SOME ASPECTS ON REINSURANCE PROFITS AND LOADINGS

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Introduction

In a note on the security loading of excess loss rates I am deducing a simple formula intended to replace some tedious calculations. In the beginning of that note I made the point that some authors recommend a loading proportional to the dispersion of the total claims amount of a treaty δ_1 while others tend to favour δ_1^2.

I also stated that a loading proportional to δ_1 or its estimate $\delta_1{}^*$ could be deduced from the statistical uncertainty in measuring the risk (section 4).

The question has been raised if and to what extent a loading system based on the dispersion is unduly punishing the smaller portfolios. This will be examined below.

The pricing concept will be analyzed from the point of view of a big dominating Reinsurer who wants to be fair in all directions. The conclusion of this study supports an affirmative answer to the question put above.

In a second part the loading is studied from a different angle bringing competition into the picture. The pricing or loading becomes a problem of operations research under the simplified assumption that profit is the only purpose of our activity. Not unexpectedly, the loading coming out from this aspect differs from those of part one.

Part two also deals with the question of how much of the loadings which we are aiming for, get lost in the competitive process. It is also shown that in most cases *the harder the competition is, the higher loadings shall be used.*

Part one and part two thus deal with the loading problem from different aspects, and illustrate the complexity of the problem. It is my hope that this note could stimulate further researches in this interesting and important area, also in a moment when some

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reinsurers are more concerned with the question of surviving than in fixing the loadings which should on the average and in the long run turn up as profits.

Part 1 — *Pricing and loading as a matter of fairness.*

1. Profit can be considered as the reward which the reinsurer should receive because of his willingness to engage his capital and free reserves, in order to take over and carry part of the fluctuation in the gross results of the ceding company.

2. Profit, or rather expectation of profit, is thus the *price for carrying variance,* i.e. possible fluctuations in the negative direction. From this basic idea we shall try to develop a pricing concept, the price being understood as an *addition* to the expected average pure loss cost. Seen from this angle the "price" or expected profit is equal to the loading. Below we will cultivate this concept mainly with regard to non-proportional reinsurance.

3. It is certainly so that reinsurers during the past years have been in such a situation that prices, defined as above, have often been negative. This might partly be unintentional and explained by the differences of measuring and forecasting the net value of the risk. Further factors are the difficulties of the insurance industry, the exaggerated competition between reinsurers and the premium prestige thinking in several quarters.

4. We agree that situations exist when an intentional underrating can be defended as a means of keeping a long-term connection which is expected to give profits later. Underrating can also be used "to come in" and secure a connection which could be made profitable in the long run.

5. When technicians under-rate, this can sometimes be explained by lack of knowledge and experience. It is also possible that the power structure within some reinsurance companies has promoted the pure selling points of view, on the expense of profitability. In other cases when there is a balance of power between marketing and technical aspects a newly-established technical unit might have to try to sell their services. This is easier when the rates which come out do appear as "reasonable".

6. Do let us leave all this aside and try to see what can come out of the rational concept introduced in the beginning. Let us

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also just mention the possibility of asking for a special price addition because of the high quality of service which a big Reinsurer can offer its ceding companies.

7. An insurance or reinsurance company possesses a certain portfolio P which has a variance $V = \delta^2$ and a loading included in the premiums which totals B. This last quantity will include in some rational way also financial revenues and administration costs, etc. Expected profit under a "normal" year is thus assumed to be B.

8. This company considers accepting and bringing into its portfolio a treaty p with b and δ_1. What is a rational price or rating? We assume that the new treaty p is stochastically independent of P.

9. Some reinsurance companies might be willing to accept the new treaty if the expected technical result is positive and if financial revenues cover administrative expenses. Others might instead look at the sum of the above three quantities which has to exceed a certain level higher than zero.

10. Below we will assume that the treaty p is accepted if the company will thereby enter into a new risk situation which is judged as unchanged or better.

11. The above criterion is too vague and has to be elaborated further. To be more precise we could say:

(a) that b shall be fixed in such a way that the risk of getting a negative result shall not increase. When defining such a negative result we could also consider the possibility of mobilizing some free reserves $= U$;

(b) that the mathematical expectation of such a negative result defined with or without U as above shall not increase.

Below we shall study under simplified assumptions what loadings will emerge from this criterion.

12. Let us consider the total portfolio of a company, P, and assume that the results of a certain year are normally distributed (B, δ). We hope that $B > 0$ which is unfortunately not always the case. However, we have available a special reserve U and can tolerate a loss of the year up to $B + U > 0$.

If the total claims amount is X the probability of "ruin" is

$$\text{Prob}\,[X > P + B + U] = 1 - \Phi\left(\frac{B + U}{\delta}\right)$$

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The addition of a treaty p with b and δ_1, leads to a new situation with the probability of ruin equal to:

$$1 - \Phi\left(\frac{B + b + U}{\sqrt{\delta_2 + \delta_1^2}}\right)$$

13. Using criterion a) as above, the probability remains unchanged if:

$$B + U = \frac{B + U + b}{\sqrt{\delta^2 + \delta_2^1}}$$

As δ_1 is small compared with δ this can be written, the terms of higher order being neglected:

$$(B + U)\left(1 + \frac{1}{2}\frac{\delta_1^2}{\delta^2}\right) \cong B + U + b$$

or

$$b \cong \frac{B + U}{2\delta^2} \cdot \delta_1^2$$

or

$$b \cong C_a \cdot \delta_1^2$$

In other words, the loading of the marginal treaty should be proportional to the variance of that treaty. The proportionality factor C_a *increases* with $B + U$ and *decreases* when the total variance of the portfolio increases. Thus the higher the part of the free reserve available, the more loading we would request on the marginal treaty.

14. If we prefer the stronger criterion in 11 above, we may first of all define

$$f(x) = \int_x (y - x)\ \varphi(y)dy = \varphi(x) - x[1 - \Phi(x)]$$

Criterion b) is satisfied when

$$\sqrt{\delta^2 + \delta_1^2}\, f\left(\frac{B + U + b}{\sqrt{\delta^2 + \delta_1^2}}\right) = \delta\, f\left(\frac{B + U}{\delta}\right)$$

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Dividing by $\sqrt{\delta^2 + \delta_1^2}$, developing and neglecting the terms of higher order we obtain:

$$f\left(\frac{B+U}{\delta}\right) + \left(\frac{B+U+b}{\delta}\left(1 - \frac{1}{2}\frac{\delta_1^2}{\delta^2}\right) - \frac{B+U}{\delta}\right) f'\left(\frac{B+U}{\delta}\right)$$
$$\cong f\left(\frac{B+U}{\delta}\right)\left(1 - \frac{1}{2}\cdot\frac{\delta_1^2}{\delta^2}\right)$$

or

$$\frac{1}{\delta}\left[(B+U+b)\left(1 - \frac{1}{2}\frac{\delta_1^2}{\delta^2}\right) - B - U\right] f'\left(\frac{B+U}{\delta}\right)$$
$$= -\frac{1}{2}\frac{\delta_1^2}{\delta^2} f\left(\frac{B+U}{\delta}\right)$$

which gives

$$b - \frac{1}{2}(B+U)\frac{\delta_1^2}{\delta^2} \cong -\frac{1}{2}\frac{\delta_1^2}{\delta}\cdot\frac{f\left(\frac{B+U}{\delta}\right)}{f'\left(\frac{B+U}{\delta}\right)}$$

Thus

$$b \cong \delta_1^2\cdot\frac{1}{2\delta}\left(\frac{B+U}{\delta} + \frac{f\left(\frac{B+U}{\delta}\right)}{-f'\left(\frac{B+U}{\delta}\right)}\right)$$

or

$$b \cong C_b\,\delta_1^2$$

Criterion *b*) is harder than criterion a) and thus leads to

$$C_b > C_a$$

As

$$f(z) = \varphi(z) - z[1 - \Phi(z)]$$
$$-f'(z) = 1 - \Phi(z)$$

and

$$\frac{f(z)}{-f'(z)} = \frac{\varphi(z)}{1 - \Phi(z)} - z$$

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Thus

$$z + \frac{f(z)}{-f'(z)} = \frac{\varphi(z)}{1-\Phi(z)} = \frac{1}{\sqrt{2\pi}\; E(z)} = \frac{1}{\frac{1}{z} - \frac{1}{z^3} + \frac{1.3}{z^5} - \frac{1.3.5.}{z^7} + \ldots}$$

Here $E(z)$ is identical to the Esscher function.

15. We thus have

$$C_b = \frac{1}{2\delta} \; \frac{\varphi\left(\frac{B+U}{\delta}\right)}{1 - \Phi\left(\frac{B+U}{\delta}\right)}$$

Tabulation

$z = \frac{B+U}{\delta}$	$\frac{\varphi(z)}{1-\Phi(z)}$	$\frac{C_b}{C_a} = \frac{\varphi(z)}{1-\Phi(z)} \cdot \frac{1}{z}$
2.0	2.37	1.18
2.5	2.82	1.13
3.0	3.28	1.09

It is easily seen from the above that C_b/C_a converges quickly towards 1 when z increases.

16. The above leads to a security loading of premiums proportional to the *variance*. One advantage with the variance is the additivity on the assumption of independence between treaties.

17. In our loading or pricing system the pricing is reduced to the fixing of the constant $C = C_0$.

18. If for a whole portfolio this C_0 is given it is easy to see that a marginal treaty could be accepted at a $C < C_0$. This point will be illustrated below.

The premium of the whole portfolio is $\pi = E + C_0\delta^2 = E + B$ and let us impose the condition $B = l \,.\, \delta$ where l might be 2, 3 or any other positive constant decided.

We then get $C_0 = l/\delta$

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In the marginal treaty we have according to criterion a)

$$C_a = \frac{1}{\delta} \cdot \frac{B}{2\delta} = \frac{1}{\delta} \cdot \frac{l}{2} = 0.5\, C_0$$

and using criterion b) we get a C_b

$$C_0 > C_b > 0.5\, C_0$$

especially if

$$l = 2.5 \qquad C_b = 1.13\, C_a = 0.57\, C_0$$

19. Let us illustrate the fixing of C_0 by a practical example of a Motor Excess portfolio. We assume that:

$$E = 75{,}000{,}000$$
$$\delta = 5{,}000{,}000$$

and that the company wants:

$$B = 2.5\, \delta$$

as $$B = C_0 \delta^2$$

we get $$C_0 = \frac{2.5}{\delta}$$

The premium for treaty i is thus $E_i + \dfrac{2.5}{\delta} \cdot \delta_i^2$

where $$B_i = \frac{2.5}{\delta} \delta_i^2$$

We have $$B = \Sigma B_i = \frac{2.5}{\delta} \Sigma \delta_i^2 = 2.5\, \delta$$

20. On the assumption that excess claims are Poisson distributed we obtain for a certain treaty

$$\frac{\delta_i}{E_i} = \frac{g(k)}{\sqrt{n}}$$

where n is the expected number of excess claims and k the relative length of the layer.

The function $g(k)$ depends on the structure of the claims distribution in the area above the first risk.

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Generally we have $g(1) = 1$.

For an exponential claims distribution we have $g(\infty) = 2$ and for a Pareto-type claims distribution $g(\infty) = \sqrt{\frac{2(\alpha-1)}{\alpha - 2}}$

21. We further have $E_i = n_i \cdot m_i$ where m_i denotes the average excess claim.

We thus have

$$\delta_i^2 = E_i^2 \cdot \frac{g^2(k)}{n_i}$$

or

$$\delta_i^2 = E_i \cdot m_i \, g^2(k)$$

This gives

$$\pi_i = E_i + C_0\delta_i^2 = E_i(1 + C_0 g^2(k) \cdot m_i)$$

The relative security loading is thus *independent of the size of the treaty.*

22. Let us illustrate the above by a simple example and assume that for a certain market the claims distribution in Motor is well described by the Pareto law with $\alpha = 3$.

We then obtain that for a layer (x, kx) the average excess claim

$$m(x, kx) = \frac{x}{2}(1 - k^{-2})$$

Thus

$$m(x, \infty) = \frac{x}{2}$$

We further get

$$g(k) = \frac{2}{1 + 1/k}$$

Thus

$$\pi_i = E_i\left(1 + C_0 \cdot \frac{x}{2}\frac{(1 - {}^1/k^2)4}{(1 + 1/k)^2}\right)$$

$$\frac{\pi_i}{E_i} = 1 + C_0 \cdot 2x\,\frac{1 - 1/k}{1 + 1/k}$$

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23. The "price" or security loading in such a case is thus

$$C_0 \cdot 2x \cdot \frac{1 - 1/k}{1 + 1/k}$$

with

$$C_0 = \frac{2.5}{5000000}$$

we obtain

$$C_0 \cdot 2x \cdot \frac{1 - 1/k}{1 + 1/k} = 0.1 \cdot \frac{x}{100\,000} \cdot \frac{1 - 1/k}{1 + 1/k}$$

This means that for the unlimited layer, $k = \infty$, we should ask for a proportional loading of 10 % in the case of a first risk of 100,000 and 20 % in the case of 200,000, etc.

The above can be illustrated in a table which shows the loading in %.

	Upper point of layer		
First risk	500′	1,000′	∞
100′	6.7	8.2	10
150′	8.1	11.1	15
200′	8.6	13.3	20
250′	8.3	15.0	25
300′	7.5	16.1	30
350′	6.2	16.8	35
400′	4.4	17.2	40
500′	—	16.7	50

24. It has already been mentioned that a loading proportionate to the dispersion δ will result from the condition that the individual loaded rate shall have a certain probability to be at least equal to the expected pure loss cost. This point combined with the result of the previous sections thus leads to a loading or pricing of the form

$$C_1\delta + C_2\delta^2$$

Since in the future we shall be more skilled in measuring the risk, the first term will diminish in importance and the second will dominate.

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These more precise estimates—extremely important because of the competition—may be available through a better knowledge of the claims distribution completed—if a rational concept can be developed—by a credibility approach.

Part 2. *Pricing and loading strategy in a competitive market*

25. The above has dealt with the pricing problems when the reinsurer tries to show the same degree of fairness in all directions. Let us see what occurs if the reinsurer tries to adapt himself to the competitive market and seeks an optimal pricing strategy on the assumption that profits are the purpose of his activity.

26. We understand that in real life a great deal of other aims determine the strategy and the pricing but in the simplified model below we assume that profit is the *only* goal.

27. Let us then study how to price a non-proportional treaty, bearing in mind that the same general reasoning could also be used for a proportional treaty.

We assume that for the cover period we know —

the *expected* burning cost	E
the costs of handling the treaty	C
and the cost of quotation and negotiation	C_0

We assume that a quoted rate r is connected with a probability of obtaining the cover equal to $p(r)$.

28. We thus have $p(r) \geqslant 0$ and if the market is rational $p'(r) \leqslant 0$.

Our rating strategy would be to maximize

$$g(r) = p(r)\ (r - E - C) - C_0$$

Differentiating we get

$$g'(r) = p(r) + p'(r)\ (r - E - C) = 0$$

which gives

$$r_0 = E + C + \frac{p\ (r_0)}{-p'(r_0)}$$

and

$$g(r_0) = p(r_0)\ \frac{p\ (r_0)}{-p'(r_0)} - C_0$$

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From this it appears that:

1. The price r_0 should exceed $E + C$.
2. That for this optimal price $g(r_0)$ can be positive, zero or even negative.

29. The rational pricing will thus depend upon the function $p(r)$. Let us especially assume

$$p(r) = ae^{-\frac{1}{b}(r-E-C)}$$

Then

$$\log p(r) = \log a - \frac{1}{b}(r - E - C)$$

and

$$\frac{p'(r)}{p(r)} = -b^{-1}$$

the optimal price is then

$$r_0 = E + C + b$$

and the maximum of the expected profit

$$g(\max) = a \cdot e^{-1} \cdot b - C_0$$

Here a represents our chance of getting the cover when we are quoting $E + C$ and b determines the sensitivity of this probability to changes in the rate. The lower b is, the higher is this sensitivity.

Let us assume that for some particular cases $b = 0.2\,E$; then we should quote $\pi = 1.2\,E + C$ and our profit expectation will be $g(\max) = a \cdot e^{-1} \cdot 0.2\,E - C_0$.

If $a = 0.1e \cong 27\,\%$ we have $g(\max) = 0.02\,E - C_0$.

In other words, our profit becomes 2 % of the expected burning cost of the treaties which we are quoting diminished by the costs of quoting and negotiation of these treaties. Of the treaties we are quoting we succeed in getting $ae^{-1} = 10\,\%$.

30. In 27 above we assumed complete knowledge of $E + C$. With such complete knowledge of the risk we would have quoted r_0. Our actual rating is however $r \neq r_0$. Let us assume that r has a distribution $f(r)$ and

$$\int f(r)dr = 1$$
$$\int rf(r)dr = r_0$$
$$\int (r - r_0)^2 f(r)dr = \delta_0^2$$

δ_0 will thus describe the precision in our risk estimates.

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31. The distribution of our outgoing quotations is characterized by $f(r)$ and the distribution of the rates on the accepted quotations by

$$\text{const. } p(r) \cdot f(r)$$

$$\text{where const.} = \frac{1}{\int p(r)\, f(r) dr}$$

Suppose that $f(r)$ is normal (r_0, δ_0) and

$$p(r) = ae^{-\frac{1}{b}(r-E-C)}$$

then

$$\text{const. } p(r) \cdot f(r) = \text{const. } e^{-\frac{(r-r_0)^2}{2\delta_0^2} - \frac{r-E-C}{b}}$$

$$= \text{const. } e^{-\frac{1}{2\delta_0^2}\left[r-\left(r_0-\frac{\delta_0^2}{b}\right)\right]^2}$$

It thus appears that the *distribution of the rates of the portfolio obtained is again normal and* with $(r_0 - \delta_0/b, \delta_0)$.

The variance thus remains unchanged and *the whole effect of the competition is on the mean* which is reduced by δ_0^2/b.

The above result again underlines the importance of

a) reducing δ_0 i.e. increase the precision in our risk estimates.
b) increasing b i.e. reduce the effect of competition and the price sensibility of the ceding company (new products, unconventional excess treaty forms).

32. It is quite possible that in certain cases the loadings which are built into the rating formulas are fully consumed by the effect of competition, which in the simplified model used above is measured by δ_0^2/b.

Some reinsurers load the rates by adding a term proportional to the dispersion of the annual excess claims cost δ_1. At one big Reinsurer such addition was $0.4\ \delta_1$.

This so-called frequency security loading will be consumed as far as $\delta_0^2/b \geqslant 0.4\ \delta_1$.

If the precision in our risk estimates is such that $\delta_0^2 = \delta_1^2/4$, the frequency severity loading is consumed if $b \leqslant 0.625\ \delta_1$. A high competition gives a small b and the above condition may thus often be satisfied.

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The loading or "price" should thus be at least δ_0^2/b.

33. We now intend to seek the size of the optimal loading h to be added to our estimation of $E + C$.

We have $r_0 = E + C + h$.

When quoting r the expected profit is $p(r) . (r - E - C)$ multiplying this with the distribution of r and integrating we obtain

$$g(h) = \int (r-E-C)\, p(r) \cdot \frac{1}{\delta_0 \sqrt{2\pi}} e^{-\frac{1}{2\delta_0^2}(r-r_0)^2} dr$$

If we put $t = r - E - C$ we obtain

$$g(h) = \int tae^{-\frac{t}{b}} \cdot \frac{1}{\delta_0 \sqrt{2\pi}} e^{-\frac{1}{2\delta_0^2}(t-h)^2} dt$$

$$g(h) = a \int \frac{t}{\delta_0 \sqrt{2\pi}} e^{-\frac{1}{2\delta_0^2}\left(t - \left(h - \frac{\delta_0^2}{b}\right)\right)^2} e^{-\frac{1}{2b}\left(2h - \frac{\delta_0^2}{b}\right)} dt$$

$$\text{or putting } u = t - \left(h - \frac{\delta_0^2}{b}\right)$$

$$g(h) = ae^{-\frac{1}{2b}\left(2h - \frac{\delta_0^2}{b}\right)} \cdot \int \left(u + h - \frac{\delta_0^2}{b}\right) \frac{1}{\delta_0 \sqrt{2\pi}} e^{-\frac{u^2}{2\delta_0^2}} du$$

$$g(h) = a\left(h - \frac{\delta_0^2}{b}\right) e^{-\frac{1}{2b}\left(2h - \frac{\delta_0^2}{b}\right)}$$

$$g'(h) = ae^{-\frac{1}{2b}\left(2h - \frac{\delta_0^2}{b}\right)} \left(1 - \frac{1}{b}\left(h - \frac{\delta_0^2}{b}\right)\right)$$

$$g'(h) = 0 \text{ gives } h = b + \frac{\delta_0^2}{b}$$

Not unexpectedly, we obtain a loading or price equal to b, i.e. the result of section 29 above, *increased* by the minimum loading δ_0^2/b which results from the lack of precision in our estimate of the risk.

34. In the above model the price h is thus equal to $h(b) = b + \delta_0^2/b$ which has a minimum $h = 2\delta_0$ when $b = \delta_0$.

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Ceding companies and reinsurers thus seem to have a common interest to try to form a reinsurance market such that b is not too far away from δ_0.

35. If $\delta_0 = \delta_1/2$ which seems to be a plausible assumption when five-year statistics are used to determine E and δ_1 we get a minimum price $2\delta_0 = \delta_1$.

36. It seems plausible that b is mostly less than δ_0. Whenever $b < \delta_0$ an *increased competition,* i.e. decrease in b, results in an *increase in the optimal loading h.*

37. The maximum value of $g(h)$

$$g(\max) = a \cdot b \cdot e^{-1-\frac{\delta_0^2}{2b^2}}$$

$$g(\max) = \frac{ab}{e}\, e^{-\frac{\delta_0^2}{2b^2}}$$

Assuming as in the above section that $a = 0.1\, e \cong 27\,\%$ and the market has an "optimal" structure ($b = \delta_0$), we obtain

$$g(\max) = 0.1\delta_0 e^{-\frac{1}{2}} \sim 0.06\ \delta_0$$

As the loading—assuming an optimal market structure—is $2\delta_0 = \delta$, the above means that we can expect to obtain in profits a maximum of 3 % of the sum of the loadings we are shooting for in our quotations. From this we will have to deduct of course costs for quoting and negotiating excess treaties.

38. The above is meant to illustrate the difficulties of making profits in a competitive market, also where the rating is brought up to a reasonable level. It thus does not primarily illustrate the troubles of several Motor Excess reinsurers which were mainly explained by some lack of technical knowledge and in cases also lack of experience which has led to substantial under-rating.

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