$$A_2T_1^2 = AT_1^2 + AA_2^2 = AT_1^2 + AT^2 - (AT^2 - AA_2^2)$$
$$= 2OA^2 + 2OT^2 - (CT^2 - CC_1^2)$$
$$(T \text{ is radical centre } \therefore\ AT^2 - AA_2^2 = CT^2 - CC_1^2)$$
$$= 2OC^2 + 2OT^2 - (CT^2 - CC_1^2)$$
$$= CT_1^2 + CT^3 - CT^2 + CC_1^2$$
$$= CT_1^2 + CC_1^2$$
$$= C_1T_1^2.$$

$\therefore$ ⊙ with centre T_1 and radius A_2T_1 passes through C_1 and hence through C_2.

Similarly, it will pass through the ends of the diameter perpendicular to T_1B.

If ρ be the radius of radical circle, and R of circumcircle of $\triangle ABC$,

$$A_2T_1^2 = AT_1^2 + AA_2^2, \text{ and } \rho^2 = AT^2 - AA_2^2$$
$$\therefore \quad A_2T_1^2 + \rho^2 = AT^2 + AT_1^2 = 2OA^2 + 2OT^2$$
$$\therefore \quad A_2T_1^2 = 2R^2 + 2OT^2 - \rho^2.$$

A. G. Burgess.

Discrimination of the Roots of a Cubic Equation by Elementary Algebra.—The following note shows how the conditions for the reality or equality of the roots of a cubic can be obtained from the similar conditions for a quadratic. The method does not involve the use of the calculus or the properties of turning points, nor of the imaginary cube roots of unity.

Suppose the general cubic equation has been reduced as usual to the form

$$x^3 + px + q = 0. \quad \text{.........................} \quad (1)$$

It is certain that this has at least one real root, α, say: reduce all the roots of the equation by α and we get

$$(\xi + \alpha)^3 + p(\xi + \alpha) + q = 0,$$
$$\text{or,} \quad (\alpha^3 + p\alpha + q) + (\xi^3 + 3\alpha\xi^2 + \overline{3\alpha^2 + p}\,\xi) = 0,$$
$$\text{or,} \quad \xi(\xi^2 + 3\alpha\xi + \overline{3\alpha^2 + p}) = 0, \quad \text{............} \quad (2)$$

so that the roots of (1) are α, $\alpha + \xi_1$, $\alpha + \xi_2$, where ξ_1, ξ_2 are the roots of

$$\xi^2 + 3\alpha\xi + \overline{3\alpha^2 + p} = 0. \quad \text{............} \quad (3)$$

The roots of (1) are therefore real and distinct, real and two equal, one real and two imaginary, according as

$$(3\alpha)^2 - 4(3\alpha^2 + p) \gtreqless 0,$$

$$i.e. \quad \text{according as } 3\alpha^2 + 4p \lesseqgtr 0.$$

Taking the first of these cases along with the condition $\alpha^3 + p\alpha + q = 0$, and writing it $3\alpha^2 + 4p = -k^2$, to avoid trouble with the odd indices and the inequality, we find

$$\sqrt{\frac{-4p-k^2}{3}}\left(\frac{-4p-k^2}{3} + p\right) = -q,$$

which, on simplifying and squaring, becomes

$$-(4p+k^2)(p^2+2pk^2+k^4) = 27q^2,$$

or, $$-4p^3 - (3pk+k^3)^2 = 27q^2,$$

or, $$4p^3 + 27q^2 = -(3pk+k^3)^2,$$

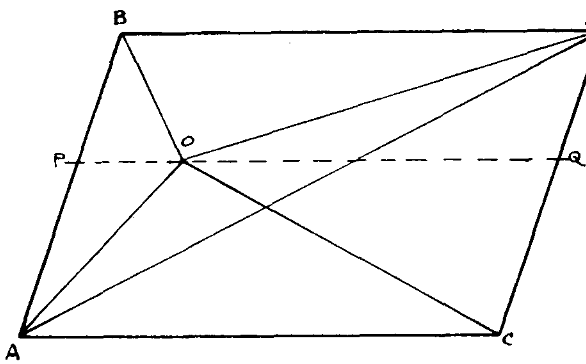
or, $$4p^3 + 27q^2 < 0.$$

Similarly, the necessary and sufficient conditions for the other two cases, viz., three real roots, two equal, and one real root only, are

$$4p^3 + 27q^2 = 0, \quad 4p^3 + 27q^2 > 0 \text{ respectively.}$$

J. M'WHAN.

The Moments Theorem.—If $ABDC$ be a parallelogram, of which AD is a diagonal, and if O be any point which, by joining



to A, B, C, D, makes three triangles, OAB, OAD, OAC, each of

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