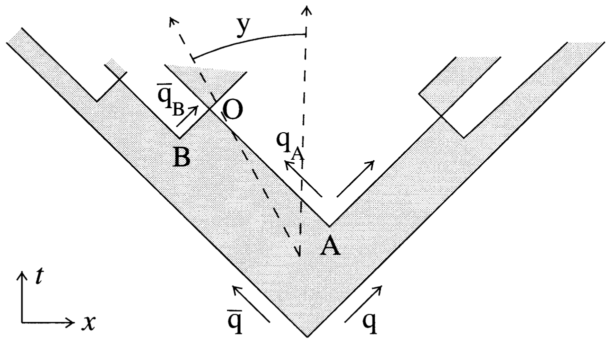

Fig. 7.1. Space-time development in a breakup situation showing some of the vertices produced together with the state S_{AB} discussed in the text. The rapidity y of the state S_{AB} is the hyperbolical angle between the broken-line directions.

We note that due to *causality* the two original endpoint particles will know nothing about the breakup vertices 'behind' them, at least not for some considerable time. As they are massless and move with the velocity of light there is no possibility of reaching them with a signal until they have turned around.

We further note that a produced pair will immediately start to separate owing to the forces exerted by the two adjoining string pieces. The new particles in that way use up the field energy between them, i.e. the string field in between them vanishes. Their parting situation is actually irrevocable – they will never meet again.

In this way the notion of *confinement* is smuggled in. A string force field is always confining in the sense that the force field vanishes at the endpoint 'charges'. This is in contrast to the situation in electrodynamics, where a newly produced electron-positron pair will continue to interact even if pulled apart by external forces.

In our case, at every vertex there will be two independent string pieces with endpoint particles moving away in opposite directions. There may be several vertices along the string, as shown in Fig. 7.1. In this way every vertex actually partitions the set of all vertices into two parts, those belonging to the string piece moving to the left and those belonging to the string piece moving to the right. This observation will later on provide us with a convenient way to order the vertices.

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7.2 The kinematics of the decay and its implications

1 Preliminary remarks

We will now consider the energy-momentum properties of one of the string pieces, the one ending in q_A and $\overline{q}_B$. We will call the state consisting of the two particles and the force field between them S_{AB} and we note that it is after formation isolated from the remaining system. The two particles are produced at adjacent vertices, at the space-time points $A = (x_A, t_A)$ and $B = (x_B, t_B)$, respectively. In order to compute the energy-momentum of S_{AB} we consider the space-time point $O = (x_O, t_O)$. This is, according to Fig. 7.1, the first meeting point of q_A and $\overline{q}_B$ and there is no field between them when they are at this point.

According to the equations of motion given in Chapter 6 the energies E_j and momenta p_j $(j = A, B)$ at this point (note that momentum is counted positive along the positive x-axis) are given by

$$\begin{aligned} E_A &= \kappa(x_A - x_O), \quad E_B = \kappa(x_O - x_B) \\ p_A &= \kappa(t_A - t_O), \quad p_B = \kappa(t_O - t_B) \end{aligned} \tag{7.1}$$

Therefore the state S_{AB} will have a *total energy-momentum depending only upon the space-time difference between the production vertices A and B*:

$$E = E_A + E_B = \kappa(x_A - x_B), \quad p = p_A + p_B = \kappa(t_A - t_B) \tag{7.2}$$

For reference we note that there is a relationship between some of the quantities in Eq. (7.1) because the positive (negative) lightcone component of the point labelled O is equal to the corresponding component for the vertex A (B):

$$t_O + x_O = t_A + x_A, \quad t_O - x_O = t_B - x_B \tag{7.3}$$

If the state S_{AB} corresponds to a meson state with mass m then the vertices A and B must lie on the two branches of the hyperbola

$$\frac{E^2 - p^2}{\kappa^2} = \frac{m^2}{\kappa^2} = (x_A - x_B)^2 - (t_A - t_B)^2 \tag{7.4}$$

Therefore there is a strong correlation between two vertices corresponding to the production of a definite mass in between. One can, assuming that one knows one of the vertices (e.g. A) draw the hyperbola branch along which B must be found (see Fig. 7.2) and vice versa.

It is also useful to note that the velocity of the 'particle' produced

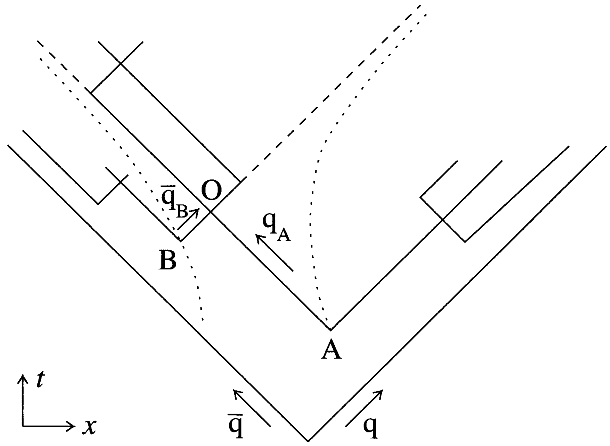

Fig. 7.2. Two neighboring vertices A and B with the requirement that they should each lie on a hyperbola. The hyperbolas are indicated for A and B.

between the vertices A and B is given by

$$v_{AB} = \frac{p}{E} = \frac{\Delta t}{\Delta x} \tag{7.5}$$

where Δ indicates the differences between the A and B coordinates. We remember from Chapter 2 that this result is to be expected in connection with spacelike vectors. The system is evidently at rest when q_A and $\bar{q}_B$ are produced at the same time. The rapidity of the system is given by the hyperbolic angle, y, shown in Fig. 7.1 and we note that the faster the system, the more tilted towards the lightcone is its velocity direction:

$$y_{AB} = \frac{1}{2}\log\left(\frac{1+v_{AB}}{1-v_{AB}}\right) = \frac{1}{2}\log\left(\frac{\Delta x + \Delta t}{\Delta x - \Delta t}\right) \tag{7.6}$$

2 *The consequences*

The distance between the vertices A and B must be spacelike in order that the mass should be real, according to Eq. (7.4). Thus the two production points are not causally related and no signal can be sent between the vertices. This has some interesting consequences, which we will now consider. According to Fig. 7.1 vertex A appears earlier than vertex B in the ordinary time sense. This is, however, a statement which depends upon the Lorentz system if A and B are spacelike with respect to each other, since then we can always, according to Chapter 2, find a Lorentz boost to another frame such that the vertex B (in its new position B_y) will seem to appear earlier than vertex A (A_y, see Fig. 7.3).

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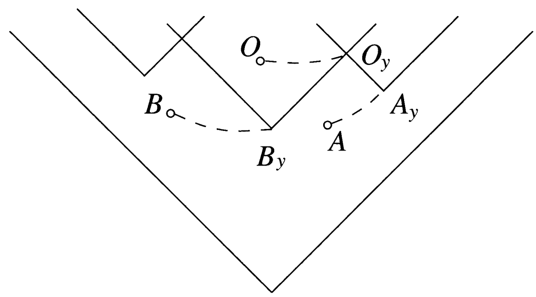

Fig. 7.3. The situation in Fig. 7.1 after a Lorentz boost along the negative direction. The points (A, A_y), (B, B_y) and (O, O_y) are shown together with the hyperbolas on which they move during the Lorentz boost.

The same considerations also apply to every other pair of adjacent vertices. We conclude that *all the vertices must be spacelike with respect to each other* for the produced states to have positive masses. Therefore no statement about (ordinary) time-ordering in the breakup process is Lorentz-invariant. There is consequently no 'first' vertex in this sense; the vertices all occur, in a relativistic setting, at the same time. We will later see that there are other possible ways to order the process and also other ways to define a useful time variable.

Thus, for the description of the decay process to be Lorentz-invariant then *there can be no vertex that is more significant than any other*. Each vertex has the same property, i.e. it divides the system into two parts, the vertices to its left and those to its right. Evidently these parts can also be described as two independent groups of particles moving apart. One often uses the term 'jet' for such a connected group. (It may then happen that a jet will contain only a single particle, *viz.* if we consider the outermost vertex on one end.)

It is an important constraint, when we provide a probabilistic description of the process, that all the vertices must be treated in the same way. This is what causality and Lorentz invariance imply.

The fact that all the vertices occur at spacelike distances with respect to each other also seems to be necessary from the point of view of ordinary common sense. It seems evident that the field cannot break up at a space time point if such a breakup has already occurred earlier, i.e. in the backward lightcone with respect to the point. In accordance with what has been said above there is then no longer any field left, and therefore there is no energy left, and so on.

7.3 Ordering of the decay process along the lightcones

Another property that we may deduce from the mass-shell condition (7.4) is that for every yoyo meson there is only a single degree of freedom. We may prescribe either the energy-momentum component $p_+ = E + p$ or the energy-momentum component $p_- = E - p$ (the positive and negative lightcone components) of the system S_{AB}. They are linked by the mass-shell condition

$$p_+ p_- = m^2 \tag{7.7}$$

(Let the reader be warned, as Carter Dickson or any other honest mystery writer would say. A very sophisticated reader might note that we are at this point introducing a slight mismatch between the ordinary space-time coordinates and the lightcone coordinates. We have already shown that the squared mass is given by the area spanned by the string during a complete period and not by a half period as Eq. (7.7) implies. The difference corresponds to using, instead of the normal metric $dxdt$, the lightcone metric $dx_+dx_- = 2dxdt$. We will go on employing this mismatch in order to avoid writing several factors of 2 or $\sqrt{2}$ in our formulas.)

From the calculations in connection with Eq. (7.1) we note that for the state S_{AB} the positive lightcone component is actually carried by the $\bar{q}_B$-particle and the negative one by the q_A-particle at the time of their first meeting to form the final-state yoyo-hadron. (It is necessary to make use of Eq. (7.3) to prove this statement.) This property is in the same sense valid for all the yoyo-hadrons, i.e. that the positive (negative) lightcone energy-momentum is, at the meeting points, carried by the corresponding $\bar{q}(q)$-particle. The assignment to the particles of positive and negative lightcones is of course related to the choice of directions of motion for the original pair.

This observation provides a useful way of ordering the process. Consider Fig. 7.4, which exhibits the decay of a whole string system stemming from an original pair $q_0, \bar{q}_0$ with lightcone energy-momenta p_{+0}, p_{-0} into many yoyo-hadrons, which go off in different directions (i.e. with different velocities). From the remarks above we conclude that *the production process is easily ordered along one of the lightcones.* Then the corresponding lightcone energy-momentum of the yoyo-meson indexed j (composed of $q_j, \bar{q}_j$ from adjacent vertices) is given by the lightcone component of either the q_j (the p_{-j} if we use the negative lightcone ordering) or the $\bar{q}_j$ (the p_{+j} for the positive lightcone ordering). The other component can be computed from Eq. (7.7). We will normally choose to number the yoyo-hadrons along the positive lightcone.

The sum of these components will, of course, add up to the lightcone components of the original pair; this corresponds to total energy-

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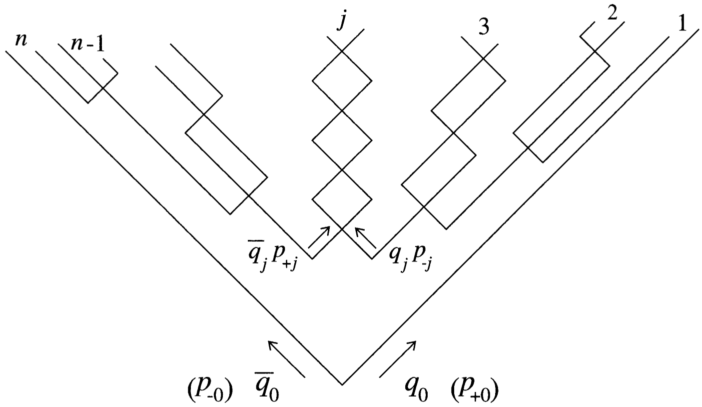

Fig. 7.4. A high-energy string breakup of a pair $q_0, \bar{q}_0$ having lightcone energy-momenta p_{+0}, p_{-0}.

momentum conservation:

$$\sum_{j=1}^{n} p_{\pm j} = p_{\pm 0} \tag{7.8}$$

Thus *the production process can be characterised as a set of choices for the lightcone components of one set of constituents of the yoyo-hadrons, i.e. of either the* q_j *or the* $\overline{q}_j$.

These lightcone components are evidently obtained from the field (remember that all the pairs are produced 'at rest'). Therefore *another way to describe the energy-momentum of the final-state yoyo-hadrons is to state the size of the space-time region within which the constituents have been acted upon by the string force field.* In order to state the energy-momentum of the system S_{AB} in Fig. 7.1 we may therefore prescribe a *lightcone distance*, either $\Delta t + \Delta x = \Delta x_+$ or $\Delta t - \Delta x = \Delta x_-$ ($\Delta t = t_A - t_B,\ \Delta x = x_A - x_B$). The other of these is then given by Eq. (7.7) rewritten as

$$\Delta x_+ \Delta x_- = -\frac{m^2}{\kappa^2} \tag{7.9}$$

In this way *the production process can be considered as a series of 'steps' along the positive (negative) lightcone. Each step corresponds to the lightcone distance between two adjacent vertices.* Then energy-momentum conservation according to Eq. (7.8) corresponds to stepping all the way from the turning point of the original q_0 ($\bar{q}_0$) back to the origin.

After each step it is necessary to go along the opposite lightcone a distance Δx_{-j} (Δx_{+j}) in order to keep the yoyo-meson on the mass shell. In that way the string decay process corresponds to a Markovian stochastic process, where each vertex in the process is determined solely by the previous starting point, i.e. the vertex already reached, and by the probability of taking a particular step along the lightcone.

It is convenient to define the scaled lightcone components z_+ and z_- by means of the equations

$$z_\pm = \frac{p_\pm}{p_{0\pm}} \tag{7.10}$$

where $p_{0\pm}$ are the corresponding lightcone components for the original q- and $\overline{q}$-particles. The quantitities $z_\pm$ are Lorentz invariants, being the ratio between two quantities which transform with the same factors $\exp(\pm y)$ under a Lorentz boost along the x-axis.

The total production process may then be looked upon as a set of steps $\{z_{+j}\}$ along the positive lightcone (or equivalently $\{z_{-j}\}$ along the negative lightcone). Energy-momentum conservation means that all the steps add up to unity. Each step corresponds to the production of a new meson containing a fraction of the original q- (or $\overline{q}$-) particle's energy-momentum that corresponds to the step size.

7.4 Iterative cascade fragmentation models

The above situation as viewed in a frame boosted along the positive x-axis with a large velocity is shown in Fig. 7.5. We note that, while in Fig. 7.4 the hadrons in the centre are the slowest and also the first to be produced in time in that system, in Fig. 7.5 it is instead the hadrons which are furthest out along the lightcone (usually the fastest in Fig. 7.4) that are the slowest and the first to be produced (cf. the discussion of velocities and rapidities in connection with Eqs. (7.5), (7.6)). This is again a very general property of all Lorentz-covariant production processes and we will return to this observation in the next section.

Up to now we have not been concerned with the conservation of internal quantum numbers, e.g. the flavor quantum numbers of the newly produced $q\overline{q}$-pairs. We will from now on assume that the pairs produced are actually a quark and its antiparticle, an antiquark with the opposite flavor, i.e. *the pairs will together have the quantum numbers of the vacuum.*

This means that it is possible to relate adjacently produced hadrons also by means of their flavor quantum numbers. We will introduce the notion of 'rank' in the following sense. The first-rank meson contains the quantum number of the original q-particle together with the antiflavor of the $\overline{q}$-particle produced at the first vertex along the lightcone.

In the same way we define a second-rank particle as the particle composed of the q-particle from the first vertex and the $\overline{q}$-particle from the

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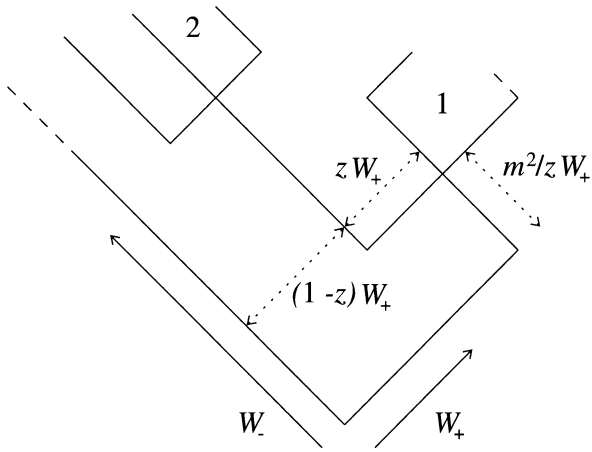

Fig. 7.5. The situation of Fig. 7.4 in a frame boosted along the positive lightcone direction in such a way that the first-rank particle is at rest. For simplicity we write $z \equiv z_{+1}$.

next, etc. It is evidently possible to introduce rank also by starting with the original $\bar{q}$-particle and the negative lightcone. Thus ordering by rank and flavor corresponds, in this kind of model, to an ordering along the lightcone(s).

From Fig. 7.5 we notice that the first vertex along the lightcone, V_1, actually divides the decay event into a single first-rank particle moving to the right and all the remaining ones as a combined jet moving to the left.

After the production of the first meson with lightcone fraction z_{+1} the remainder of the system will share the fraction $1 - z_{+1}$. This means that the remaining system will have a squared mass s_1 equal to (using for simplicity z for z_{+1})

$$s_1 = (1-z)W_+\left(W_- - \frac{m^2}{zW_+}\right) = (1-z)\left(s - \frac{m^2}{z}\right) \tag{7.11}$$

where we suppose the original system to have squared mass $s = W_+W_-(= p_{+0}p_{-0}$, due to Lorentz invariance).

The different parts of this formula have simple geometrical interpretations. The first term, i.e. the scaled-down mass-square is immediately recognised. For the second term it is easy to convince oneself that the area of the region below the first vertex, $V_1 = \kappa(x_{+1}, x_{-1})$, and above the production point of the original pair, is

$$\Gamma_1 = \kappa x_{+1}\kappa x_{-1} \tag{7.12}$$

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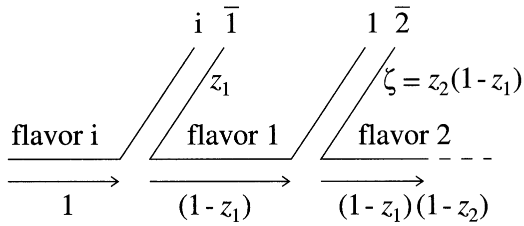

Fig. 7.6. An iterative cascade chain.

This is (apart from the factor κ^2) the squared proper time $\tau_1^2 = t_1^2 - x_1^2$ of the vertex V_1. The positive lightcone component of the vertex V_1 (with respect to the origin) is what is left of the original q's energy-momentum, $\kappa x_{+1} = (1-z)W_+$. The negative lightcone component is similarly what was taken by the first particle, i.e. $\kappa x_{-1} = m^2/zW_+$. Therefore the quantity Γ_1 is equal to minus the second term in Eq. (7.11):

$$\Gamma_1 = (1-z)W_+\frac{m^2}{zW_+} = (1-z)\frac{m^2}{z}. \tag{7.13}$$

In the Lund model formulas both terms are taken into account and the model therefore exhibits complete energy-momentum conservation, i.e. every new particle takes away not only its forward lightcone energy-momentum zW_+ but also the negative fraction needed to put it on the mass shell.

We will later see that the proper times of the vertices are generally of a limited size. For large values of s we may then neglect Γ_1 and approximate the *remainder system* as being the same as the original one apart from a scaling down of the positive lightcone component by the factor $1 - z \equiv 1 - z_1$.

The basic idea of regarding particle production at high energies as a scaling process was conceived many years ago, [90], to describe the fragmentation regions in hadronic interactions. Later similar ideas were used in partonic scenarios as *iterative cascade fragmentation schemes*, [13]. Then one assumes that there is a certain probability

$$f_{i1}(z_1)dz_1 \tag{7.14}$$

of producing the first-rank hadron (indexed by the original q's flavor i and the produced $\bar{q}_1$'s antiflavor) with fractional energy-momentum z_1, leaving the system with a q_1-particle at the endpoint and with a scaled down energy-momentum $1 - z_1$ (see Fig. 7.6).

Then the process can be repeated, with a probability

$$f_{12}(z_2)dz_2 \tag{7.15}$$

of producing a second-rank meson with flavors 12 (the second flavor-index refers to a $\bar{q}$-particle) and with energy-momentum fraction

$$\zeta_2 = z_2(1 - z_1) \tag{7.16}$$

After that the system is left with a q_2-particle at the end and with a scaled down energy-momentum equal to

$$1 - z_1 - \zeta_2 = (1 - z_1)(1 - z_2) \tag{7.17}$$

Thus at each step a new flavor is produced, a certain probability distribution is applied to find the fraction z_j and the remainder system is scaled down by a further factor $1 - z_j$.

In this way the problem has been reduced to finding a set of probability distributions $f_{ij}(z)$ and then repeatedly applying them to the situation at hand. This is the basis of what is often referred to as the *iterative cascade jet* or *Feynman-Field model* in honor of two of the main contributors. We will consider some of their main features in section 9.4

In the next chapter we will see that *there is a unique form for the distribution(s) f in the Lund model.* To prove that we will require that the final-state meson production process should be statistically the same if we describe it in terms of steps along the positive or along the negative lightcones (*left-right symmetry*).

We will end this chapter with a few remarks on a possible problem, to my knowledge first raised by Bjorken for the iterative cascade models, in the well-known Landau-Pomeranchuk 'formation time' concept.

7.5 The formation time and iterative cascade jets

Landau and Pomeranchuk considered the notion of a formation time in the context of QED bremsstrahlung. In its simplest setting the problem is as follows:

- at what time can one distinguish between a state containing a single charged particle and a state containing the particle accompanied by a photon?

They pointed out that in a Lorentz frame where the particle moves along one axis and the photon is moving transversely to this axis then it it is necessary to wait at least a time corresponding to the photon's wavelength to make a measurement that can distinguish the photon. Since the wavelength is inversely proportional to the transverse momentum of the photon $k_\perp$, it is thus necessary to wait a time

$$\tau_0 \simeq k_\perp^{-1}. \tag{7.18}$$

In a frame where the photon has energy E there will be a time-dilation factor $\gamma(v) = E/k_\perp$ and one obtains

$$\tau = \tau_0 \frac{E}{k_\perp} \simeq \frac{E}{k_\perp^2} \tag{7.19}$$

With the wavelength exchanged for some rest frame typical production time, i.e. with $k_\perp$ exchanged for some 'virtuality' Q (e.g. the transverse mass of a hadron), this formation time should, in any relativistically covariant and causal setting, provide a time-ordering of the process. Therefore it is always the slowest particles which will be the first to be emitted while the higher-energy particles will take a time proportional to their energy.

In the iterative cascade models the first-rank particle, according to the considerations above, will take a fraction z_1 of its energy-momentum leaving a fraction $1 - z_1$ to the remaining ones. The second-rank particle then takes $z_2(1 - z_1)$, etc. The values z_j are assumed to be given stochastically by means of a distribution $f(z)dz$.

As we will later see, one basically obtains a geometrical series for the final-state particle energy-momentum fractions. Therefore the first-rank particle is generally faster than the rest, i.e. it will have a longer formation time. Bjorken's question was: 'how can it then be the first to be produced in the chain?'

In the Lund model there is evidently a simple answer to this problem. Rank-ordering, as we have seen, corresponds to an ordering along the lightcone of the production vertices. There is no contradiction to an ordinary time-ordering with respect to the original $q\bar{q}$ production point, which is in accordance with the Landau-Pomeranchuk prescription. In any frame it is always the slowest mesons which are the first to be produced, according to the Lund model.