

as a readable (with some caveats, as above) and refreshingly detailed account of the whole sweep of ‘infinitesimal methods’ from antiquity to the 1990s, this book is highly recommended.

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**Complex analysis** by Andrei Bourchtein and Ludmila Bourchtein, pp. 346, £54.99 (paper), ISBN 978-9-91159-221-8, Springer Verlag (2021)

There being no shortage of textbooks on complex analysis, some justification is needed to add another one to what has been a crowded market for many years. The preface of this one states that it “covers all the traditional topics of complex analysis, starting from the very beginning—the definition and properties of complex numbers, ... —and ending with properties of conformal mappings, including the proofs of the fundamental results of conformal transformations.” There are only five chapters, with the titles: 1. Introduction; 2. Analytic functions and their properties; 3. Singular points, Laurent series, and residues; 4. Conformal mappings; elementary functions; 5. Fundamental principles of conformal mappings; transformation of polygons.

Since it is meant to start from the beginning, it is peculiar that there is no mention of the triangle inequality for complex numbers at all in the whole book—it is indispensable, of course, but it is just applied throughout without any comment. To paraphrase Littlewood: *Equations are just balancing acts, but inequalities are something else, because ...* Some of the familiar terms are defined in unusual ways; for example, on page 12, the notion of a curve is made difficult by not including continuity in the definition, so that it is necessary to say a ‘continuous curve’, whereas a ‘simple and closed curve’ is presumed to be continuous. Also, instead of its usual definition, the residue of  $f(z)$  at  $z_0$  is defined by the value of the integral

$$\frac{1}{2\pi i} \int_{|z-z_0|=\rho} f(z) dz.$$

A lengthy remark is then required on the relevance of the unquantified number  $\rho$ , leaving the feeling of having ‘the cart before the horse’ for most readers.

The usual material associated with the topics in Chapter 3 are dealt with thoroughly, and the properties of elementary functions are covered in some detail in Chapter 4, which begins with the geometric interpretation of the derivative and the notion of a conformal mapping. Thus, linear and fractional linear functions, power and root functions, exponential and logarithm functions, the basic trigonometric functions and the Joukowski function are covered. Analytic continuation and the specification of domains of conformality, and the consideration of inverses, including multi-valued functions and the construction of the corresponding Riemann surfaces, are included in the chapter. Proofs of the main results on conformal mappings are provided in the last chapter. For the Riemann mapping theorem, the authors choose the modern approach based on the calculus of variations, and the proof is essentially that by F. Riesz and L. Fejér using Montel’s theory of normal families of functions. This is fine for a reader already familiar with function theory, otherwise some background material, such as that in [1], will be necessary. The chapter also includes Schwarz’s lemma, the Schwarz reflection principle, and the Schwarz-Christoffel mapping.

The Index seems rather inadequate, because many terms mentioned in the text, such as Green's formula, do not have entries. Indeed, entries corresponding to several leading letters have only single entries—for example, 'Exterior point' is the only entry for E. The book does not compare well with many of the items in its Bibliography.

#### Reference

1. J. L. Walsh, History of the Riemann Mapping Theorem, *Amer. Math. Monthly* **80** (1973) pp. 270-276

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**The best writing on mathematics 2021** by Mircea Pitici (ed.), pp. 288, £20.00, ISBN 978-0-691-22570-8, Princeton University Press (2022)

Readers of *The Mathematical Gazette* will be pleased to see Mike Askew's 2020 Presidential Address in this 'compendium of good mathematical writing'. It is a plea for raising the profile of reasoning, rather than procedural competence, in mathematics education. In his exposition, Askew explores what he calls 'the language of mathematical reasoning', distinguishing between types such as deductive, inductive, abductive (if you want to know what than means you will need to read the article), analogical and relational. While this appeal is very welcome, it is rather a pity that it is necessary, and I get the impression that it would be taken for granted in mathematical education in many countries outside the UK.

Roger Howe's essay on 'knowing and teaching elementary mathematics' covers similar ground, comparing the mathematical understanding of US and Chinese teachers. He concludes that his own country has a good way to go before it can even compete with its rival. Even at the elementary level, most of the Chinese teachers are specialists who can concentrate on their subject and spend significant time outside the classroom working with individual students and sharing ideas with their colleagues. This is underscored by comparing the PISA results in the two countries.

Moving from school to university, there are some thought-provoking essays on how to organise research. Stephan Garcia compiles a list of 24 recommendations for academic supervisors, ranging from 'Pivot when returns diminish' to 'Turn lemons into lemonade'. A short but piquant essay by Melvyn Nathanson asks if you can 'own' a mathematical truth, and thereby prevent others from using it in their own research, even with acknowledgment, before they have published it. Terence Tao confesses to almost flunking an exam at Princeton by inadequate revision of the topics which he had chosen. This, apparently, is a significant source of comfort to the more recent graduate students at the college.

The earlier section of the compendium contains the usual assortment of interesting topics, many of which explore the relevance of mathematics to other disciplines, and in particular architecture and design. There is a fascinating discussion by Michael Duddy which focuses on the clash between logic, intellect and truth on the one side and intuition, feeling and beauty on the other. Specifically, he surveys the work of Palladio in Renaissance Florence which necessitates compromises when 'turning the corner' of an Ionic portico. A related article by Steve Pomerantz explores the work of the Cosmati family of marble workers in Rome who produced intricate non-regular tilings of hexagons, triangles and rhombi.