PART I

PLANETARY THEORY AND ANALYTICAL METHODS

PLANETARY THEORIES AND OBSERVATIONAL DATA

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ABSTRACT

A brief review is given of planetary theories from Leverrier to Newcomb to the age of computers. The presently used planetary theories are discussed and the process of replacing these theories with new ones is described. Some difficulties in preparing new planetary theories and the observational discrepancies which have been encountered previously are discussed.

HISTORY

The first systematic application of dynamical principles to the motions of planetary bodies was made by Leverrier for the theories of the motions of the planets Mercury, Venus, Earth, and Mars for the epoch 1850. (Leverrier 1858, 1859, 1861a, 1861b). These theories suffered from two defects: (1) they were not based on a consistent set of planetary masses, and (2) they did not satisfy the observed longitudes of the planets. Leverrier used only Newtonian mechanics and was unable to account for the secular motions of the periheli Therefore, he augmented his dynamical theories by nondynamica terms in order to achieve agreement with the observations. Leverrier's theories formed the basis for the tables of Mercury, Venus, Earth and Mars in the various national ephemerides until 1900 and in the Connaissance de Temps until 1960.

In 1900, the planetary theories of Simon Newcomb were generally adopted. Newcomb's theories of Mercury, Venus, Earth and Mars (Newcomb 1895) were also based on Newtonian dynamics, but they incorporated a consistent system of planetary masses. Again the theories did not satisfy the observational data with respect to the observed motions of the perihelia, and so Newcomb was forced to assume that the

V. Szebehely (ed.), Dynamics of Planets and Satellites and Theories of Their Motion, 3-14.

gravitational forces toward the Sun did not vary exactly according to the inverse square law of the distance. Thus, in the expression for the gravitation, $f = mm'/r^n$, where m and m' are the masses of the two bodies and r is the distance between them, the exponent n of r was assumed to be of the form $n = 2 + \Delta$, where Δ is a small quantity. In this case the perihelion of each planet will have a direct motion found by multiplying its mean motion by $\frac{1}{2}\ \Delta$. Newcomb used the value of $n = 2.0000001612$ and augmented the secular motion of the perihelia of the four planets by the corresponding amount. Shortly after the introduction of Newcomb's tables of Mars into the national ephemerides, it was realized that they failed to represent the observations of the planet by amounts large enough to indicate errors in the tables. F. E. Ross in 1912 concluded that the eccentricity of the orbit adopted by Newcomb was in error. He compared the observations of Mars with Newcomb's tables and obtained new values of the elements and calculated corrections to Newcomb's tables, which have been incorporated in the ephemerides of the planet since (Ross 1917). In 1950 Eckert, Brouwer, and Clemence (1951) produced a simultaneous numerical integration of the orbits of the five outer planets, which replaced the ephemerides based on the tables of Hill for Jupiter and Saturn and Newcomb for Uranus and Neptune. By that time the observations of Saturn and Neptune had departed significantly from the ephemerides. In 1960 these special perturbation theories were introduced as the basis for the tables of the outer planets. To this day the tables of the inner planets printed in the national ephemerides are based on the theories of Newcomb with Mars revised by Ross, and the integration by Eckert, Brouwer and Clemence is the basis for the tables of the outer planets.

PRESENT STATUS

In the meantime Clemence (1949, 1961) has used Hansen's method to calculate a new general theory for Mars and R. E. Laubscher (1971) has derived definitive constants for this theory based on the observations of Mars from 1750 to 1971. Improved elements for Mercury, Venus and Earth have also been determined by comparison of Newcomb's theories with observations (Clemence 1943, Duncombe 1958, Morgan 1933). Sharaf (1955, 1964) has prepared a numerical general theory of the motion of the planet Pluto, while Cohen, Hubbard and Oesterwinter (1967) performed a numerical integration to calculate an improved ephemeris of Pluto. Numerical integrations have been used by numerous individuals and groups to calculate improved ephemerides, particularly for special purposes or limited periods of time (e.g. Ash et al, 1971, Oesterwinter and Cohen 1972, Standish et al, 1976).

It is recognized that before planetary theories can be significantly improved some of the underlying constants must be corrected. For this reason in 1970 Commission 4 of the IAU appointed Working Groups on units and time scales, precession, and planetary ephemerides. A Joint Report of these working groups will be submitted to the IAU for adoption in Grenoble this year. This report includes recommendations for a new set of planetary masses, a new value for the precessional constant, a definition of time scales for dynamical theories, and a consistent set of basic constants. Once these recommendations are adopted by the IAU it is anticipated that new planetary theories will be prepared at various institutions and by several different methods. These new theories will permit the preparation of ephemerides which can be fitted to the observations and which can be intercompared to ensure their accuracy. For this purpose at the U. S. Naval Observatory, in cooperation with other organizations, the preparation of new general theories, particularly for the inner planets, are being undertaken. The methods of Musen and Carpenter are being applied to determine numerical general theories (Musen and Carpenter 1963, Carpenter 1963, 1965, 1966, 1966). Numerical integrations of all the planets of the solar system are being planned and observations for all the planets are being collected and reduced to the FK4 system. The observations will be compared with the general theories and the numerical integrations in order to rectify the constants. It is hoped that these efforts will result in accurate ephemerides for all the planets that will satisfy the observations over an extended period of time.

However, before the impression is given that all the problems with respect to the formation of planetary theories are solved and that the process is perfectly straightforward, it is advisable to describe some of the difficulties which have been experienced with the theories and some of the observational problems that have been encountered.

PROBLEMS

In calculating a general theory for a planet there is an ever present problem of convergence. With classical methods of calculating general theories, as higher orders are calculated, the amplitude of the changes of the coefficients reduces, and it was assumed that convergence was being achieved. The practical considerations of hand computations limited the process to somewhere between second and third order and the numerical contributions at that order were small. While there was no method of testing for convergence, it was assumed that contributions from higher orders would be smaller and that they would not be cumulative.

With the availability of computers for calculating general theories, either numerical or analytical, it has become feasible and advisable to depart from the concept of orders and instead to use an iterative approach. With iterative methods the expressions for the motion of the planet are substituted into the fundamental equations of motion and a new theory for the planetary motion is generated. This new theory can then be substituted back into the equations of motion and the process repeated. The assumption is made that eventually, when the theory is substituted into the equations of motions, identical expressions are derived and thus the theory has converged. Experience has shown that this process apparently works well except when dealing with resonant terms. When resonance is involved, several possibilities arise; either the values of the coefficients may oscillate about some value, or the coefficients change from iteration to iteration, normally finding a minimum or maximum and then diverging from those values with each successive iteration in a parabolic manner, or the values of the coefficients in each successive iteration assume an asymptotic approach to some value. (Figure 1) Where there is a single predominant resonant term, such as in the case of the Earth perturbed by Venus, it appears possible to isolate the term and determine its value independently. When this value of the resonant term and the other coefficients are substituted into the equations of motion, the results are equivalent to the input values. Thus a form of convergence is achieved. In more complicated cases such as the Jupiter-Saturn resonance, it is not certain that convergence by such manipulations can be achieved.

The second problem arising in the calculation of general theories, and possibly related to the first problem, is the question of accuracy. General planetary theories have in most cases failed to deliver the positional accuracy to which they were calculated. This could be due to a lack of convergence, neglected terms, or other causes and indicates that extreme care must be exercised to guard against these sources of error. Therefore, careful investigation of the accuracy of a general theory should be made, in addition to comparing the theory to observations.

The basic purpose of a planetary theory is to represent the motion of the planet, particularly as observed from the Earth, so the third problem relates to the comparison of the theories with the observational data. This is particularly evident in regard to the latitude residuals of Uranus and Neptune. Having fitted the numerical integrations to observations, in the case of Uranus to normal points and in the case of Neptune to the individual observations, a systematic effect remains in the residuals. (Figures 2 and 3) For Uranus

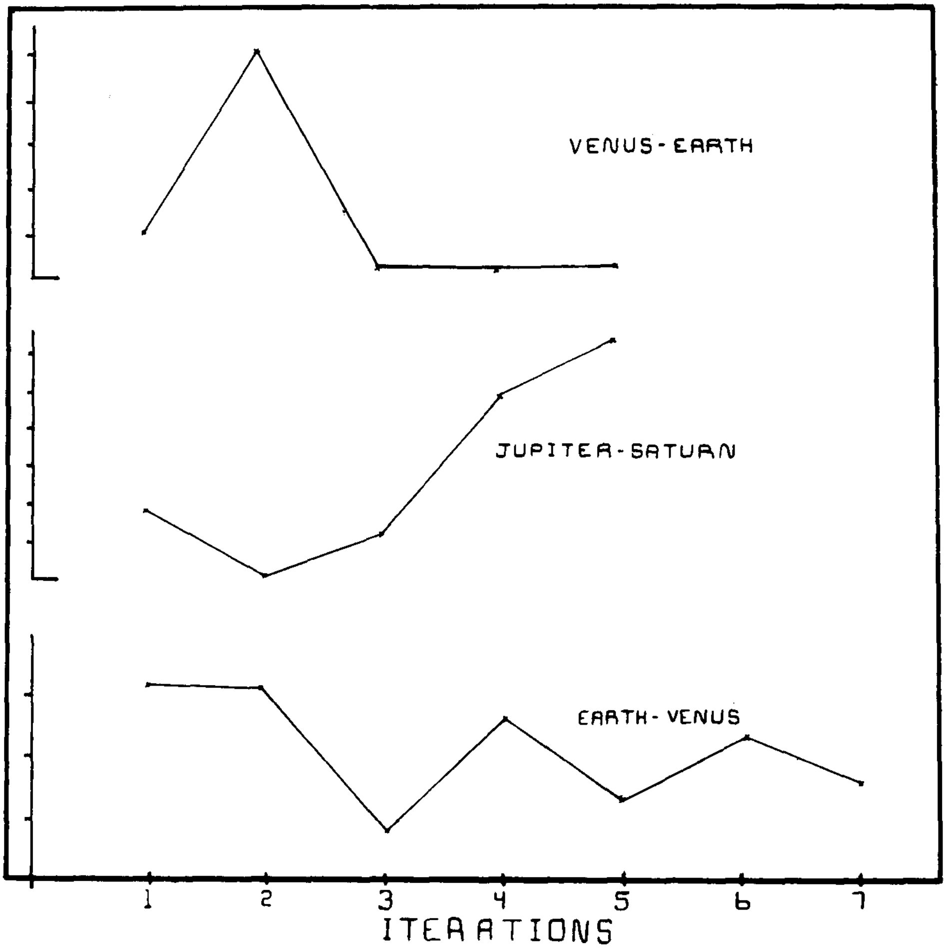

Figure 1. Plot of convergence characteristics of resonance terms.

more than two orbital periods are covered by the observations, and the residuals in latitude have the characteristic of a secular change. The effect was observed by Newcomb (1898), Wylie (1947) and Seidelmann, Duncombe and Klepczynski (1969). The latitude residuals of Neptune show a periodic effect, but it must be remembered that a complete period of Neptune's orbit has not been observed (Seidelmann et al 1971). The residuals, after a least squares fitting to data with a secular effect but not covering the whole orbital period, would have the periodic signature observed. Thus, it appears that there is a secular effect in the observations of both Uranus and Neptune, which is either due to an omitted effect in the calculation of the theories, or due to a systematic error present in the observations of these planets. Further,

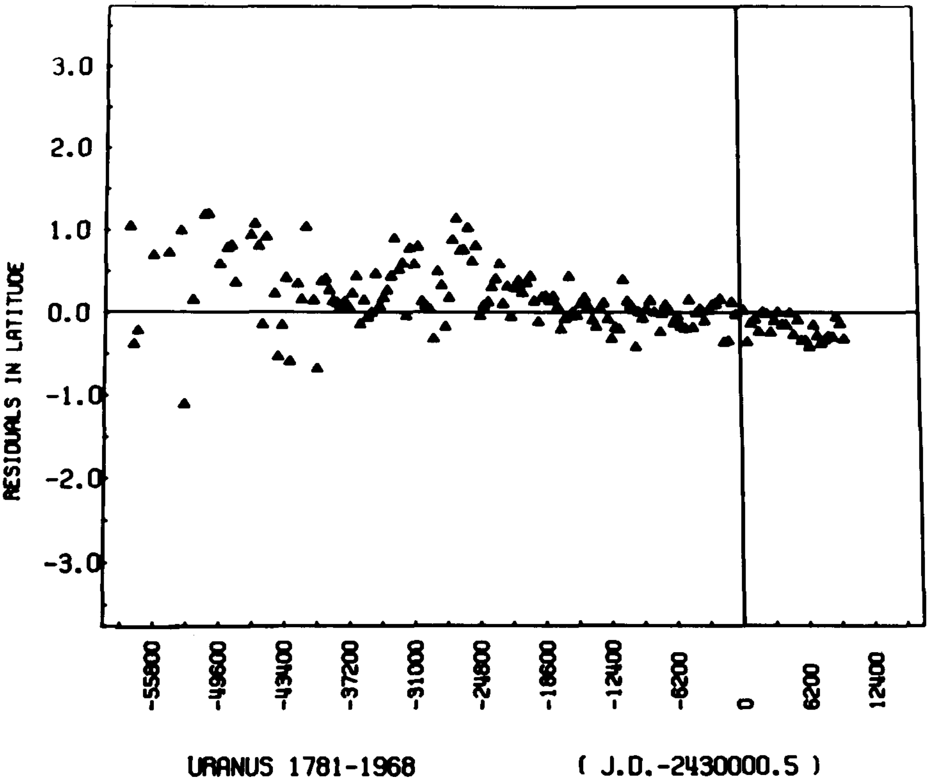

Figure 2. Latitude residuals of Uranus.

when the best fit of the observations to the ephemerides of the outer planets is performed and the residuals are analyzed, significant periodic variations are found in the residuals. Unfortunately no logical hypothesis appears to satisfy the data. There is also the disturbingly large difference between the mass determinations for Uranus. The reciprocal mass of Uranus, determined from the observations of Saturn, is 22693 ± 33 (Klepczynski et al 1970), while the determination from the satellite observations is 22945 ± 15 (Dunham, 1971).

The fourth problem might be described as the opposition, or phase, effect. When the optical observations of the brighter planets are compared with ephemerides, a systematic effect is present which is a function of the time before and after opposition (Standish et al 1976). (Figures 4 and 5) This systematic effect in the residuals is found to be present when observations are compared to several different theories. Thus, this is most likely an observational error, perhaps due to an incomplete phase correction to the

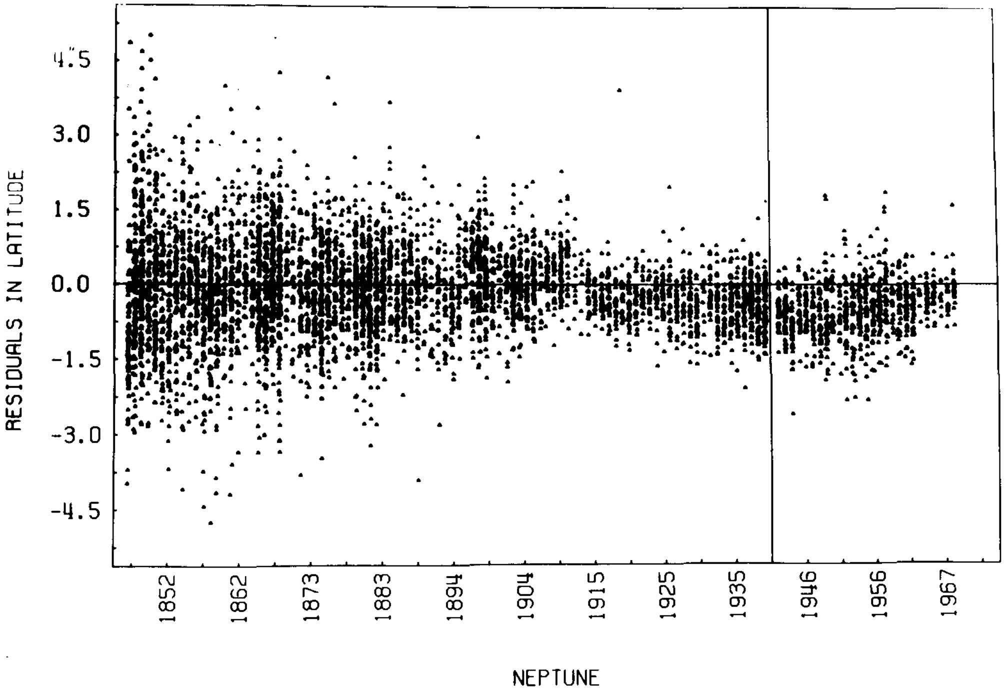

Figure 3. Latitude residuals of Neptune.

observations, or the effect of irradiation in the observer's personal equation, or some effect which is a function of the brightness of the object.

The fifth problem is the secular variation of the obliquity of the ecliptic as discussed by Duncombe and Van Flandern (1976). The planetary observational data has indicated a correction to Newcomb's secular variation of the obliquity of (-0".277±.18) T where T is in centuries from 1900.0. Further there seems to be a difference between the observations made before 1900 and those made after 1900, which may indicate how rigorously the observations have been reduced to the FK3, FK4 system. (Figure 6) If this is truly an observational problem it could be related to the observational effects observed in the residuals of Uranus and Neptune. The evidence seems to indicate that the theoretical value for the secular variation for the obliquity of the ecliptic is correct, but that care must be exercised in the use of pre-1900 observations, particularly of the inner planets which are observed as day objects.

The sixth problem may be described as the many aspects of that peculiar planet Pluto. The knowledge of Pluto is

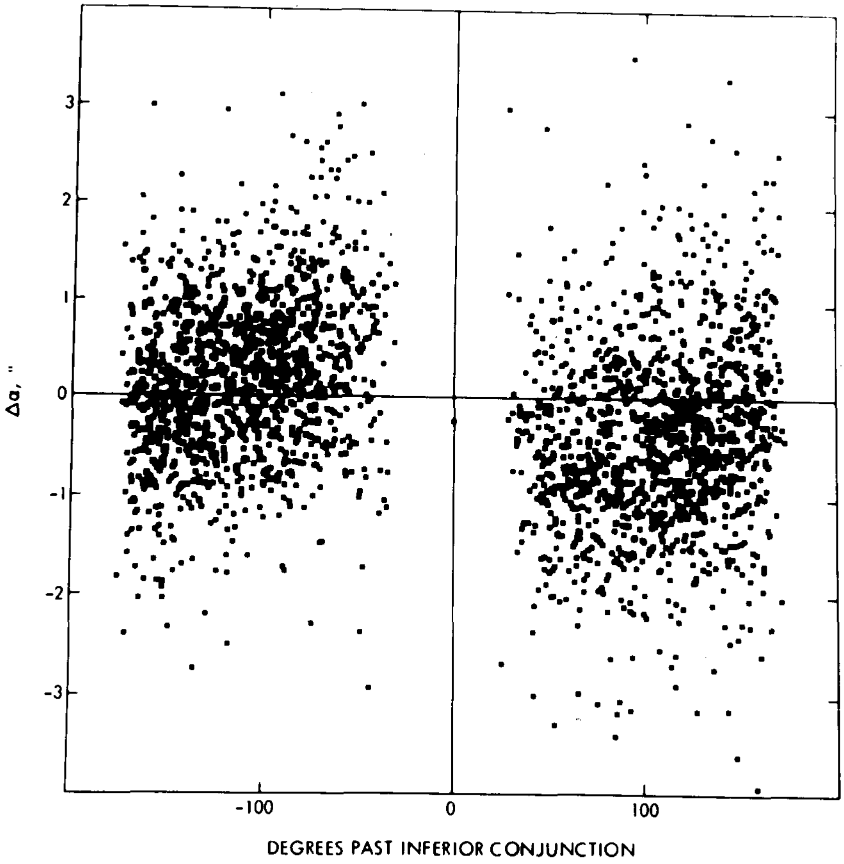

Figure 4. Optical residuals illustrating "opposition effect" for Mercury. (Standish et al, 1976)

very limited in part due to the fact that observations for only about one fourth of its orbital period are available. There are estimates of its mass and radius, but they lead to an uncomfortably large density. Even so the mass estimates are so low that the prediction of the existence of Pluto based solely on its effects on the orbit of Uranus is impossible. Yet Pluto was discovered very close to its predicted position. Either its discovery was a case of serendipity or perhaps only one of many objects in that part of the solar system has been discovered. Assuming that the mean elements of Pluto are sufficiently well known to merit the preparation of a general theory, there remains the problem that the orbit of Pluto crosses the orbit of Neptune. This makes it impossible to use harmonic analysis on the distances between those two bodies and thus prohibits the calculation of a general theory of Pluto, except by techniques such as that

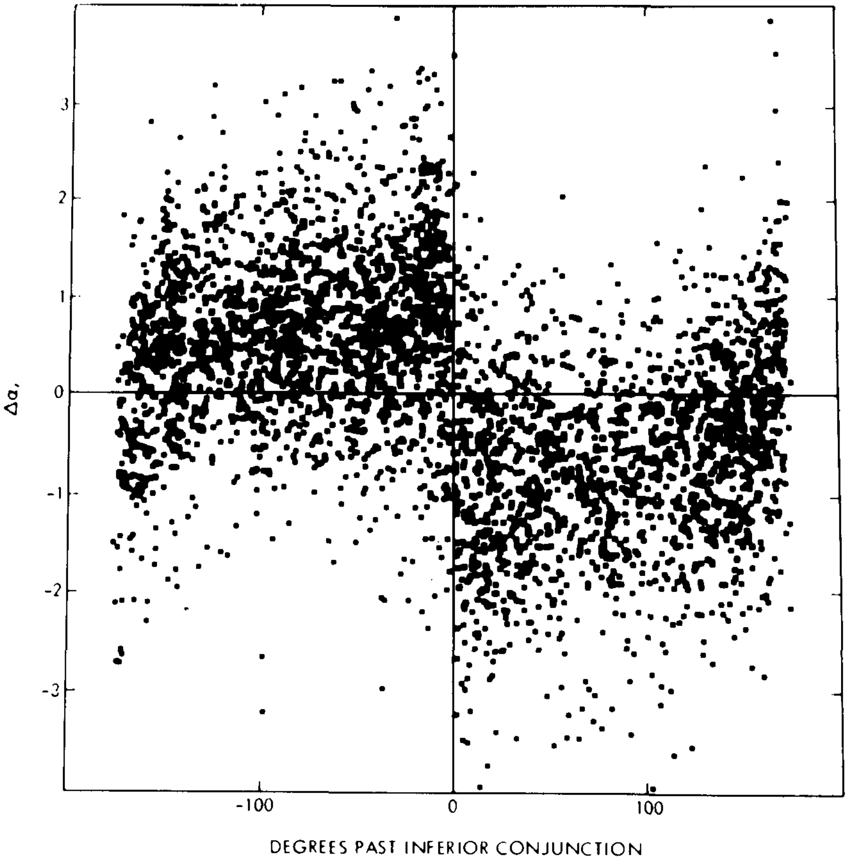

Figure 5. Optical residuals illustrating "opposition effect" for Venus. (Standish et al, 1976)

of Goodrich and Carpenter (1966) where an exact resonance is assumed, or that of Sharaf (1955, 1964) where numerical integration is used for the Neptune-Pluto terms, or that of Petrovskaya (1972) where special expansions are used.

The seventh problem concerns the use of mixtures of the various types of observational data; namely optical, radar and spacecraft observations. Optical observations provide an angular determination in two coordinates of the position of the object as projected on the celestial sphere. With transit circle instruments one observation per day can be made at a given observatory. Radar observations provide an accurate observation of the distance from the Earth to the object and back in a short span of time, and many observations can be made on a given day at a given observatory. Spacecraft data also provide accurate range or range-rate

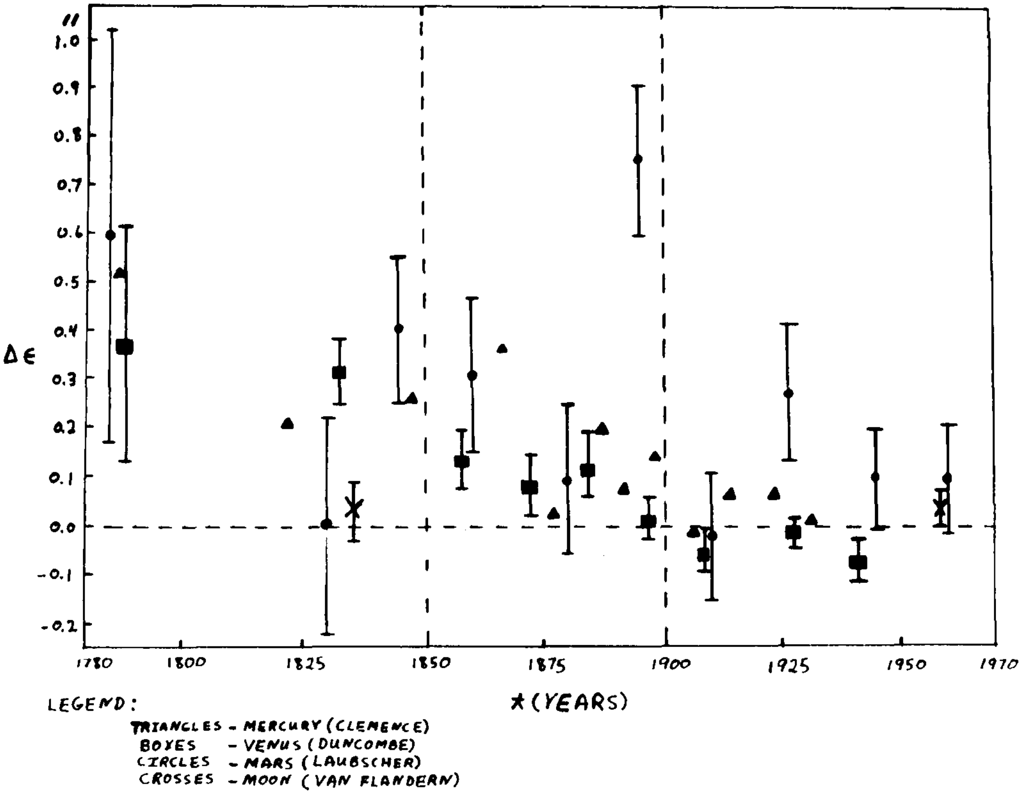

Figure 6. Determinations of the obliquity of the ecliptic.

determinations and many observations are possible in a short period of time from a given observatory. The optical data covers a long period of time with some improvements in the accuracy of the observations over that period, although the possibility remains of the presence of systematic errors, particularly in the older observations. The spacecraft and radar data cover a shorter time period with significant changes in the accuracy of the observations over that period. The important question here concerns the manner in which these observations are combined. What is the best way of determining the relative weights to be applied to these different types of observations in order that the best possible ephemeris for the object may be determined?

The eighth problem concerns the implications of the lunar observational data. Van Flandern (1975, 1976) has hypothesized a secular change in the gravitational constant based on occultation observations of the Moon. Whether the hypothesis of such a change in the gravitational constant is correct or not, there is some systematic effect present in the lunar observations. The Moon, due to its short period, tends to magnify effects which may also be present in planetary data. Thus it can be anticipated that whatever is affecting the observations of the Moon will also affect planetary observations with a smaller amplitude, or over a longer period of time.

These problems, and perhaps others as yet undetected, face us as we commence the task of improving the theories of motion of the principal planets. It will require the cooperation and effort of many investigators in the fields of astrometry, dynamical astronomy and celestial mechanics to bring this project to a successful conclusion.

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