linear groups) is a comparatively new area of research. The subject might be considered to have started in the 1950s with the classification of finite subgroups of division rings by Herstein and Amitsur, then foundations of a general theory were laid by Zalesskii in 1967. However, only in the last ten years have many major results on skew linear groups appeared. The aim of the present book is to allow the subject to "come of age" by presenting a systematic development. The authors have succeeded admirably and this book will provide both a vital work for new researchers in the area and act as a standard reference work.

The authors begin by showing how familiar ideas of irreducibility, absolute irreducibility and unipotence extend from the commutative setting to the non-commutative case. These results are fundamental to the rest of the book yet some are from 1983 and 1984 papers showing how new is the subject matter of the book. General methods are given for constructing examples of skew linear groups using Ore domains and Goldie's theorem. The finite subgroups of division rings (both zero and non-zero characteristic) are classified in the second chapter which also deals with finite and locally finite skew linear groups.

The third chapter considers skew linear groups over a division ring D which is a locally finite-dimensional division algebra over a field F. Properties of skew linear groups over D are related to properties of linear groups over F. Chapter 4 studies skew linear groups over division rings D of the following type: $D=F(G)$ where F is a central subfield of D, G is a polycyclic-by-finite subgroup of D and D is generated as a division ring by F and G. This chapter requires the reader to know quite a lot about polycyclic groups and, in particular, about their group algebras. Generalisations are given of Mal'cev's theorem showing that a finitely generated linear group is residually finite and of results of Wehrfritz on residually nilpotent subgroups of finite index in linear groups. Evidence is given to suggest that the Tits alternative for finitely generated linear groups (which does not extend to skew linear groups in general) may extend to the class of skew linear groups considered in this chapter.

The main sections of Chapter 5 deal with locally finite normal subgroups and soluble normal subgroups of skew linear groups. Stronger results are obtained in the absolutely irreducible case. The shortness of the final chapter on applications shows that, as yet, the theory has found few applications but given the recent nature of the work this is not too surprising.

An unavoidable weakness of the book is the wide demand made on the background knowledge of the reader: finite group theory, representation theory, number theory, the theory of soluble and nilpotent linear groups, group algebras over polycyclic groups and results from ring theory. The authors, aware of these difficulties, have provided very full references and this is a valuable feature of this useful book.

E. F. ROBERTSON

Pietsch, A. *Eigenvalues and s-numbers* (Cambridge studies in advanced mathematics 13, Cambridge University Press, 1987), pp. 360, 0 521 32532 3, £35.

This book is a detailed account of a subject that has developed rapidly in the last ten years. The author is well qualified to write about it, since he has been one of the principal contributors to this development.

The notion of "s-numbers" of an operator is designed to incorporate the approximation, Gelfand and Kolmogorov numbers; further examples are Weyl, Chang and Hilbert numbers. The emphasis here is on quasi-norms formed from the various sequences of s-numbers, and on their relationship with the summing norms. For operators between Hilbert spaces, the s-numbers all coincide, and $(\sum a_n(T)^2)^{1/2}$ equals the 2-summing norm $\pi_2(T)$. Various results of this type have been proved for operators between Banach spaces, often involving mixed summing norms of the type $\pi_{p,2}(T)$. Chapter 2 of the book describes the state of the art in this area, including a number of specific calculations for diagonal operators between sequence spaces.

The starting point for eigenvalue theorems is the theorem of Weyl (1949) that for operators between Hilbert spaces, $\sum|\lambda_k(T)|^r \leq \sum a_k(T)^r$ for any $r>0$. In 1978, König showed that the same

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holds, with an intervening constant, for operators between Banach spaces. Chapter 3 gives an account of this result, together with further refinements and connections with r-summing norms.

The next topics considered are the notions of trace and determinant. The fact that there are Banach spaces without the approximation property amounts to saying that the obvious way to try to define trace on the class of nuclear operators does not work. In fact, it can be shown that there is no continuous function with the properties of a "trace" on this class. In recent years, the author and others have shown that it is possible to define a trace on certain smaller operator ideals described in terms of s-numbers or eigenvalues. Somewhat similar considerations apply to determinants.

Chapters 5 and 6 are devoted to showing how the abstract machinery developed earlier can be applied to "concrete" matrix and integral operators. This has its origins in Schur's theorem of 1909, but again there has been a spate of results in the last few years. The general idea is to show how smoothness properties of a kernel translate into convergence properties of the eigenvalues. Results of this kind are described first for matrices, particularly of "Hille-Tamarkin" and "Besov" type, and analogous results are then presented for integral operators.

Finally, the author devotes a complete chapter to a historical survey. This is conducted with characteristic thoroughness, starting from Gauss and Cauchy. To test the reader's level of cultural attainment, there are extended quotations in six different languages.

In order to keep the volume self-contained, there is a certain amount of overlap with the author's earlier book *Operator Ideals.* In particular, there is an opening chapter on r-summing and (r,s)-summing operators. Before this, there is a preliminary section setting out the prerequisites. These amount mostly to fairly basic Banach space theory, except that it will be daunting for some to discover that familiarity with interpolation theory is assumed. Readers who do not possess this familiarity will in fact find that they can manage without it most of the time, and in some cases the language of "interpolation couples" could be replaced by the ordinary Hölder inequality.

This book is the first unified account of a new and exciting subject, and it contains a wealth of information for the reader who is sufficiently determined. However, it must be said that a high degree of determination is needed. One useful concession is made to readers who would like to settle for something less than the entire book: the sections pertinent to the most important results are identified. This apart, the book is not one to read casually or dip into. The material is densely packed, with a minimum of motivation and discussion. For the reviewer at least, it would be helpful if new definitions were accompanied by a few simple examples. The author relies uncompromisingly on fixed notations introduced somewhere in the book: typically, these are gothic characters surrounded by superscripts and subscripts, and the list of them runs to three double-column pages. None of this makes for easy reading. However, the primary purpose of the book is to give a full and thorough treatment of the subject area, and in this it has undoubtedly succeeded. It will surely become a standard reference.

G. J. O. JAMESON

SUNDER, V. S., *An invitation to von Neumann algebras* (Universitext, Springer-Verlag, Berlin–Heidelberg–New York, 1986), xiv + 171 pp., 3 540 96356 1, DM 68.

Rings of (Hilbert space) operators, later to become known as von Neumann algebras, were subjected to a deep and penetrating analysis by Murray and von Neumann in a series of papers from 1936 to 1943. They concentrated on the so-called factors (algebras with trivial centre), which may be regarded, via a direct integral theory, as the building blocks for more general von Neumann algebras. Factors were classified into types I_n $(1 \leq n < \infty)$, I_∞, II_1, II_∞ and III, and constructions for examples of each type were given. However, it was known that the classification was not fine enough to detect $*$-isomorphism, and it was realized that the level of complexity and mystery was an increasing function of the Roman numeral!

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