The Journal of Navigation (2024), 1–19
doi:10.1017/S0373463324000183

CAMBRIDGE UNIVERSITY PRESS

RESEARCH ARTICLE

Error analysis of dead reckoning navigation system by considering uncertainties in an underwater vehicle's sensors

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Received: 9 August 2023; **Accepted:** 25 April 2024

Keywords: navigation; dead reckoning; error analysis; sensor uncertainties

Abstract

In this paper, a complete introduction to the dead reckoning navigation technique is offered after a discussion of the many forms of navigation, and the benefits and drawbacks associated with each of those types of navigation. After that, the dead reckoning navigation solution is used as an option that is both low-cost and makes use of the sophisticated equations that are used by the system. Moreover, to achieve the highest level of accuracy in navigation, an investigation of navigation errors caused by dead reckoning is calculated. Employing the suggested dead reckoning navigation system, the final position of an underwater vehicle can be established with a high degree of accuracy by using experimental data (from sensors) and the uncertainties that are associated with the system. Finally, to illustrate the correctness of the dead reckoning navigation process, the system error analysis as uncertainty that was carried out using experimental data using the dead reckoning navigation technique is compared with GPS data.

1. Introduction

Determining a vehicle's position is essential for every object tracking and navigation systems. A tracking system can be more successful, which provides a more accurate location at any moment. First, complete control of the system must be prepared to achieve this purpose. Then, one of the navigation systems for tracking and obtaining the final error of the system can be employed. To specify a position in autonomous underwater vehicle (AUV) systems, various methods, such as using the Global Positioning System (GPS), Inertial Navigation System (INS) and dead reckoning (DR) navigation system, are generally applied, each with its limitations (Kayton and Fried, 1997; Grewal et al., 2007; Chan and Baciu, 2012; Noureldin et al., 2012; Leonard and Bahr, 2016). With the passage of time, navigation systems have evolved. The newer ones enable engineers to navigate faster and more accurately. These navigation systems have advantages and disadvantages that make none the best for all situations. GPS is a satellite-based radio navigation system. GPS is one of the global navigation satellite systems (GNSSs) that provides geolocation and time information to a GPS receiver anywhere on or near the Earth where there is an unobstructed line of sight to four or more GPS satellites. It operates independently of any telephonic or internet reception. GPS provides users with real-time data based on location, and its signal is available worldwide. The GPS calibration on its own makes it easy for everyone to use. It provides critical positioning capabilities to military, civil and commercial users worldwide. Despite these advantages, the accuracy of GPS depends on a sufficient signal quality received. The GPS signal is affected by the atmosphere, electromagnetic interference, ionosphere, etc., which results in an error in the GPS signal of approximately 10 m (Bakhoum, 2010; Gharib and Moavenian, 2014; Gharib et al., 2024). Additionally, GPS does not penetrate solid structures and is affected by large constructions, which means it is useless indoors and underwater or underground. Moreover, the position can occasionally be significantly in error when the number of satellites is limited (Bakhoum, 2010; Gharib and Moavenian, 2014; Gharib et al., 2024). Since the inertial navigation method is independent of any external input, it is an autonomous system that does not radiate outside energy. Therefore, it has adequate concealment, and external electromagnetic interference does not affect it (Titterton and Weston, 2004). One of the major benefits of inertial navigation systems is that they do not need any contact outside the vehicle with the environment (Titterton and Weston, 2004). However, the positional error increases in this method and the long-term precision is low. Furthermore, a long initial alignment period is required before each use and details regarding time cannot be given (Titterton and Weston, 2004). The exact values of many parameters can be unknown or have uncertainties since they cannot be measured accurately (Amaral et al., 2022; Campos et al., 2023). Therefore, some researchers include uncertainties in their investigations. Junratanasiri et al. (2011) proposed a navigation system for a mobile robot to navigate through an uncertain environment while focusing on dynamic obstacles. To increase the awareness of users about uncertainties, Pankratz et al. (2013) used a controlled mixed reality environment to conduct a pilot study. Hacohen et al. (2017) developed a motion planning method for uncertain dynamic environments. Zhai et al. (2020) developed a robust modification strategy for an inertial navigation system to overcome the impacts of unpredictable stochastic disturbances in unmanned ground vehicles.

A DR navigation system determines a position by developing a given situation, displacement and distance. This specified position is called dead reckoning. It is generally agreed that the DR position can be determined by displacement and velocity (Cotter, 1978; Fifield, 1979). A DR navigation system operates while the Earth's surface and/or sky is out of reach (Tsakiri et al., 1999; Bevly et al., 2006). The inclusion of inertial sensors and advanced DR measurements offers a means to keep the navigation and positioning system on track in some instances where satellites do not reliably measure location (Tsakiri et al., 1999; Bevly et al., 2006). A DR navigation system does not need internet or infrastructure, so it works both outdoors and indoors, and is easy to incorporate into any design (Tsakiri et al., 1999; Bevly et al., 2006). Despite these advantages, a major drawback of DR is that the errors of the method are cumulative as new positions are determined entirely from previous positions, so the error in the fixed position increases over time (Gebre-Egziabher et al., 2003; Kumar and Das, 2004; Bevly et al., 2006). Additionally, due to its cumulative existence, the error may theoretically be high after a long time (Gebre-Egziabher et al., 2003; Kumar and Das, 2004; Bevly et al., 2006).

Different methods have been developed for position estimation in DR navigation methodology. Aghili and Salerno (2013) calculated a mobile robot's geographical location and orientation using a low-cost inertial sensor and a GPS. Doostdar and Keighobadi (2012) designed a sliding mode observer for a nonlinear MIMO attitude and heading reference system in land vehicle applications. Sabet et al. (2017) designed a low-cost aided DR navigation system for accurate estimation of attitude and heading in the presence of external disturbances, including external body accelerations and magnetic disturbances. An indoor pedestrian DR system based on a pocket-worn smartphone is presented by Zhao et al. (2019). The proposed system tracks a person's location through DR calculation by using the sensors embedded in smartphones. Kuang et al. (2022) designed a magnetic field matching algorithm based on the relative trajectory and attitude of the smartphone generated by pedestrian DR. This algorithm used the pedestrian DR algorithm to estimate the attitude of the mobile phone in real time. In recent years, machine learning-based techniques have also been integrated with DR method to enhance the accuracy (Brossard et al., 2020; Saksvik et al., 2021; Yan et al., 2023). A deep-learning approach to aid DR navigation using a limited sensor suite was developed by Saksvik et al. (2021). Yan et al. (2023) proposed a coarse-to-fine geomagnetic indoor localisation method based on deep learning.

This paper introduces the DR method to navigate a vehicle position once no external data such as GPS are received. The proposed system is used to determine the position of an autonomous underwater vehicle. The basis of this technique is Newton's equations in classical mechanics. This simulator determines the final location of a vehicle based on the adjustment of its primary position, motion angle

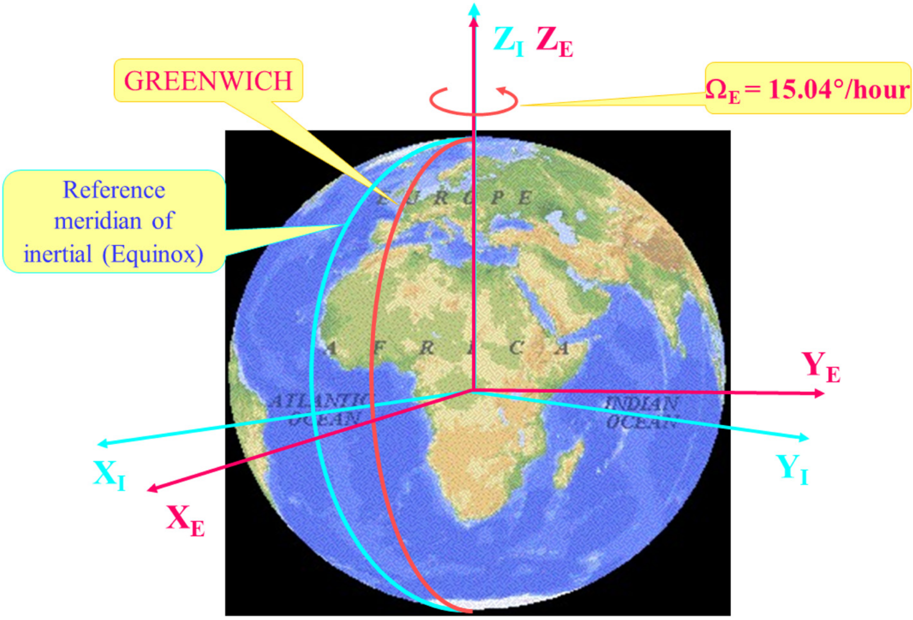

Figure 1. Inertial and Earth coordinate systems.

and speed data inputs. Furthermore, the proposed algorithm can be used to analyse DR navigation errors based on the definition of error of the gyro sensor, speedometer and different movement scenarios.

2. Navigation and localisation

2.1. Coordinate systems

The final navigation output that most users need to receive includes the coordinates of a point called latitude and longitude as well as their altitude and accuracy (Noureldin et al., 2012). Susceptible measurements are performed by an inertial navigation system. For instance, there are three components perpendicular to body rotation axes and three accelerometers, each of which is not directly related to any coordinate system (linear or curvature). These measurements must be integrated analytically and eventually. Therefore, they are converted to the elliptical coordinate system (Wrigley, 1977; Noureldin et al., 2012). As a result, all coordinate systems must be well defined in the measurement transformations and integral outputs. The definition of different coordinate systems associated with an inertial navigation system is given in the next section.

2.2. Inertial coordinate system

A graphic sample for an inertial coordinate system is displayed in Figure 1. This is a summarised definition of an inertial coordinate system for computational purposes. The directions of the inertial coordinate system axes are demonstrated in Figure 1.

An inertial coordinate system is defined as follows:

Centre: in the centre of the Earth
X_I axis: towards the average vernal equinox at time T_0
Y_I axis: a complete right-turn inertial coordinate system
Z_I axis: towards the north celestial pole at time T_0

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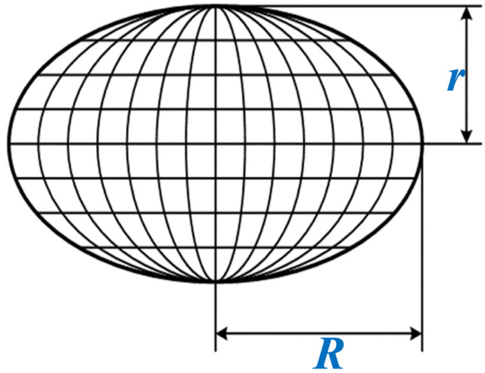

Figure 2. *Essential parameters in the Earth ellipsoid model (Noureldin et al. 2012).*

2.3. Earth-Centred-Earth-Fixed (ECEF)

Earth-Centred-Earth-Fixed (ECEF) is a coordinate system that contains the mapping system coordinates. It is not an inertial navigation system and orbits around the Sun at the rate of 7.292115×10^{-5} rad/s (Kayton and Fried, 1997; Noureldin et al., 2012; Ramesh et al., 2016).

The directions of the ECEF axes are demonstrated in Figure 1.

ECEF is defined as follows:

Centre: in the center of Earth's mass
X^e axis: towards the Greenwich meridian on the equator plate
Y^e axis: 90° east of the Greenwich meridian on the equator plate
Z^e axis: the rotation axis of the basis elliptic

The coordinates in ECEF can be converted with an inertial navigation system by a negative rotation around the Z axis. Essential parameters in the Earth ellipsoid model are shown in Figure 2. The equations related to these parameters will be as follows:

$$\begin{aligned} R &= 6378137\,\text{m} \\ r &= R(1-f) = 6,356,752.3142 \end{aligned} \tag{1}$$

$$f = 1 - (r/R) = \frac{1}{298.257223563} \text{ flattening} \tag{2}$$

$$e = [1-(r/R)^2]^{0.5} = 0.0818191908426 \text{ eccentricity} \tag{3}$$

In the above equations, 'f' is the flattening coefficient of the Earth and 'R' is the Earth's equatorial radius. Also, Radios East-West 'R_{EW}' and Radios North-South 'R_{NS}' can be defined as (Kayton and Fried, 1997; Noureldin et al., 2012; Ramesh et al., 2016)

$$R_{NS} = \frac{R(1-e^2)}{(1-e^2\sin^2\Lambda)^{3/2}} \tag{4}$$

$$R_{EW} = \frac{R}{(1-e^2\sin^2\Lambda)^{1/2}} \tag{5}$$

where 'Λ' is latitude and 'λ' is longitude, 'e' is the eccentricity of the Earth, 'R_{EW}' is Radios East-West, and 'R_{NS}' is Radios North-South.

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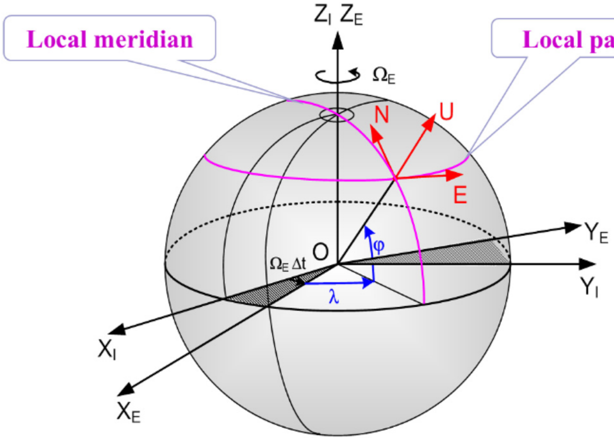

Figure 3. *Local horizon system (NEU).*

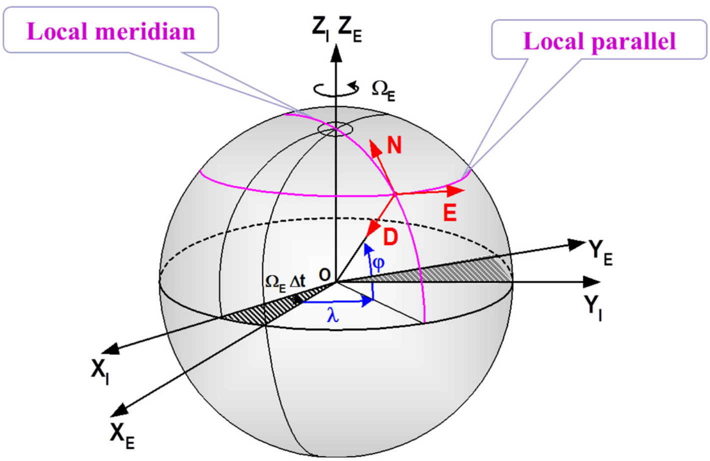

Figure 4. *Local horizon system (NED).*

2.4. Local horizon system

The local horizon system is a type of coordinate system known to surveyors as Earth's coordinate system (Kayton and Fried, 1997; Noureldin et al., 2012). If a change is made in 'X' and 'Y' directions, the velocity of the inertial mapping system is usually defined as components along the axes of the local horizontal coordinate system (Noureldin et al., 2012). The directions of the local horizontal coordinate system axes are demonstrated in Figures 3 and 4.

The definition of a local horizon system is as follows:

Centre: upper centre
X axis: east of the elliptical curve (also displayed as E axis)
Y axis: north of the elliptical curve (also shown as N axis)
Z axis: upwards along the upright ellipsoid (also expressed as U axis, or Z axis downward displayed as D axis in some Russian books and references)

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The conversion matrix between the local horizon and ECEF coordinate systems is as follows:

$$\begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} = \underbrace{\begin{bmatrix} -\sin\lambda & -\mathrm{snin}\Lambda\cos\lambda & \cos\Lambda\cos\lambda \\ \cos\lambda & -\sin\Lambda\sin\lambda & \cos\Lambda\sin\lambda \\ 0 & \cos\Lambda & \sin\Lambda \end{bmatrix}}_{C^e_{\mathrm{LL}}} \begin{bmatrix} E \\ N \\ UP \end{bmatrix} \tag{6}$$

The angular velocity of the local horizontal coordinate system relative to the Earth can be displayed as follows:

$$\omega_E = \frac{V_N}{R+h} \tag{7}$$

$$\omega_N = \frac{V_E}{R+h} + U\cos\Lambda \tag{8}$$

$$\omega_T = \frac{V_E}{R+h}\tan\Lambda + U\sin\Lambda \tag{9}$$

U: Earth's rotation rate
R: Earth's radius
V_{N} and V_{E}: relative velocities of the local horizontal coordinate system with respect to the Earth
h: Earth's ellipsoid height

3. Dead reckoning navigation

Determining the position is the most essential need to be obtained in a moving vehicle permanently and carefully. This is more critical for marine vehicles, which are more restricted than vehicles on the surface (cars, trains, etc.). Thus, several navigation systems must work simultaneously so that the position of a marine vehicle will still be available in case of a system failure (Cestone, 1971; Tsakiri et al., 1999; Leonard and Bahr, 2016).

DR navigation is a method that can estimate the location of a moving vehicle without needing external data and only based on internal sensors (Filaretov et al., 2015; McIntire et al., 2018). The location of a taken course in this method is estimated by starting from a specific point and using classical mechanics rules (Claus and Bachmayer, 2015; Filaretov et al., 2015; McIntire et al., 2018).

DR navigation has been in use since the early days of submarine development. Even if there were no other navigation systems today, an AUV would be able to navigate underwater using this method. DR depends on the primary precise fixed position, velocity and passing time parameters (Cotter, 1978). This method determines an AUV's approximate position by continuing the course from the last precise fixed point using the taken courses and calculating the distance by the vehicle speed (Bevly et al., 2006). Gyro is needed to achieve the course and speedometer in DR for the submarine's location (Carlson et al., 2004). One of the factors affecting this method is navy currents. In some systems, the speed of these currents is very effective in the desired movement. Suppose the directions of these currents are completely opposite of a vehicle's movement. In that case, the vehicle's speed will be decreased. In contrast, the vehicle's speed increases when the directions of these currents are in the vehicle's movement direction (Filaretov et al., 2015).

Figure 5 demonstrates the status of an underwater vehicle in the presence of steady currents. It moves from its first position at a 75° angle to the north. The point P_2 will be the position of the vehicle which is measured by the DR navigation method without the effect of sea currents. The position of P_2 is determined by velocity and time. In Figure 5, the line connecting P_1 and P_2, the direction of the initial vector, and the current speed are called the primary vector and SE current secondary vector, respectively. In the presence of a sea current, P_2 moves towards the secondary vector and is transferred to the DR

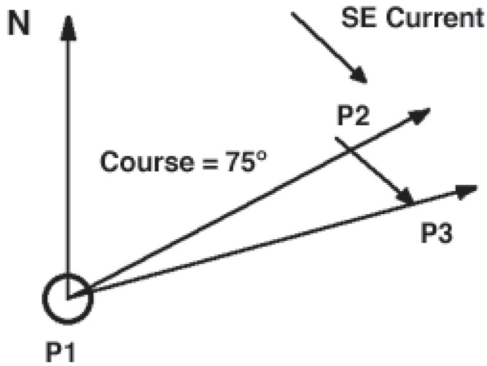

Figure 5. *Underwater vehicle dead reckoning navigation.*

position (P_3), which is affected by the vehicle's angle, speed, passing time and sea current. The vector P_1P_3 is the result of adding vectors P_1P_2 and P_2P_3.

It is worth noting that the reliability of DR depends on determining the error at constant body speed and error at heading (Carlson et al., 2004; Bevly et al., 2006; Filaretov et al., 2015). These errors are dependent on the accuracy of measuring instruments such as echo sounders, speedometers and gyro (compass).

3.1. Dead reckoning navigation equations

Dead reckoning navigation method equations of an AUV are the equations whose solution leads to an estimate of the current position of the vehicle (Kayton and Fried, 1997). Another method is inertial navigation, which estimates the current position by taking the initial position of the moving object as well as the accelerations of the object over time (Grewal et al., 2007; Noureldin et al., 2012; Gharib et al., 2024). These methods are based on the famous kinematic relations between acceleration, speed and position. Based on these relations, the acceleration integral produces speed, and the speed integral generates position (Leonard and Bahr, 2016). One of the advantages of DR and inertial navigation methods is that they do not need external data for mobile vehicles from the outside. However, the accuracy of the estimation will increase by correcting navigation errors (Carlson et al., 2004). Most of the navigation methods can be considered as the processes in which an integral operator leads to provide the outputs (Leonard and Bahr, 2016). Table 1 displays this issue. In GPS-based position stabilisation navigation systems, integration is unnecessary to convert the sensor output to the position. However, GPS sensitivity can be affected by a number of factors, including the following.

1. Signal blockage: GPS signals can be blocked or weakened by tall buildings, mountains, trees and other obstacles.
2. Atmospheric conditions: GPS signals can be affected by atmospheric conditions such as solar flares, ionospheric disturbances and weather conditions like heavy rain or snow.
3. Satellite position: The accuracy of GPS signals can be influenced by the position of the satellites in the sky.
4. Multi-path errors: GPS signals can be reflected off surfaces like buildings, water and mountains, causing the receiver to receive multiple signals that can lead to errors.
5. Receiver errors: GPS receivers can introduce errors due to factors like clock drift, signal processing and antenna placement.

Integrating once in DR and twice in inertial navigation converts the sensor output to the position. These methods to find the position of an object have an essential problem. In these methods, the precision of identifying the vehicle's ultimate position decreases as time passes during navigation.

The practical procedure for solving DR equations involves several steps, which will be described later.

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Table 1. *Integral process in navigation systems.*

Navigation system	Number of integrals	Error growth rate
GPS	0	–
DR	1	t
INS	2	t^2

To better understand the concepts in DR, it is necessary to briefly review the relations between the coordinate systems used in this navigation first.

The main equation of DR navigation is the velocity vector equation, which can be displayed as $\overrightarrow{p}(t) = \int_0^t \overrightarrow{v}(\tau)d\tau$. Its input is the velocity vector '$\overrightarrow{v}(t)$' whose components are the velocity components north, south and down. The output of the navigation equation is the position vector '$\overrightarrow{p}$', which in NED coordinates is the equation $\overrightarrow{v} = \begin{bmatrix} V_N & V_E & V_D \end{bmatrix}$ and the result after integration will be $\overrightarrow{p} = \begin{bmatrix} p_n & p_e & h \end{bmatrix}$, where the components of this vector are the coordinates of north, east and height, respectively. The velocity vector in the body coordinate system has to be converted to the velocity vector in the geographical coordinate system. This is done with the use of direction cosine matrices. Suppose that the velocity vector in the body coordinate systems ($\vec{V}_{\text{Body}}$) and in the geographical coordinate systems ($\vec{V}_{\text{NED}}$) are as follows:

$$\vec{V}_{\text{Body}} = [V_x, V_y, V_z] \tag{10}$$

$$\vec{V}_{\text{NED}} = [V_N, V_E, V_D] \tag{11}$$

In these relations, velocity 'V_N' is towards the geographical north and velocity 'V_E' is towards the east.

For converting the velocity vector in the body coordinate system to the velocity vector in the geographical coordinate system, we use the following relation:

$$\vec{V}^T_{\text{NED}} = T \cdot \vec{V}^T_{\text{Body}} \tag{12}$$

$$\boldsymbol{T} = \begin{bmatrix} \cos(\psi)\ \cos(\theta) & -\cos(\varphi)\ \sin(\psi) + \sin(\varphi)\ \sin(\theta)\ \cos() & \sin(\psi)\sin(\varphi) + \sin(\theta)\ \cos()\cos(\varphi) \\ \sin(\psi)\cos(\theta) & \cos(\psi)\cos(\varphi) + \sin(\psi)\sin(\varphi)\ \sin(\theta) & \sin(\psi)\cos(\varphi)\sin(\theta) - \sin(\varphi)\cos(\psi) \\ -\sin(\theta) & \cos(\theta)\sin(\varphi) & \cos(\theta)\cos(\varphi) \end{bmatrix} \tag{13}$$

The following relations will be achieved by using the navigation formulae:

$$\begin{cases} \dot{\Lambda} = \dfrac{V_N}{R_{\text{NS}} - h} \\ \dot{\lambda} = \dfrac{V_E}{(R_{\text{EW}} - h)\cos\Lambda} \\ \dot{h} = -V_D \end{cases} \tag{14}$$

where 'h' represents the height, which is negative for altitude above sea level and positive for below sea level.

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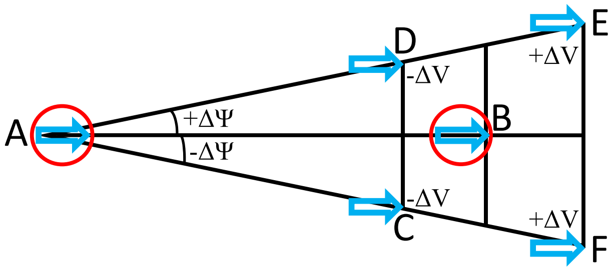

Figure 6. *Determining the probable domain.*

Using these components, latitude ‘Λ’ and longitude ‘λ’ at any time can be calculated:

$$V = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2} \tag{15}$$

$$\begin{aligned} \Lambda(t) &= \int_{t_0}^{t} \frac{V_N}{(R_{\mathrm{EW}} - h)} dt + \Lambda(t_0) \\ \lambda(t) &= \int_{t_0}^{t} \frac{V_E}{(R_{\mathrm{EW}} - h)\cos\Lambda} dt + \lambda(t_0) \\ h(t) &= -\int_{t_0}^{t} V_D\, dt + h(t_0) \end{aligned} \tag{16}$$

3.2. Sensor deviations in dead reckoning navigation

DR points show approximate underwater vehicle positions. During the movement of the vehicle from the origin point to the destination point, some factors affect movement and cause the DR navigation points to not exactly match the designated points along the way, so the final point is not obtained precisely (Donovan, 2012; Filaretov et al., 2015). Generally, dead reckoning navigation errors can be divided into the two following categories: errors of the measurement instruments and errors due to sea current (Filaretov et al., 2015).

The general state of an underwater vehicle’s possible situations is shown in Figure 6, considering the errors of the speedometer and gyro navigation sensors. If its sensors are not faulty, the vehicle starts moving from point A and arrives at point B over time. Suppose errors in the range of ‘$\pm\Delta\Psi$’ and ‘$\pm\Delta V$’ in gyro and speedometer sensors have occurred. In that case, the vehicle will be at a point in the domain of CDEF instead of point B. Navigation values (velocity and north angle) are needed to achieve accurate and momentary navigation, as displayed in Figure 6. We need the actual values of the speedometer and heading angle sensors, as well as their error rates, as illustrated in Figure 6.

3.3. Error analysis for conventional dead reckoning navigation

The accuracy of dead reckoning navigation depends on the sensors’ precision in measuring the vehicle’s heading and speed. We can use the error propagation theory to analyse the navigation error caused by the sensor errors. This theory states that the error in the estimated position is proportional to the errors in the measured parameters and their derivatives. Generally, the navigation error caused by sensor errors can be reduced using sensor fusion techniques, such as Kalman filtering or particle filtering, which combine the measurements from multiple sensors to estimate the vehicle’s position more accurately. These techniques can also take into account the correlation between the sensor errors and the vehicle’s motion dynamics, which can improve the accuracy of the estimated position.

While sensor fusion techniques can improve the accuracy of dead reckoning navigation, there are some potential problems and negative points that should be considered.

1. Sensor limitations: The accuracy of sensor measurements can be limited by factors such as environmental conditions, sensor drift and hardware limitations. If the sensors used in the sensor

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fusion system are not accurate, or if they are not properly calibrated, then the accuracy of the estimated position will be reduced.
2. High computational complexity: Sensor fusion techniques are computationally intensive and require significant processing power. This can be an issue in systems with limited computing resources, such as embedded systems or mobile devices.
3. Modeling errors: The accuracy of the estimated position depends on the accuracy of the models used to represent the system dynamics and the sensor errors. If these models are not accurate, then the estimated position will be less accurate and the navigation error may increase.
4. System complexity: Sensor fusion systems can be complex and may require significant expertise to design, implement and maintain. This can increase the cost and complexity of the navigation system.
5. Integration with other navigation systems: Sensor fusion techniques are often used in conjunction with other navigation systems, such as GPS or inertial navigation systems. The integration of these systems can be challenging and may require careful calibration and coordination to ensure that the estimated position is accurate.
6. Limited applicability: Sensor fusion techniques may not be applicable in all navigation scenarios. For example, dead reckoning navigation may be the only option in environments with limited or no access to external sensors, such as underground tunnels or underwater environments. In such scenarios, the accuracy of dead reckoning navigation can be limited by the sensor's accuracy and sensor fusion techniques may not be effective.
7. Cost: Implementing a sensor fusion system can be expensive, particularly if high-quality sensors are needed or if custom hardware and software are required.
8. Maintenance: Sensor fusion systems require regular maintenance and calibration to continue operating accurately. This can be time-consuming and costly.
9. Security: Sensor fusion systems can be vulnerable to cyber attacks, compromising the estimated position's accuracy and reliability.

Overall, while sensor fusion techniques can improve the accuracy of dead reckoning navigation, they are not without their challenges and limitations. These considerations should be carefully weighed when deciding whether to use sensor fusion techniques for a given navigation application.

Considering the problems mentioned in the preceding section in obtaining accurate navigation for a vehicle (marine), we will address this problem by employing a novel and practical method that considers the uncertainties in the data collection from sensors.

4. New analysis of dead reckoning navigational error

Due to the different selection of gyro and speedometer, and their different errors in the designed simulator, the gyro sensor error coefficient 'A' and speedometer sensor error coefficient 'B' are considered as one of the input parameters. Thus, latitude and longitude errors are achieved as follows:

$$\Delta\psi = A\sec(\mathrm{Lat}) \tag{17}$$

$$\Delta V = BV \tag{18}$$

$$\delta\dot{\Lambda} = \frac{1}{(R_{\mathrm{NS}} - h)}\,\delta V_{\mathrm{North}} + \frac{V_{\mathrm{North}}}{(R_{\mathrm{NS}} - h)^2}\,\delta h \tag{19}$$

$$\delta\dot{\lambda} = \frac{1}{(R_{\mathrm{EW}} - h)\cos\Lambda}\delta V_{\mathrm{East}} + \frac{V_{\mathrm{East}}}{(R_{\mathrm{EW}} - h)^2\cos\Lambda}\,\delta h - \frac{V_{\mathrm{East}}\ \tan\Lambda}{(R_{\mathrm{EW}} - h)\cos\Lambda}\delta\Lambda \tag{20}$$

$$\delta\dot{h} = -\delta v_D \tag{21}$$

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According to accurate calculations, the possibility of error (system uncertainties) in determining the final position of latitude and longitude after integrating from the sides is as follows:

$$\begin{aligned}
\delta\Lambda(t) &= \int_{t_0}^{t} \left(\frac{\partial\ (\dot{\delta})}{\partial t}\right) dt \\
&= \int_{t_0}^{t} \left(\frac{1}{(R_{\text{NS}}-h)}\ \delta V_N + \frac{V_N}{(R_{\text{NS}}-h)^2}\ \delta h\right) dt + \delta\Lambda(t_0) \\
&= \int_{t_0}^{t} \left(\frac{\cos\Psi}{(R_{\text{NS}}-h)}\ \delta V - \frac{V_E}{(R_{\text{NS}}-h)}\ \delta\Psi + \frac{V_N}{(R_{\text{NS}}-h)^2}\ \delta h\right) dt + \delta\Lambda(t_0)
\end{aligned} \tag{22}$$

$$\begin{aligned}
\delta\lambda(t) &= \int_{t_0}^{t} \left(\frac{\partial\ (\dot{\delta\lambda})}{\partial t}\right) dt \\
&= \int_{t_0}^{t} \left(\frac{1}{(R_{\text{EW}}-h)\cos\Lambda}\delta V_E + \frac{V_E}{(R_{\text{EW}}-h)^2\cos\Lambda}\ \delta h + \frac{V_E\ \tan\Lambda}{(R_{\text{EW}}-h)\cos\Lambda}\delta\Lambda\right) dt\delta + \lambda(t_0) \\
&= \int_{t_0}^{t} \left(\frac{\sin(\Psi)}{(R_{\text{EW}}-h)\cos\Lambda}\delta V + \frac{V_N}{(R_{EW}-h)\cos\Lambda}\delta\Psi + \frac{V_E}{(R_{\text{EW}}-h)^2\cos\Lambda}\ \delta h\right) dt + \delta\lambda(t_0) \\
&\quad + \int_{t_0}^{t} \left(\int_{t_0}^{t} \frac{V_E\ \tan\Lambda}{(R_{\text{EW}}-h)\cos\Lambda} \left(\frac{1}{(R_{\text{NS}}-h)}\ \delta V_N + \frac{V_N}{(R_{\text{NS}}-h)^2}\ \delta h\right)\right) d^2t + \delta\lambda(t_0)
\end{aligned} \tag{23}$$

To use the above relations practically, they are required to be in the discrete format as follows:

$$\delta(\Lambda_n) \ = \Lambda\delta_0 + \sum_{k=1}^{n} \left(\frac{1}{(R_{\text{NS}}-h_k)}\ \cos(\Psi_k)\delta V_k - \frac{1}{(R_{\text{NS}}-h_k)} V_k \sin(\Psi_k)\delta\Psi_k + \frac{V_k\cos(\Psi_k)}{(R_{\text{NS}}-h_k)^2}\ \delta h_k\right) \Delta t_k \tag{24}$$

$$\begin{aligned}
(\lambda_n) = \delta\lambda_0 + \sum_{k=1}^{n} \Bigg(&\frac{1}{(R_{\text{EW}}-h_k)\cos\Lambda_k}\ \sin(\Psi_k)\delta V_k \ + \frac{1}{(R_{\text{EW}}-h_k)\cos\Lambda_k} V_k \cos(\Psi_k)\delta\Psi_k \\
&+ \frac{V_k\sin(\Psi_k)}{(R_{\text{EW}}-h_k)^2\cos\Lambda_k}\delta h_k + \frac{V_k\sin(\Psi_k)\ \tan(\Lambda_k)}{(R_{EW}-h_k)\cos(\Lambda_k)}\delta\Lambda_k\Bigg)\Delta t_k
\end{aligned} \tag{25}$$

4.1. Induced frequency difference

In this section, we investigate solving induced frequency differences between input data of velocity and angle sensors. Gyro and speedometer sensors have different frequencies between input and output data; however, they do not have much of an impact due to the underwater vehicle system, which is inertial (Noureldin et al., 2012). Furthermore, the minimum rate of sensor data retrieval can be chosen for more accuracy and optimal use of navigation sensors, and the remaining data obtained from other sensors can be assumed to be fixed. Additionally, the interpolation methods (linear or nonlinear) were applied to obtain more accuracy. Considering the short time interval of sensor data retrieval in the navigation, using the Bisection numerical method is suggested to achieve the sensor's data when needed.

4.2. Flowing water error

This section explores the errors caused by flowing water and how to correct them. Deviation of the movement of water is the volume of deviation that happens in the direction of the float while traveling. The water flow on the sea maps is calculated by the angle of the real water flow direction. The intensity of the current in miles per hour is the magnitude of the current in inches per hour. If the difference between the two deviation points is greater than one hour (or less than one hour), the flow intensity

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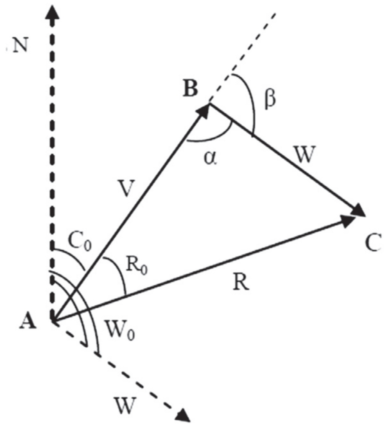

$$R^2 = V^2 + W^2 - 2VW\cos\alpha$$

$$where, \alpha + \beta = 180, and\ \beta = W_0 - C_0 \xrightarrow{yields}$$

$$\alpha = 180 - (W_0 - C_0) \xrightarrow{yields}$$

$$R^2 = V^2 + W^2 - 2VW\cos(180 - (W_0 - C_0))$$

Figure 7. *Correcting water flow errors in the DR method.*

should be measured on a scale of one hour; for example, the flow intensity should be multiplied by 1.5 if it is one and a half hours. Based on Figure 7, assume that the float moves with the course of 'C_0' and the velocity of 'V'. The water movement often moves with the course of 'W_0' and the velocity of 'V'. The purpose is to measure the vector 'R' (the velocity vector of the vessel influenced by the water flow) and the angle 'R_0' (angle between 'V' and 'R'). Calculations of the vector and the laws of the triangle are used to measure 'R_0'. So, the vector 'R' can be determined first, and then the angle 'R_0' can be calculated using the sine rules and the values 'R' and 'α'.

$$\frac{W}{\sin R_0} = \frac{R}{\sin \propto} \tag{26}$$

$$\sin R_0 = \frac{W \times \sin(\propto)}{R} \tag{27}$$

$$R_0 = \sin^{-1}\left(\frac{W \times \sin(\propto)}{R}\right) \tag{28}$$

If $W_0 > C_0$, the R_0 value should be subtracted from C_0 to fix the volume of water flow variance R_0. This implies that the vessel can travel down the course: $C_{01} = C_0 - R_0$ to be influenced by the flowing water at point B. If $C_0 > W_0$, the amount of R_0 is to be applied to C_0, i.e. $C_{01} = C_0 + R_0$; if $W_0 = C_0$, it is $C_{01} = C_0$.

Table 2 shows the various scenarios to address the issue of water flow in the dead reckoning method.

4.3. Software simulation of underwater vehicle's dead reckoning navigation

As an illustration, we are going to simulate software with experimental data to determine the dead reckoning navigation of a submarine under the following conditions.

In our simulation, the following assumptions are considered.

- The underwater vehicle maintains a constant speed and heading throughout the journey.
- The underwater vehicle does not encounter any winds that affect its movement.
- The underwater vehicle does not change its course or speed to avoid obstacles or perform manoeuvres.

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Table 2. *Various scenarios to address the issue of water flow in the dead reckoning method.*

C_O	W_0	W_0, C_0 (Comparison)	α	C_{O1}
$C_O \leq 90$	$W_0 \leq 90$	$W_0 < C_0$	$\alpha = 180 - (C_O - W_0)$	$C_{O1} = C_O + \lvert R_0 \rvert$
$C_O \leq 90$	$W_0 \leq 90$	$W_0 > C_0$	$\alpha = 180 - (W_0 - C_O)$	$C_{O1} = C_O - \lvert R_0 \rvert$
$0 < C_O \leq 90$	$90 < W_0 \leq 180$	$W_0 > C_0$	$\alpha = 180 - (W_0 - C_O)$	$C_{O1} = C_O - \lvert R_0 \rvert$
$0 < C_O \leq 90$	$180 < W_0 \leq 270$	$W_0 < C_0 + 180$	$\alpha = 180 - (W_0 - C_O)$	$C_{O1} = C_O - \lvert R_0 \rvert$
$C_O \leq 90$	$180 < W_0 \leq 270$	$W_0 > C_0 + 180$	$\alpha = -180 + (W_0 - C_O)$	$C_{O1} = C_O - \lvert R_0 \rvert$
$C_O \leq 90$	$270 < W_0 \leq 360$	$W_0 > C_0 + 180$	$\alpha = -180 + (W_0 - C_O)$	$C_{O1} = C_O + \lvert R_0 \rvert$
$90 < C_O \leq 180$	$W_0 \leq 90$	$W_0 < C_0$	$\alpha = 180 - (C_O - W_0)$	$C_{O1} = C_O + \lvert R_0 \rvert$
$90 < C_O \leq 180$	$90 < W_0 \leq 180$	$W_0 < C_0$	$\alpha = 180 - (C_O - W_0)$	$C_{O1} = C_O + \lvert R_0 \rvert$
$90 < C_O \leq 180$	$90 < W_0 \leq 180$	$W_0 > C_0$	$\alpha = 180 - (W_0 - C_O)$	$C_{O1} = C_O - \lvert R_0 \rvert$
$90 < C_O \leq 180$	$180 < W_0 \leq 270$	$W_0 > C_0$	$\alpha = 180 - (W_0 - C_O)$	$C_{O1} = C_O - \lvert R_0 \rvert$
$90 < C_O \leq 180$	$270 < W_0 \leq 360$	$W_0 < C_0 + 180$	$\alpha = 180 - (W_0 - C_O)$	$C_{O1} = C_O - \lvert R_0 \rvert$

- The Earth is assumed to be an ellipsoid model, and the underwater vehicle's movement is calculated using ellipsoid trigonometry by using Equations (1)–(5).

$$\begin{aligned} &v_x = 3, v_y = 4, v_z = 0.1,\ V_N = 4.33\,\text{m/s},\ V_E = 2.5\ \text{m/s}, \\ &\Lambda = 27^\circ, \lambda = 54^\circ, h = 20\ \text{m}, \theta = 5^\circ, \varphi = 5^\circ,\ \Psi = 30^\circ, \delta\Psi = 0.7\ \text{seclat} \\ &\delta\Psi = 0.7\ \text{seclat} = 0.786^\circ{=}0.01372^{\text{rad}} \end{aligned} \tag{29}$$

$$\begin{aligned} &\delta V_N = \cos(\psi)\delta V - V\sin(\psi)\delta\psi = -0.0145\,\text{m/s} \\ &\delta V_E\ \sin(\psi)\delta V + V\cos(\psi)\delta\psi = 0.0708\quad \text{m/s} \end{aligned} \tag{30}$$

$$\begin{cases} \dot{\Lambda} = \dfrac{V_N}{R_{\text{NS}} - h} = 3.92\ \times 10^{-7} \\ \dot{\lambda} = \dfrac{V_E}{(R_{\text{EW}} - h)\cos\Lambda} = 4.4\ \times 10^{-7} \\ \dot{h} = -v_D \end{cases} \tag{31}$$

Now if 10 h is considered for the navigation time, we are going to specify the longitude and latitude of the desired endpoint by using real data from sensors.

$$\begin{aligned} \dot{\Lambda} &= \frac{V_N}{R_{\text{NS}} - h} \\ \dot{\lambda} &= \frac{V_E}{(R_{\text{EW}} - h)\cos\Lambda} \end{aligned} \tag{32}$$

$$\begin{aligned} \Lambda &= \int_{t_1}^{t_2} \frac{V_N}{R_{\text{NS}} - h} dt = 27.0304\ \text{degree} \\ \lambda &= \int_{t_1}^{t_2} \frac{V_E}{(R_{\text{EW}} - h)\cos\Lambda} dt = 54.0205\ \text{degree} \end{aligned} \tag{33}$$

Knowing these components, we can calculate latitude 'Λ' and longitude 'λ' at any time. The essential issue in numbering is to consider this: $h = -20$ m.

$$\begin{cases} \dot{\Lambda} = \dfrac{V_N}{R_{\text{NS}} - h} \overset{\text{yields}}{\rightarrow} \Lambda = \int_{t_1}^{t_2} \dfrac{V_N}{R_{NS} - h} dt \\ \dot{\lambda} = \dfrac{V_E}{(R_{\text{EW}} - h)\cos\Lambda} \overset{\text{yields}}{\rightarrow} \lambda = \int_{t_1}^{t_2} \dfrac{V_E}{(R_{EW} - h)\cos\Lambda} dt \\ \dot{h} = -v_D \overset{\text{yields}}{\rightarrow} h = -\int_{t_1}^{t_2} V_D\, dt \end{cases} \tag{34}$$

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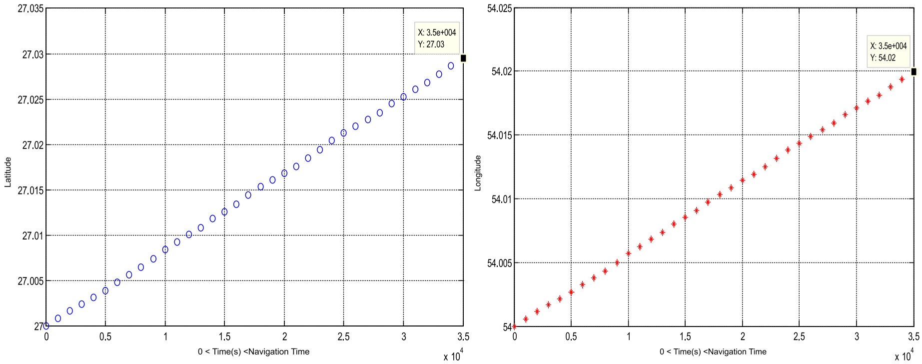

Figure 8. *Changes in the longitude and latitude of an underwater vehicle over time.*

To solve the above integral by using experimental data, discretisation should be used:

$$\delta(\Lambda_n) = \delta\Lambda_0 + \sum_{k=1}^{n}\left(\frac{1}{(6,378,137+20)}\cos(\Psi_k)\delta V_k - \frac{1}{(6,378,137+20)}V_k\sin(\Psi_k)\right.$$
$$\left.\times\, 0.7\text{ seclat } + \frac{V_k\cos(\Psi_k)}{(6,378,137+20)^2}\,0.21\right)\Delta t_k \tag{35}$$

$$\delta(\lambda_n) = \delta\lambda_0 + \sum_{k=1}^{n}\left(\frac{1}{(+20)\cos\Lambda_k}\sin(\Psi_k)\delta V_k + \frac{1}{(6,378,137+20)\cos\Lambda_k}V_k\cos(\Psi_k)\right.$$
$$\left.\times 0.7\text{ seclat } + \frac{V_k\sin(\Psi_k)}{(6,378,137+20)^2\cos\Lambda_k}\,0.21 + \frac{V_k\sin(\Psi_k)\ \tan(\Lambda_k)}{(6,378,137+20)\cos(\Lambda_k)}\delta\Lambda_k\right)\Delta t_k$$

According to calculation, the following is the tolerance of error for identifying a location's latitude and longitude of the underwater vehicle during 10 h of traveling:

$$\delta(\Lambda) = -3.2\times10^{-5}\Delta t = -0.0066\text{ degree} \tag{36}$$
$$\delta(\lambda) = 1.55\times10^{-8}\Delta t = 0.032\text{ degree} \tag{37}$$

5. Result and discussion

In the subsequent figures, the simulation results and error analysis for DR navigation are displayed. The values for the taken course and heading angle, as well as the amount of possible error in the taken course and the heading angle at the endpoint (during time t), can be determined based on the use of DR navigation equations (achieving the desired endpoint) and the consideration of DR navigation error equations.

Figure 8 depicts the evolution of the underwater vehicle's longitude and latitude over time. Based on DR navigation equations, which account for the distance traveled and the direction traveled, this figure presumably depicts the actual path taken by the underwater vehicle.

Figure 9 depicts the possible navigational errors in latitude and longitude over time. The X axis presumably represents time in seconds, whereas the Y axis indicates the possible error in the latitude and longitude coordinates. This graph likely illustrates the uncertainty associated with DR navigation, as errors can accumulate over time due to various factors, including flowing water error, induced frequency difference and sensor measurement error.

Figures 10 and 11 represent the underwater vehicle's latitude and longitude over time in the presence of system uncertainties. These numbers indicate how the underwater vehicle's actual course deviated

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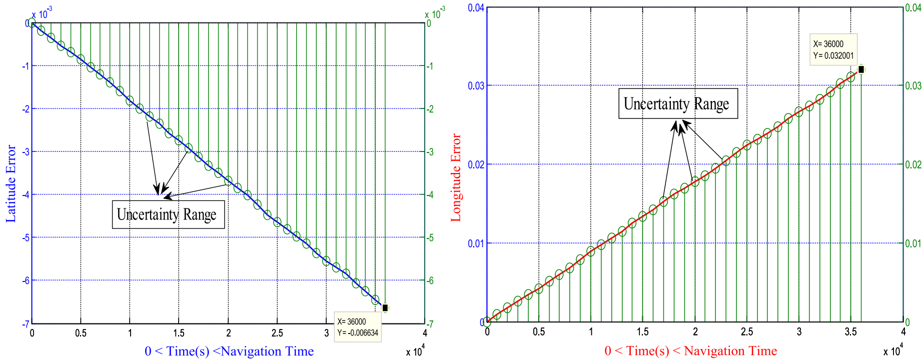

Figure 9. *Latitude and longitude navigation possible errors (X: time in seconds and Y: latitude error and longitude error).*

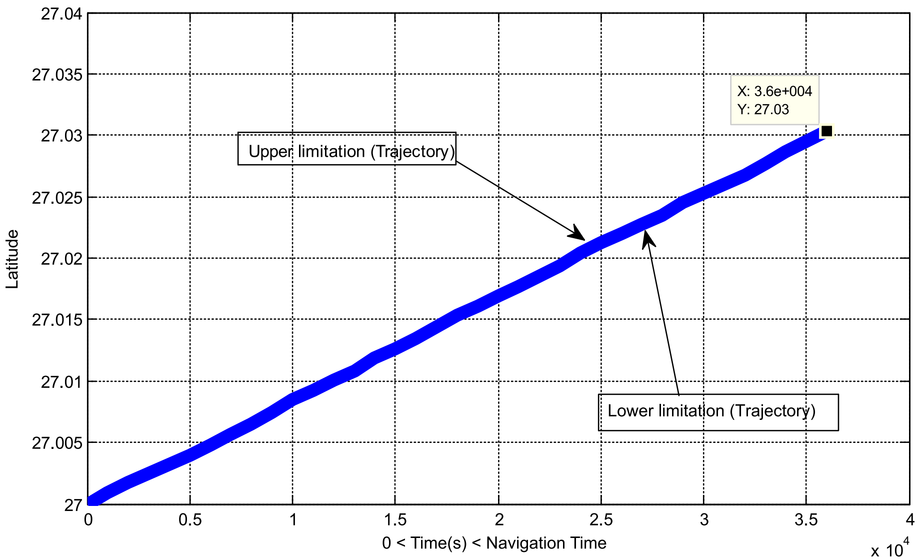

Figure 10. *Tracking the change in submarine latitude in the presence of system uncertainties.*

from the calculated path due to factors such as measurement errors and environmental factors. The deviation probably corresponds to the difference between the actual and calculated paths.

Figure 12 displays the position navigation of an underwater vehicle while system uncertainties are available. The *X* axis shows longitude, while the *Y* axis shows latitude. The graph presumably reflects the actual position of the underwater vehicle over time, as determined by DR navigation equations, compared with the desired position.

The actual position of the underwater vehicle may deviate from the intended position in the presence of system uncertainties, such as measurement errors and environmental factors. The figure most likely illustrates the deviation between the actual and desired positions, the deviation represented by the difference between the two positions.

The accuracy and limitations of DR navigation in the presence of system uncertainties can be determined by examining these figures. The quantity of deviation between the actual position and the desired position can be used to assess the navigation system's performance and identify areas for enhancement. In addition, the deviation trend over time can reveal the effect of environmental factors and measurement errors on the navigation system's accuracy. For instance, if the deviation is growing over time, it may indicate that environmental factors have a greater impact on the navigation system

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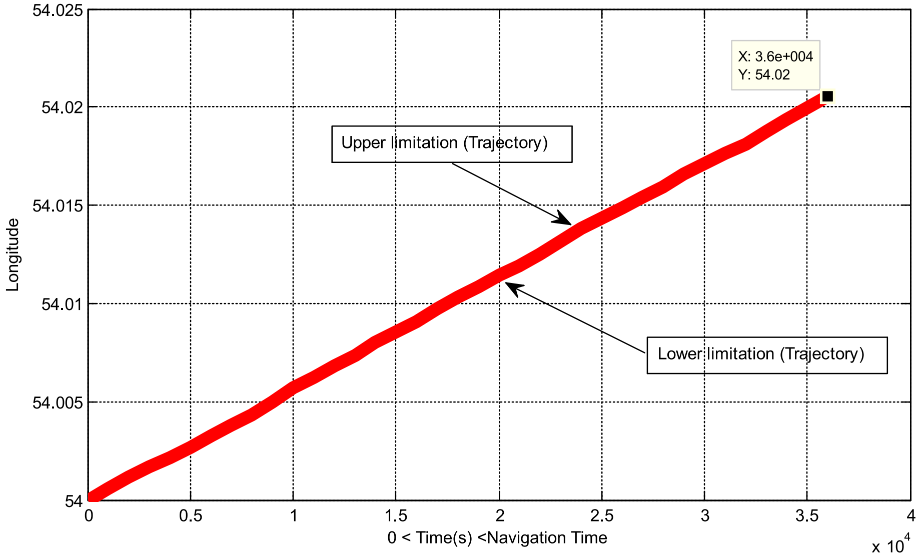

***Figure 11.** Tracking the change in underwater vehicle longitude in the presence of system uncertainties.*

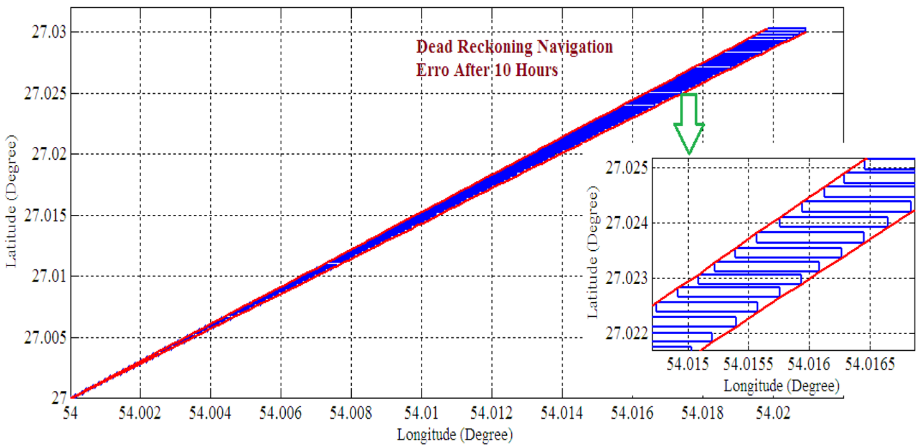

***Figure 12.** Underwater vehicle's position navigation in the presence of system uncertainties (X: longitude amd Y: latitude).*

as time passes. These figures provide a visual representation of the effectiveness of the underwater vehicle's navigation system in the presence of system uncertainties, and can be used to evaluate and enhance the system's accuracy.

To examine the simulation, we practically test the software with experimental data from sensor data. The position of the underwater vehicle at any time can be determined by having experimental data such as the velocity in different directions, and yaw, pitch and roll angles; and to make sure of the results of DR navigation software, they can be compared with the results of the GPS. It is pointed out that when using GPS, it is important to keep in consideration that regular GPS receivers cannot be relied upon when it is possible to send interference signals to them. Next, we model the system to achieve more precise results by incorporating the different kinds of uncertainty and possible errors mentioned in the paper. Finally, the DR navigation method results are compared with the online GPS results, taking into consideration the uncertainties, demonstrating the high level of accuracy of the method proposed in this paper.

Figure 13 displays the tracking underwater vehicle's position test performance which is done by the DR navigation system. Figures 14 and 15 display the comparisons of the underwater vehicle's position

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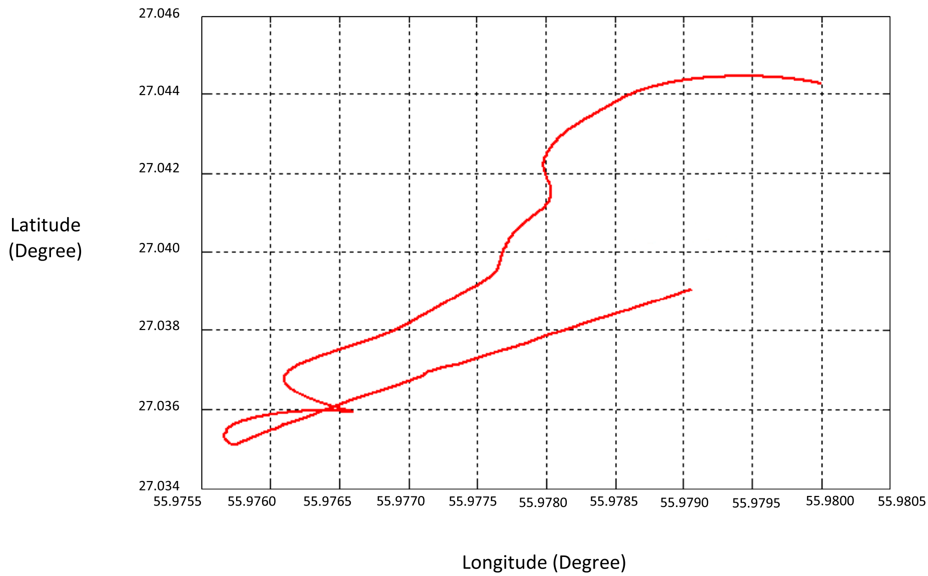

Figure 13. *Performance test, underwater vehicle's position tracking by DR method (X: longitude and Y: latitude).*

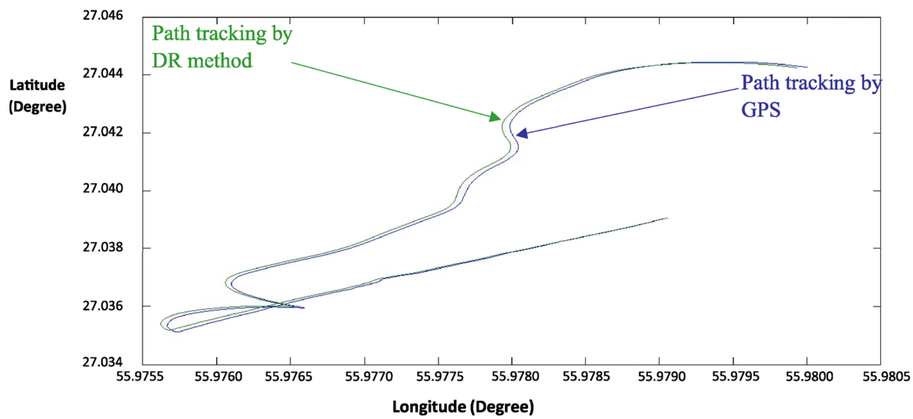

Figure 14. *Performance test, comparison of underwater vehicle's position tracking by DR and GPS.*

tracking by DR and GPS; they are achieved without system uncertainties and in the presence of system uncertainties, respectively. The practical results demonstrate very high tracking accuracy using the DR navigation method.

6. Conclusion

The information that was provided in the research suggests the dead reckoning (DR) navigation technique is a low-cost solution that is dependent on the dynamic equations of the system. In this study, a detailed examination of the use of DR navigation in underwater vehicles is presented, taking into consideration the possibility of sensor inaccuracy. Through the use of the DR navigation technology, the eventual location of the underwater vehicle may be estimated with a high degree of accuracy. Additionally, the findings of the system error analysis performed on the DR navigation technique using data from the actual world are presented in this study. These findings were compared with GPS data, and the comparison showed that there were only minor variations between both of them. This demonstrates that the DR navigation system properly records the precise position of the underwater vehicle. When taken as a whole, the

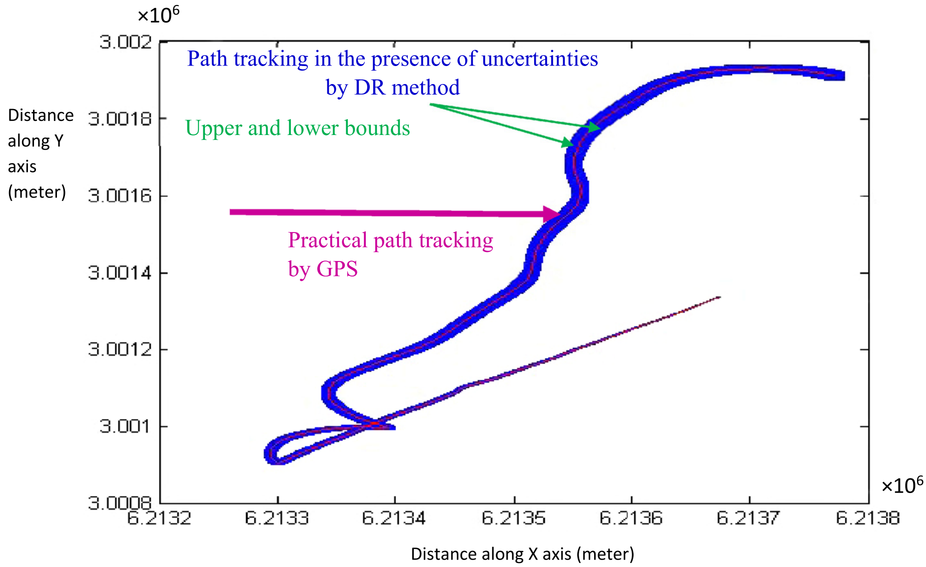

Figure 15. *Performance test, underwater vehicle position tracking by DR software in the presence of system uncertainties and its comparison with the tracked course by GPS.*

research illustrates that the DR navigation approach is capable of achieving a high degree of accuracy, despite the fact that errors and uncertainties are present. According to the findings, the DR navigation approach seems to be an effective tool for accurately establishing the precise position of the underwater vehicle. Furthermore, it has the potential to be used in other applications that need accurate positioning.

Funding statement. The authors disclosed receipt of the following financial support for the research, authorship and/or publication of this paper: This work has been financially supported by the University of Torbat Heydarieh. The grant number is 1402/04/17-174.

Competing interests. The authors declare that they have no conflict of interest.

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Cite this article: Gharib MR, Fard MA, Koochi A (2024). Error analysis of dead reckoning navigation system by considering uncertainties in an underwater vehicle's sensors. *The Journal of Navigation* 1–19. https://doi.org/10.1017/S0373463324000183