PARTICLE ENERGIZATION IN STOCHASTIC DOUBLE LAYERS

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Electrostatic turbulence develops in current carrying plasmas when the relative electron-ion drift exceeds the critical value for laminar current flow. Recent 2D computer experiments (Barnes, 1982) indicate that many weak ion acoustic double layers form in such turbulence when the plasma is strongly magnetized ($\omega_{ce} \gtrsim \omega_{pe}$), the electron/ion temperature ratio is large ($\gtrsim 10$), and the relative electron-ion drift is comparable to or less than the electron thermal speed. The double layers emerge from the incoherent spectrum of electrostatic ion cyclotron and ion acoustic waves as intense localized electric field structures propagating subsonically relative to the ion bulk flow. The occurrence of weak ion acoustic double layers, excited by field-aligned currents in the Earth's auroral regions, has also been reported from in situ spacecraft measurements (Temerin et al., 1982). An important question concerns the effect of these coherent electric fields on plasma transport properties such as bulk heating and acceleration. For example, one might expect nonlinear diffusion processes, manifested as distinct non-thermal features in the particle spectra, to accompany the quasilinear diffusion of ions as they traverse turbulent regions in space. This idea motivates the work presented here.

A test particle approach is used to determine the spatial evolution of ion velocity distributions through a model turbulent layer containing randomly distributed, relatively propagating electric field pulses of a single polarity. Results of the aforementioned spacecraft observations and numerical simulations are used to construct a reasonable (though non-self-consistent) model for the random double layer electric fields. The basic phenomenology of weak ion acoustic double layers in a magnetized plasma is described briefly, a model turbulent layer containing many weak double layers is then formulated, and the evolution of test ion distributions through the turbulent layer is discussed. Since the calculation does not include quasilinear effects, it should be interpreted as a qualitative model complementing the more traditional approach to turbulent heating and acceleration.

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MODEL TURBULENT LAYER

Weak ion acoustic double layers ($e\phi \lesssim T_e$) appear to evolve randomly from current driven turbulence in both 1D simulations (Sato and Okuda, 1981; Hudson and Potter, 1981; Kindel et al., 1981) and 2D simulations (Barnes, 1982). Their mean separation along the magnetic field is about 100-1000 λ_D (Debye lengths), depending on the boundary conditions. Their transverse dimension varies but is generally comparable to or larger than the parallel dimension - typically $10\lambda_D$. The parallel potential profile is nonmonotonic with a negative potential pulse preceding (in space) the double layer potential transition (Hasegawa and Sato, 1982). The transitional part of the waveform evolves from the negative pulse, which resembles a small but finite amplitude wave packet propagating initially with velocity $\frac{1}{2}\, c_s \lesssim v_p \lesssim c_s$. The localized disturbance intensifies by exchanging momentum with reflected electrons (Chanteur et al., 1983; Lotko, 1983). As $e\phi$ approaches T_e, strong ion trapping in the negative pulse produces an inertial drag, causing a rapid deceleration of the wave to $v_p = 0$. At this time, the ion starved double layer decays within a few ion plasma periods. The net lifetime of the DL is a few ion trapping periods - typically 10 - 100 ω_{pi}^{-1}. Subsequently, another double layer forms at a random location in the system (Okuda and Ashour-Abdalla, 1982; Barnes, 1982).

In order to model this phenomenon, we consider a model system of length 100 $N\lambda_D$ partitioned into N unit cells of length 100 λ_D. The cell length corresponds to the lower limit on the mean double layer spacing and is chosen for computational expediency. At any given instant, each cell contains one DL located at a random position, so the rms spacing differs from the mean spacing. The propagating DL electric field profile is taken to be a delta function whose parameters vary stochastically with time:

$$E_j = \phi_j \; \delta\left[x - r_j - s_j \; (t - t_j \;)\right] \tag{1}$$

ϕ_j, s_j and r_j are uncorrelated random variables corresponding to the DL potential, velocity, and position in the cell at time $t = t_j = 10j\omega_{pi}^{-1}$, where $j = 0,1,2,...$ To account for the finite DL lifetime, the values of ϕ_j, s_j and r_j are changed every $10\omega_{pi}^{-1}$, whereupon j is incremented by one. The j^{th} values of the random variables are selected from a random number sequence uniformly distributed over the intervals, $0 < e\phi \lesssim \alpha T_e$, $0 \lesssim \bar{v} - s \lesssim c_s/2$, and $0 \leq r \lesssim 100\lambda_D$, where T_e, c_s, and $\bar{v}$ are the electron temeperature, ion acoustic speed, and ion bulk flow velocity, and $0 < \alpha \leq 1$ determines the mean DL amplitude. The parameter α is varied in order to determine the effect of stronger or weaker double layers on the ion energization.

The previously mentioned numerical simulations indicate that the double layer propagates more slowly as its amplitude increases, so at first glance, one might choose these two variables to be correlated. In 2D simulations, however, the nonplanar DL propagates along the magnetic field with the maximum amplitude varying in the transverse direction. Thus, when an ion encounters a double layer, the effective potential depends on the relative position at the time of encounter. For this reason, ϕ and s are taken to be uncorrelated.

The delta function approximation (1) assumes that (i) the double layer spacing is large compared to the double layer width, and (ii) the ion transit time through the double layer is small compared to the double layer lifetime. The first condition is certainly satisfied in view of the simulation results and spacecraft observations. The second is equivalent to assuming that the ion motion is adiabatic during the encounter, i.e.,

$$\tfrac{1}{2} M (v - s)^2 + e\phi = \text{constant} \tag{2}$$

If W and T denote, respectively, the double layer width in λ_D and lifetime in ω_{pi}^{-1}, then (ii) requires that

$$W/T < c_s (e\phi/T_e)^{\frac{1}{2}} \tag{3}$$

This condition is at best marginally satisfied, so nonadiabatic effects can be expected to modify somewhat the results presented here. As a final caveat, it should be noted that the waveform (1) does not include the negative potential pulse that precedes the double layer and that locally traps ions. This neglected feature is presumably important for only a small fraction of the ions in any given unit cell.

SPATIAL EVOLUTION OF TEST ION DISTRIBUTIONS

The problem is to determine the velocity increments of test ions traversing the model turbulent layer. As formulated, the process is Markovian and can be characterized by a transition probability, i.e., the probability $P(v_0, \Delta)$ that a particle with initial velocity v entering a unit cell leaves the cell with velocity $v = v_0 + \Delta$. This probability function is constructed numerically from an ensemble of 1000 trials for each initial velocity with each trial characterized by a different random number sequence for the DL parameters. The change in ion velocity after a DL encounter, if one occurs, is determined from (2), taking into account the three distinct types of phase space trajectories corresponding to ions overtaken by, reflected by, or overtaking the double layer.

The transition probability after N unit cells may be determined by iterating the so-called Smoluchowski equation (Wang and Uhlenbeck, 1945). Alternatively, one can iterate directly the integral equation for the particle distribution function:

$$F_{n+1}(v) = \int_0^v du\, F_n(u)\, P(u, v - u) \tag{4}$$

$F_n(v)$ is the velocity distribution of ions entering the n^{th} cell or alternatively, leaving the $(n-1)^{th}$ cell. N iterations of (4) determines the evolution through a turbulent layer N cells long.

A few illustrations would greatly fascilitate the interpretation of the following discussion, but, unfortunately, space limitations preclude their inclusion in this brief report. A complete discussion including detailed diagnostics will be published elsewhere. The main features of the spatial evolution are as follows:

(1) After a few unit cells, a double peaked distribution forms. The low velocity peak is the remnant of the incoming distribution. The higher velocity peak is the accelerated component. Both have comparable densities at this stage.

(2) Since the double layers are all polarized in the same direction ($E > 0$) and propagate in the direction antiparallel to the electric field vector (negative velocities), they always accelerate ions in the positive direction. Thus, the accelerated component appears at positive velocities in the rest frame of the incoming distribution.

(3) The peak energy and thermal spread of the accelerated component scale with the product of the mean double layer amplitude and the number of unit cells transversed, and, by comparison, depend only weakly on the mean double layer velocity and thermal spread of the incoming distribution.

(4) The slope of the remnant distribution at negative velocities increases with the number of cells traversed. This feature would presumably help sustain the instability that causes double layers. On the other hand, incoherent waves may also propagate at these velocities, leading to a quasilinear type diffusion of the velocity distribution. This effect would tend to suppress any instabilities via enhanced ion Landau damping.

(5) After about 10-20 cells, depending on the mean double layer amplitude, the density of the accelerated component exceeds that of the remnant component. At these distances into the turbulent layer, one might expect the nature of the turbulence to differ significantly from that near its leading edge. Double layers may cease to form at this point, and continued iteration of Eq. (4) becomes questionable. This effect may determine in part the spatial extent of the turbulent layer.

In summary, it has been shown that randomly distributed double layers that arise in current driven turbulence accelerate and heat ions. Their effects on the ion distribution function differ substantially from those predicted by quasilinear processes and can be expected to influence significantly the turbulent properties of current carrying plasmas.

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DISCUSSION

R. Smith: I'm worried about the ion-crossing-time problem in Markovian statistics.

Lotko: The ion crossing time problem and the use of Markovian statistics are separate issues. Markovian statistics are applicable here because the successive double layer fields encountered by a particle are uncorrelated after a mean inter-double layer transit time, which, for the bulk, is greater than 10 DL lifetimes. The physical difficulty using Markov's method is to determine the transition probability, wherein the ion crossing time problem enters. As indicated, this effect is important when the ion transit time across a single double layer exceeds the DL lifetime. The δ-function DL model neglects this effect, and so, must be considered as a first approximation. It is, of course, least accurate for particles occupying trajectories near the phase space separatrix, although the stochastic nature of particle/DL encounters ameliorates this problem to some extent. A truly self-consistent calculation (which is currently beyond the memory capability of large scale computers) would presumably yield more bulk heating and less bulk acceleration.

Vlahos: I believe that the test particle orbits by definition cannot be the whole distribution because in this case there is an obvious lack of self consistency. Of course things will be different if you work with 10^{-2} or less particle on the tail, for example.

Lotko: Test particle calculations certainly do not describe the self-consistent evolution of the particle distribution. Nevertheless, they provide useful information about nonlinear interactions between particles and fields and about evolutionary trends in the distribution function, which is the spirit of this calculation.

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