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**Letter to the Editor**

# Dark and grey electromagnetic electron-cyclotron envelope solitons in an electron-positron magnetoplasma

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**Abstract.** We present an investigation of the amplitude modulation of an external magnetic field-aligned right-hand circularly polarized electromagnetic electron-cyclotron (EMEC) wave in a strongly magnetized electron-positron plasma. It is shown that the dynamics of the modulated EMEC wave packet is governed by a cubic nonlinear Schrödinger equation. The latter reveals that a modulated wave packet can propagate in the form of either a dark or a grey envelope soliton. This result could have relevance to the transport of electromagnetic wave energy over long distances via envelope solitons in the magnetospheres of pulsars and magnetars.

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A pair plasma is composed of electrons and positrons, which have the same mass but opposite charge. It exists in the early universe [1–3], in bipolar outflows (jets) in active galactic nuclei [4, 5], in the polar regions of neutron stars [6–13], in magnetars [14, 15], in the inner regions of the accretion disks surrounding black holes [16], at the centre of our own Galaxy [17], in the Solar atmosphere [18], as well as in plasmas in intense laser fields [19–22], and in tokamaks [23]. Furthermore, due to the production of copious amounts of positrons in some laboratories, one is able to perform experiments on a pair plasma [24–29].

In a pair plasma, there are numerous new linear and nonlinear waves [30–41]. The presence of an external magnetic field in a pair plasma greatly affects the dynamics of the pairs via the Lorentz force. Hence, the propagation characteristics of both electrostatic and electromagnetic waves in a pair magnetoplasma are significantly modified [42–57]. In particular, the linear dispersion relations for the non-relativistic circularly polarized electromagnetic and the non-relativistic elliptically polarized extraordinary electromagnetic waves, which propagate along and across the external magnetic field direction respectively, are the same. Investigations of nonlinear wave–wave interactions in a magnetized pair plasma might play an important role in the description of anti-matter and dark energy of our Universe.

In this letter, we consider the nonlinear propagation of an external magnetic field-aligned negative group dispersion right-hand circularly polarized EMEC wave in a strongly magnetized pair plasma. The external magnetic field is $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is a unit vector along the $z$ axis in a Cartesian coordinate system and $B_0$ is the strength of the magnetic field. At equilibrium, we have $n_{e0} = n_{p0} \equiv n_0$, where $n_0$ is the unperturbed plasma number density. The wave electric field is $\mathbf{E}_\perp = E(\hat{\mathbf{x}} + i\hat{\mathbf{y}})\exp(-i\omega t + ikz)$ + complex conjugate, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors along the $x$ and $y$ axes, respectively. The frequency $\omega$ and the wave number $k$ are related by the cold plasma dispersion relation [44]

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_{p-}^2}{\omega(\omega\gamma_- - \omega_c)} - \frac{\omega_{p+}^2}{\omega(\omega\gamma_+ + \omega_c)}, \tag{1a}$$

which is valid for $|\omega\gamma_\mp \mp \omega_c| \gg kV_{Te,Tp}$. Here $c$ is the speed of light in vacuum, $\omega_{p\mp} = (4\pi n_\mp e^2/m)^{1/2}$ is the electron plasma frequency, $n_\mp$ is the sum of the unperturbed and perturbed electron number densities, $e$ is the magnitude of the electron charge, $m$ is the electron rest mass, $\omega_c = eB/mc$ is the electron gyrofrequency, $B$ is the sum of the ambient and perturbed (along $\hat{\mathbf{z}}$) magnetic fields, and $V_{Te}$ ($V_{Tp}$) is the electron (positron) thermal speed. The factor $\gamma_\mp = (1+\nu_\mp^2)^{1/2}$ accounts for the relativistic electron and positron mass increase in the wave field, where $\nu_\mp$ is given by [45–47]

$$\nu_\mp^2 = \frac{\Omega_E^2\gamma_\mp^2}{(\omega\gamma_\mp \mp \omega_c)^2}, \tag{1b}$$

where $\Omega_E = eE/mc$.

Two comments are in order, First, in the non-relativistic limit, we have $\nu_\mp \ll 1$ and one can simply approximate $\gamma_\mp$ by unity. Then (1a) gives

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{2\omega_p^2}{(\omega^2 - \omega_c^2)}, \tag{2a}$$

which for $\omega \approx \omega_c$ and $\omega \ll kc$ yields

$$\omega \approx \frac{kc\omega_c}{(k^2c^2 + 2\omega_p^2)^{1/2}}, \tag{2b}$$

where $\omega_p = \omega_{p-} = \omega_{p+}$. The wave, given by (2b), is referred to as an EMEC wave, which has a negative group dispersion (that is, the rate of change of the group velocity $\partial\omega/\partial k$ with respect to the wave number is negative).

Second, in the ultra-relativistic limit, we have $\gamma_{\mp}^2 \gg 1$, and one can approximate $\gamma_-$ by $(\pm\Omega_E + \omega_c)/\omega$ and $\gamma_+$ by $(\pm\Omega_E - \omega_c)/\omega$. Here, one obtains from (1a) that

$$\frac{k^2c^2}{\omega^2} = 1 \mp \frac{2\omega_p^2}{\omega\omega_E}, \tag{2c}$$

which admits the solution

$$\omega = \frac{2\omega_p^2}{\omega_E}\left(1 + \frac{k^2c^2\omega_E^2}{4\omega_p^4}\right), \tag{2d}$$

with the minus sign in (2c), and a whistler/helicon-like mode [58]

$$\omega = \frac{k^2c^2}{2\omega_p^2}\omega_E, \tag{2e}$$

with the plus sign in (2c) and with $\omega\omega_E \ll 2\omega_p^2$. Both modes, given by (2d) and (2e), have a positive group dispersion.

It is remarkable that the dispersion relation (2c) is independent of the external magnetic field. In addition, (2e) shows that the frequency is proportional to the wave amplitude [45–47]. Furthermore, we observe that in a pair plasma, the whistler/helicon-like mode frequency is half the frequency of ultra-relativistic CPEM waves in a pure electron magnetoplasma [42, 43] without positrons.

A finite amplitude weakly relativistic carrier wave $(\omega_0, k_0)$ interacting with quasi-stationary non-resonant fluctuations (composed of the density and magnetic field perturbations) would create an envelope of the wave. The dynamics of a weakly non-relativistic ($\gamma_{\mp} \approx 1$) envelope is governed by a nonlinear Schrödinger equation [30, 52]

$$i\left(\frac{\partial}{\partial\tau} + V_g\frac{\partial}{\partial z}\right)E + \frac{V_g'}{2}\frac{\partial^2 E}{\partial z^2} - \Delta E = 0, \tag{3}$$

where $\partial E/\partial\tau \ll \omega_0 E, \omega_0 = k_0 c\omega_{c0}/(k_0^2c^2 + 2\omega_{p0}^2)$, $\omega_{c0} = eB_0/mc$, and $\omega_{p0} = (4\pi n_0 e^2/m_e)^{1/2}$. The group velocity and the group dispersion are $V_g = \partial\omega_0/\partial k_0 = 2c\omega_{c0}\omega_{p0}^2/(k_0^2c^2 + 2\omega_{p0}^2)^{3/2}$ and $V_g' = \partial V_g/\partial k_0 = -6\omega_{c0}\omega_{p0}^2 k_0 c^3/(k_0^2c^2 + 2\omega_{p0}^2)^{5/2}$, respectively. The nonlinear frequency shift $\Delta$ for $\gamma_{\mp} \approx 1$ reads [59]

$$\Delta = \frac{V_g\omega_{p0}^2}{k_0c^2}\left[\frac{2\omega_0^2\omega_{c0}^2}{(\omega_0^2-\omega_{c0}^2)^2}\frac{B_1}{B_0} - \frac{e^2\omega_0^2(\omega_0^4 + 6\omega_0^2\omega_{c0}^2 + \omega_{c0}^4)|E|^2}{m^2c^2(\omega_0^2-\omega_{c0}^2)^4}\right], \tag{4}$$

where $B_1$ is the external magnetic field-aligned magnetic field perturbation created by the pressure of the wave envelope due to the inverse Cotton–Mouton effect [60]. The expression for $B_1$ is [59]

$$\frac{B_1}{B_0} = -\frac{4\omega_{p0}^2\omega_{c0}^2|E|^2}{(\omega^2-\omega_{c0}^2)^2B_0^2}. \tag{5}$$

We stress that the contribution of the radiation pressure driven density perturbation to the nonlinear frequency shift (4) is insignificant when $\omega_0 \approx \omega_c$, and can be neglected. One can thus imagine that in a strongly magnetized tenuous plasma with $\omega_c \gg \omega_{p0}$, it is reasonable to postulate that the radiation pressure would generate finite magnetic field fluctuations instead of finite plasma number density perturbations.

Inserting (5) into (3), we have [61]

$$i\left(\frac{\partial}{\partial \tau} + V_{\rm g}\frac{\partial}{\partial z}\right)E + P\frac{\partial^2 E}{\partial z^2} + Q|E|^2 E = 0, \tag{6}$$

where we have denoted the group dispersion coefficient $P = V_{\rm g}'/2$ and the coefficient of the nonlinearity

$$Q = \frac{V_{\rm g}\omega_{\rm p0}^2\omega_0^2\omega_{\rm c0}^2}{B_0^2 k_0 c^2 (\omega_0^2 - \omega_{\rm c0}^2)^4}\left(8\omega_{\rm p0}^2\omega_{\rm c0}^2 + \omega_0^4 + 6\omega_0^2\omega_{\rm c0}^2 + \omega_{\rm c0}^4\right).$$

Since the product $PQ$ is negative, (6) depicts that the modulated wave packet is modulationally stable [62]. The stable wavepacket, however, would propagate in the form of a dark/grey envelope soliton [63], the profile of which in a stationary frame has been displayed in various papers [63–67].

Furthermore, we note that the amplitude modulation of the whistler-like mode in our pair magnetoplasma can be readily investigated following the analysis in [53]. Here, however, one has to account for the ponderomotive force driven density fluctuation and relativistic plasma particle mass increase in the wave field. The results in [53] would apply directly, except that for the present case one has to replace $\omega_{\rm p0}$ by $\sqrt{2}\omega_{\rm p0}$.

To summarize, we have presented an investigation of the amplitude modulation of a weakly relativistic EMEC wave in a strongly magnetized pair plasma, accounting for the radiation pressure driven external magnetic field-aligned quasi-stationary magnetic field perturbation and relativistic electron and positron mass increase in the wave field. It is found that the amplitude modulated wave packet, which has a negative group dispersion, is modulationally stable. A stable nonlinear wave packet can propagate in the form of either a dark or grey envelope soliton. The latter is associated with a magnetic field hole that traps the localized envelope having an insignificant electric field at the center of the envelope. In conclusion, we stress that the wave energy transport could occur via a dark or grey envelope soliton in strongly magnetized pair plasmas, such as those in pulsars and magnetars where the electron gyrofrequency is much higher than the electron plasma frequency.

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