Jean Cardinet

ANALYSIS OF INTELLECTUAL PERFORMANCE

Everyone knows of the existence of intelligence tests. This is, in fact, the only aspect of applied psychology that is familiar to the general public. However, not so many people know that these tests have been the object of protracted studies and are integrated into a highly mathematized conceptual system. Our intention is to give a summary outline of this area of research which attempts to analyze man's performance, particularly in the area of the intellect.

The normal process of thought is to go from the concrete to the abstract. Therefore it seems natural to base this outline on an actual example of research carried out in the domain of intelligence and motor co-ordination tests and to use these as a point of departure for more general principles. In the first place, the material used—the actual tests—has a history that is in itself revealing. Second, the method of analysis employed is also deserving of study because it implies a certain number of postulates that must be understood in order to interpret the results it makes possible. Let us familiarize ourselves, then, with a typical factor analysis like the one made at the University of Neuchâtel in 1956.[1]

Translated by Elaine P. Halperin.

1. J. P. Boss, "Recherches sur la validation de la batterie d'orientation professionelle G.A.T.B.," *Revue suisse de psychologie* (in press).

https://doi.org/10.1177/039219215700502007 Published online by Cambridge University Press

A total of 330 fifteen-year-old boys attending primary and secondary schools and hailing from both urban and rural areas participated in this experiment. They took the fifteen tests that make up the well-known General Aptitude Test Battery, perfected by the United States Employment Service. These tests were translated into French and adapted by the Institut de Psychologie of Neuchâtel. They included intelligence tests of arithmetic reasoning, knowledge of vocabulary, numerical computation, perception of relationships in space, the matching of geometrical forms, and a comparison of lists with an eye to detecting divergences. There were also manual tests: precise drawings with a pencil, assembling small mechanical pieces, and rearranging of pegs.

Let us examine, for illustrative purposes, examples of the questions on some of these tests:

1. *Arithmetical reasoning*

It takes a half-hour to complete a task. How many times can one accomplish the same task in eight hours? *Answer:* 16 times.

2. *Knowledge of vocabulary:*

Indicate the two words that have either the same or the opposite meaning: (*a*) great; (*b*) vast; (*c*) dry; (*d*) slow. *Answer:* "great" and "vast."

3. *Perception of relations in space:*

On the left is a plane geometric figure and on the right are four drawings in perspective, one of which represents the volume which the figure on the left executes in pliant congruity. Check that figure.

On the left, a star; on the right, these figures: (*a*) pyramid; (*b*) cube; (*c*) truncated cone; (*d*) cylinder. *Answer:* pyramid.

4. *Matching of geometric forms:*

At the top of the page appears a large number of complex geometric figures. The same figures are reproduced at the bottom, but in disorder and in any position whatsoever. Locate in the bottom group each figure that appears at the top.

5. *Comparison of lists:*

Two lists of names are written side by side. Tell whether the names are the same or different.

(*a*)	(*b*)	*Answer*
Barron, S. A.	Barron, S. A.	The same
Henri Stenier	Henri Steiner	Different
Meunier, J. P.	Meunier, J. F.	Different

6. *Series to complete:*

Find the letter which completes a series. The law of succession is different each time.

CBACBAC. . . . . B
AZBYCXD. . . . . W
CBAFEDI. . . . . H

The fifteen scores obtained from each boy were correlated in an effort to determine in what measure the classification of the subjects in one test resembled their classification in each of the other tests. A statistical index, the coefficient of correlation, expresses the degree of resemblance. In all, a table of 105 coefficients of correlation was thus computed. From that table four factors were extracted (the significance of the word "factor" will be explained later) which presented in a condensed form all the information revealed by the correlations. The interrelationship of the tests could thus be attributed to four underlying "aptitudes":

1. Scholastic: relating to verbal tests and mental calculation

2. Technical: enabling one to perceive quickly the characteristic aspects of objects or of their appearance and to analyze geometric forms on a plane surface or in space

3 and 4. Motor: the first relating to movement in the pencil drawings and the second to the manipulation of objects

Here, in a few words, we have a summary of the main points of a theoretical study on mental tests that required a year of work. The uninitiated will perhaps be surprised at the extent of the effort involved. Furthermore, because he is ignorant of the context of the research, the amount of information he will derive from it will be very slight indeed. Therefore, we shall now attempt to review the various aspects of such a study in order to evaluate its contribution in terms of our earlier knowledge, to discover in what direction we can expect these studies to develop, and to suggest what conception we can have at present in regard to individual differences in intellectual functioning.

I. TESTS OF PERFORMANCE

The tests chosen for this study are the result of a long evolution. It was only at the beginning of the present century that the first socially useful intelligence test was devised—the Binet-Simon. Doubtless a certain number of tests had already been employed during the nineteenth century, based on diverse theories about intelligence, but what stood out was their lack of relationship to intelligence—something universally assumed yet so difficult to discern. The Binet-Simon, instead of using ready-made ideas as a point of departure, attempted for the first time to conform to experimental realities. From 1905 on, Binet selected a certain number of tests on the basis of three criteria. They had to take into account the age of the children, differentiating between the oldest and the youngest; correspond to their scholastic achievements, differentiating between students with the highest grades and the rest; and, finally, be in accord with the spontaneous judgment of people who knew the child in his family setting. These tests were for the most part drawn from everyday situations. It became quickly apparent that they were altogether superior to hitherto available devices for evaluating intelligence.

This experimental attitude and orientation in regard to the problems of daily life characterized the second phase of intelligence tests. These were modeled as closely as possible on situations relating to professional or school life. For example, tests given to bus-drivers required them to react simultaneously to very varied stimuli, while those given to office workers involved arithmetic, classification, a knowledge of spelling, etc. This method produced good, practical results which made possible sound predictions about the future performance of the examinees. But protracted research on the validation of tests for each category of work was needed, and this limited their applicability to large firms, where jobs were numerous enough to make statistical studies possible. On the other hand, this method also led to an abundance of tests, because each new situation required at least one new test. Thousands, therefore, were published before 1930, although nobody, not even their authors, knew what they measured but only that they had some special prognostic value.

Obviously, some simplification was necessary. The systematic studies of validation for the selection of tests having proved impractical in many situations, it was now up to factor analysis to select tests that measured a certain type of behavior. All these profusely improvised tests had, necessarily, many points in common. The same basic capacities, although varying in degree, were required in most of them. Factor analysis, invented by Spearman and generalized by Thurstone, made it possible to determine experimentally the "fundamental aptitudes," the "dimensions of the mind," as well as to choose the best tests for each dimension.

The study which we have selected by way of illustration utilized a factorial battery. The tests chosen were those that showed the maximum relationship to certain of the principal factors of intelligence. They attempted to measure ten areas. These areas were, on the specific mental level, reasoning, verbal comprehension, numerical aptitude, spatial visualization in two or three dimensions, perception of forms, and speed of perception; and, on the level of motor co-ordination, accuracy of aim, motor rapidity, dexterity of fingers, and manual skill.

The advantage of these factorial tests is, in the first place, that we know what they measure. Nothing is more difficult than to apprehend subjectively the function required for a specific mental task. For example, many examinations that were supposed to measure reasoning proved to be tests of verbal comprehension only because the instructions were difficult to understand. Inversely, tests designed to measure the comprehension of difficult texts provided evidence of the reasoning necessary to deduce the true meaning of the words employed. Only factor analysis made it possible to relate individual differences in the showing of the examinees to the known sources of variation.

The second advantage of these tests is that, with the help of a relatively small number of questions, they enable us to range over the same area covered by almost all the earlier tests.

Finally, the analytical character of such tests makes it possible to subdivide mental performance into rather narrow and well-defined categories. This in turn enables us to establish profiles for each examinee; and, since it is likewise possible to fix the requisite profiles for each category of endeavor, rational vocational guidance becomes conceivable. Instead of employing a different test for each profession, the same ones can be used for all occupations, provided only that the coefficients of importance assigned to each test are modified according to the profession in question.

The reader may quite legitimately be intrigued by these diverse applications of the statistical method, which is itself a mystery to him. Therefore this is perhaps the place to attempt an explanation of the elements and postulates of factor analysis. Such an explanation is necessary in order to appreciate the actual evolution of these investigations.

II. FACTOR ANALYSIS

Essentially, factor analysis is a method that enables us to express in simple form the relations that exist between the various tests.

The basic experimental phenomenon is the similarity that at times exists between the showings of a same group of examinees in different tests. Those that perform the best in one test sometimes do likewise in another. This kind of similarity between two tests is expressed statistically by a coefficient of correlation that is so much the greater as the relationship between the two examinations is closer, as the classification of the examinees in the two tests is more alike.

If each of these two tests shows as well a marked correlation with a third, then we can see the beginnings of a grouping of tests, and it is natural to presuppose the existence of a source of common variation. We can postulate a same underlying aptitude for the three tests which would explain the uniformly good or bad performance of those taking all of them. Since the concept of aptitude raises many theoretical questions, we shall confine ourselves to speaking of a "factor" common to the three tests. Factor analysis is the search for these sources of hypothetical variation, or factors.

It proceeds, therefore, from a table of coefficients of correlation interpreting the relationships of each test with all the others, which is what we call a "table of correlations." It attempts to determine quantitatively the number of common factors present in the makeup of the table and the influence which each of these factors exerts in each of the tests.

A geometrical picture of the table of correlations is possible. For example, tests that measure only three different aptitudes could be portrayed in three dimensions, perhaps by pins inserted into a small ball. The tests that have a high degree of correlation would then be placed quite close to each other. Tests corresponding to different aptitudes would, on the contrary, be placed at right angles. In the optimum case of tests which measure only one of the three factors, the geometrical picture would show three plainly distinct groups of pins. Each group would form a slender bunch (corresponding to tests that measure the same factor), and the three bunches would be at right angles to the others (indicating the independence of the three corresponding aptitudes).

The two ways of interpreting the connection between the tests—algebraic, on the one hand (table of correlations), and geometrical, on the other (configuration of vectors)—are both equally valid. Therefore we will use the geometrical pattern, which is more comprehensible, although in actual practice correlations are used.

The determination of factors is generally worked out in two stages. First we determine how many dimensions are necessary to interpret in a satisfactory way the relations perceived experimentally between the tests. If all the pins lie in the same plane, like the segments of an open fan, then two dimensions are sufficient to characterize the factors common to the test battery. If all the pins are grouped together like a closed fan, a single dimension exists; there is but one common factor. Frequently, on the contrary, three dimensions are not sufficient, and the geometrical pattern can no longer be materially represented, although it preserves all its meaning from the mathematical point of view in which spaces of N dimensions are perfectly conceivable.

Once the number of dimensions has been discovered, the exact position of the factors must be determined by means of an appropriate "rotation of axes."

From the point of view of the geometrical model, the factors constitute the axes of coordinates to which the tests are related. In the table of correlations each test was related to all the others. Thanks to factor analysis, the same information is presented in a far more concise manner. Knowing the projections of each test upon the small number of reference axes (or, in other words, knowing the coordinates of each test in relation to the factors) enables us to reconstruct the configuration being studied.

This summarized formulation of our knowledge is, in itself, a scientific advance; yet it does not entirely satisfy the psychologist. From the mathematical point of view, to be sure, it is possible to describe the same configuration from the standpoint of very different axes of reference, because all these diverse descriptions illustrate the relationships between the tests. However, the psychological significance of factors varies according to whether the axes of corresponding references are situated closer to some tests than to others. If an axis is fixed in the middle of a group of vocabulary tests, this would seem to be evidence of an aptitude for verbal comprehension. But if a different rotation brings it closer to tests of logic, then we must speak of an aptitude for verbal reasoning, the significance of which is obviously quite different. Still another rotation might indicate an aptitude for verbal fluency and so on. The number of possible positions for axes of reference is unlimited.

Descriptive factors therefore might be chosen in an arbitrary way, on the basis of the psychological theories of whoever is making the study, and it is quite clear that we can scarcely attribute any scientific value to entities that are solely the result of the experimenter's free choice.

It was Thurstone who suggested a criterion for resolving this indeterminism. Just as the number of factors required to account for the correlations must be as small as possible, so the position of axes of reference must also be such that each test is related only to a minimum number of factors. This principle of "simple structure" is thus an extension of the scientific principle of parsimony that holds that a hypothesis must not be made of many variables when a single one is enough to explain the facts. The ideal of the simple structure is that each test should correspond only to a single factor—should measure only a single aspect of behavior at a time. This would be the culmination of the analysis of mental activity and the prolongation of the theory of faculties but, this time, with a solid experimental foundation and a powerful mathematical model to justify its scientific status.

To summarize, factor analysis is a means of expressing, in condensed form, the information contained in a table of correlations. It proceeds by two stages: (1) the extraction of factors which determines the number of dimensions necessary to account for the correlations, but provides an entirely arbitrary system of orthogonal reference axes, and (2) the rotation of axes, thanks to which these axes of reference are situated in order to allow the simplest possible description of the total configuration.

Having thus outlined the method and its principles, we must now see to what extent it satisfied the hopes that were reposed in it.

III. RESULTS OF FACTOR ANALYSIS

From a practical point of view, the first positive result was to set up some order among the existing tests. Some factors were found again and again in successive factor analyses, like the numerical factor or the factor of verbal comprehension. From then on a small number of tests was enough to measure them. Their definition having become clear, it was possible to foresee their appearance in new tests. Now, other tests could be purified of their influence and directed toward measuring various aspects of behavior, thus methodically enlarging our knowledge of different areas of intellectual activity.

From the point of view of psychotechnical applicability, factors constitute the equivalent of the preceding epoch's criterion for professional success, in the sense that they represent a reference for professional behavior as well as for performance in tests. The evaluation of professional success is introduced in the battery of tests on the same level as the other variables. Factor analysis enables us to ascertain the psychological exigencies of work in terms of factors. The search for new, more valid tests can be guided by these results.

But the ambitions of factorialists embrace essentially the theoretical level. Many psychologists believe that from now on a "map of the mind" can be established. Guilford, basing his opinion mainly upon his own works, thinks that the factors correspond to the intersection of three principles of classification: (1) the mental function, in the broad sense of the word, like memory, discovery, production, symbolization, and divergent

https://doi.org/10.1177/039219215700502007 Published online by Cambridge University Press

thought; (2) the material that the test embraces, which can be perceptive, verbal, or relational; and (3) the type of problem or content of the test, depending upon whether it involves analogies to be found, series to complete, relations to educe, etc.[2]

In 1952 French published a much broader compilation of the principal factorial studies made up to that time.[3] Regrouping to the best of his ability the findings presented in these various studies, he stressed the following factors: attention, deduction, ideational fluency, flexibility of perceptual closure, speed of perceptual closure, induction, judgment, rote memory, mechanical experience, numerical aptitude, speed of perception, spatial aptitude, verbal comprehension, visualization of movements in space, verbal fluency, and visual memory. Each of these is precisely defined by the tests over which he exerted a repeated influence in many studies.

We cannot fail to observe the marked similarity between this list and the list of factors measured by the battery of the United States Employment Service, which we mentioned earlier as an example. This is typical of the contemporary psychological practice which tends to stabilize itself and to hold that the exploration of the dimensions of the mind has almost been completed. In fact, the great majority of existing intelligence tests stems from one or another of these fifteen or so factors.

But, as always in science, no sooner has a theory been accepted than new facts are presented to discredit it and to pave the way for a fresh creative upheaval. It would be an error to believe that the problem of the dimensions of the mind has been resolved. Great difficulties arise, in effect, if the oversimplified doctrine is held that a small number of unitary aptitudes is responsible for the totality of manifestations of intellectual activity.

Let us return to our first example, the General Aptitude Test Battery. Analysis makes it possible to extract only four factors from all the fifteen tests. After rotation, a simple, very clear structure appeared, the interpretation of which has already been given. It included only two specifically intellectual factors, one relating to scholarly abilities and the other to technical ones. However, prior factorial analyses of broader batteries which included the same tests had revealed that the latter ranged over at least six intellectual factors. Here they seem to have been fused.

Is this due to error or to mere chance? Not at all. Other examples could

2. J. P. Guilford, *L'Analyse factorielle et ses applications* (Paris: Éditions du Centre National de la Recherche Scientifique, 1955).

3. J. W. French, *The Description of Aptitude and Achievement Tests in Terms of Rotated Factors* ("Psychometric Monographs," No. 5 [Chicago: University of Chicago Press, 1952]).

be cited of the fact that, from one study to another, the same tests are found, saturated in factors of very different generality. The phenomenon of the fission of factors is evidently the corollary of the phenomenon of their fusion. If we start with the definition of a factor and create a large number of tests that correspond with it, the factor analysis of this battery, instead of disclosing a single dimension, as was to be expected, reveals a number of orthogonal dimensions. A new analysis of these subfactors would lead to an even more pronounced splintering. This is what happened to the factor of reasoning, among others, which was successively subdivided into twenty-odd subfactors.

Doubtless the defenders of the unicity of factors would maintain that factors of a more general level appear only when the number of tests is too small to define each of the stricter factors but that it is the latter which can be defined with the greatest validity. Unfortunately, it seems that even the narrower factors can still be subdivided and that therefore no definition of "basic aptitudes" will ever be possible.

To this difficulty we must add the indeterminism of the axes' position and the possibility of different rotations as regards the same data. And it is all the more difficult, naturally, to discover the same factors when we start from different factorial analyses. The principle of rotations itself is not universally accepted by all researchers.

Under the circumstances it is not surprising that the factor analyses made everywhere should have led to the definition of an ever increasing number of factors, surely amounting to more than a thousand if we are to take into account all the published results. This proliferation of factors since 1930 reminds us curiously enough, and ironically, of the abundance of tests before this date. A new principle of order must be found for the factors themselves. Researchers like Guilford and French found it in a very rigid selection of studies which they believed to be sound, but the arbitrariness of this method lends itself to criticism. Moreover, to use mere judgment in order to sort out and reclassify diverse factors under a same rubric reminds us of the efforts at a logical classification of tests made before the advent of factor analysis and whose value proved to be doubtful.

In order to avoid this kind of arbitrariness, many psychologists have attempted to establish a relationship between the various factorial studies. Ahmavaara's transformation analysis represents a promising solution.

IV. AHMAVAARA'S TRANSFORMATION ANALYSIS

In order to make the principle of this method clear in a few words, we must remember that, if the tests (vectors) can be situated in space in relation to factors (axes of reference), it is just as possible to situate factors in relation to the tests. When two factor analyses join in utilizing certain tests, it becomes possible to compare the position of the factors in one study to that which they occupy in the other. This method was first suggested by Ahmavaara in 1954,[4] and in a second volume, which appeared in 1957,[5] he gave a rather large number of examples. From these several important conclusions can be deduced.

First of all, it is possible to view the multidimensional space of the various factorial researches as forming a whole, with each of them embracing only one limited aspect. Just as all the tests improvised at the beginning of the century were concerned with the sources of common variation that factor analysis was expected to break down, so the factors discovered thereafter can be resituated within a common frame of reference. The inherent error of these complicated mathematical transformations can be ascertained, and we can make sure that it remains within acceptable limits.

Consequently, it is possible to compare the factors found in various studies and to evaluate their degree of similarity. From a group of studies unfortunately still too small, Ahmavaara was able to show that there was satisfactory agreement as regards the following factors (in the order of decreasing invariance): numerical aptitude, word fluency, verbal comprehension, visualization of the relations between several parts of a configuration, perceptual closure, deduction, and speed of perception. We must hope that this study will be enlarged and that it will enable us to put order into the multitude of factorial results already published in the intellectual domain.

One fact perhaps even more interesting than this simple classification of the dimensions of the mind is that the method provides proof of the fission and fusion of the factors that we have already mentioned. For example, the factor of reasoning, treated as indivisible in one study, is separated into three parts in another. Although fission is still denied by certain factoralists, it is of great importance because it allows us to affirm the existence of a hierarchy among the factors of the mind. In contrast to a uniquely horizontal classification of independent factors all possessing the same status, a vertical classification likewise appears to hold greater promise for an explanation of the relations between the various factors discovered up to date. It opens up interesting perspectives of synthesis between the two

4. Y. Ahmavaara, *Transformation Analysis of Factorial Data* ("Annales Akademiae Scientiarum Fennicae," Ser. B, Vol. LXXXVIII, No. 2 [Helsinki, 1954]).

5. Y. Ahmavaara, *On the Unified Factor Theory of Mind* ("Annales Akademiae Scientiarum Fennicae," Ser. B., Vol. CVI [Helsinki, 1957]).

important current trends in factor analysis, the English, which has always been attached to a hierarchical concept dominated by the factor *g* of general intelligence, and the American, to which Ahmavaara adheres.

V. A CONCEPTION OF INTELLIGENCE

Two remarks of a general nature are perhaps useful in approaching this final phase of the discussion. First of all we must remember that the word "intelligence" is not used here in its broad sense, which includes the faculty of judging or the function of adaptation. We are not attempting to define how man thinks, which would be a problem for general psychology, but rather what constitutes the difference between the ways various individuals think. The two questions are in part independent of each other, and, in any case, we can logically differentiate between them. In the one instance, the laws of thought, which, by definition, apply to all men in the same way, are involved. The German *Gestalt Psychologie* has contributed a great deal in this area. In the other instance, individual differences between men are examined. Because of its social philosophy America has shown the greatest interest in this problem.

Furthermore, we must understand that intelligence does not exist; only intelligent behavior exists. "Intelligence" is but a convenient word that embraces but does not explain a totality of behavioral characteristics. Two attitudes are possible in the face of this kind of concept: either to postulate the existence in each individual of a totality of characteristics that one presupposes to be amenable in theory to a direct approach (the weight of the brain in relation to that of the body, for example) or to be satisfied with a conception of intelligence as a variable in a formula, like acceleration in mechanics—something that has no reality in itself but merely participates in the law that links the reaction of the organism to the stimulus with which it is provided. It is clear that factors correspond to this second logical status. They have no need of a physiological or underlying genetic reality in order to justify their value. It might very well be that they correspond only to the statistical effect of numerous and complex causes. This problem is entirely unrelated to their scientific importance, which resides in the explicit simplification which they introduce into the extraordinarily complex domain of individual differences in behavior.

Even if we must not attribute a physical reality to them, factors nonetheless must, in order to be useful, describe the behavior to which they are related in a stable and unvarying way. When factor analysis began, doubts were expressed on the subject. Ahmavaara's contribution seems to con-

https://doi.org/10.1177/039219215700502007 Published online by Cambridge University Press

firm Thurstone's confidence in the invariability of configuration, in a stability of the saturations of tests in factors independently of the sampling of examinees or of the tests that happened to be chosen.

Yet at the same time factorial invariability assumes a less oversimplified meaning following Ahmavaara's studies than it did among the founders of factor analysis. A hierarchical structuration must be accepted. Depending upon the number of tests introduced into the battery, the factors are rearranged or divided. The example chosen in the beginning illustrates the following fact: only two common factors appear in a battery that was planned to include more. If the selection of tests had been somewhat different, it might even have been possible to discover only one factor—the general factor to which Spearman sought to restrict the dimensionality of the mind. The two dimensions found correspond to the two most important poles, apart from the general factor, the scholarly pole and the practical pole, or the pole of non-verbal intelligence. There is no doubt that, if the number of tests had been a little larger, these two factors would have been replaced in the first instance by the verbal and numerical factors and by the spatial and perceptive in the second. With an even greater choice of tests, the factors that served to establish this battery would have been discovered; with an increase in the number of tests, more and stricter factors would have appeared. Ahmavaara's transformation analysis shows that at every level a factor, in its geometrical pattern, is fixed in the center of diverse factors among which it will be subdivided if the number of tests is increased. Thus, each factor must be considered as a regrouping of narrower factors until we reach the general factor, which covers all the particular individual abilities.

In short, everything occurs as if a factor analysis were a more or less powerful microscope which could reveal a crystalline structure at various levels, every crystal being composed of narrower, smaller crystals but of the same form. Should we acknowledge that the number of factors is a statistical function of the number of tests because of the sampling of tests which can never be completely controlled? Then factor analysis would be but the projection of a space with a large number of dimensions on a basis whose number of dimensions would be determined by the number of tests selected. Real dimensionality would consequently lose all scientific meaning. Only the degree of generality chosen for description would determine this dimensionality, somewhat in the way we regulate an enlargement of the microscope by the lens we use. This seductive conception does not seem as yet to have a solid statistical foundation.

If we refuse to deny the significance of the number of dimensions, we are forced to acknowledge the constant increase in the number of factors whenever a particular area of performance is examined in greater detail. Therefore we tend to draw closer to Thomson's theory. He believed that factors were the statistical result of the action of a great many links between some very narrow aspects of tasks.

Up to the present this theory has not been accepted because it clashes with the principle of parsimony, but we are forced to acknowledge the equivalent arbitrariness of the decision always to minimize the number of dimensions. It seems probable that the number of factors which intervene in any task is much larger than the small number of common factors that are evident in each battery.

But if the number of postulated factors increases, we must admit that their individual influence is reduced proportionately. In this way we arrive at the conception of a very large number of factors, very limited and of slight influence which, by addition, would constitute the hierarchical pyramid of known factors, as total space would be projected over a more and more limited number of dimensions.

Such is the general idea that we can have today of the interindividual sources of variation in intellectual performance. The first theories about intelligence, mainly philosophical elaborations, could not help us to understand these individual differences. Only recourse to experience, by tests that were first validated and later factorialized, and the reconciliation of these factors themselves by transformation analysis, enable us to establish a certain order in this complex area.

If we may venture to formulate a judgment about an evolution that is still uncertain, it might be said that the conception of the dimensions of intellectual performance is oriented toward a greater structuration. Horizontally, factors of the same degree of generality begin to become classified in a predictable fashion in a tridimensional pattern, according to the psychological function at play, the material employed, and the form of the problem raised. Vertically, hierarchical relations appear, leading to a pyramidal organization which would constitute the synthesis of the theories of the four great pioneers of factor analysis: Spearman, who stressed the primacy of the factor of general intelligence; Burt, who developed the hierarchical theory of factors; Thomson, who formulated the hypothesis of the sampling of relationships as the explanation of factors; and Thurstone, who provided the mathematical tool of multiple factor analysis and the essential methodological principle of simple structure.