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ORIGINAL RESEARCH PAPER

Capital requirement modeling for market and non-life premium risk in a dynamic insurance portfolio

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Abstract

For some time now, Solvency II requires that insurance companies calculate minimum capital requirements to face the risk of insolvency, either in accordance with the Standard Formula or using a full or partial Internal Model. An Internal Model must be based on a market-consistent valuation of assets and liabilities at a 1-year time span, where a real-world probabilistic structure is used for the first year of projection. In this paper, we describe the major risks of a non-life insurance company, i.e. the non-life underwriting risk and market risk, and their interactions, focusing on the non-life premium risk, equity risk, and interest rate risk. This analysis is made using some well-known stochastic models in the financial-actuarial literature and practical insurance business, i.e. the Collective Risk Model for non-life premium risk, the Geometric Brownian Motion for equity risk, and a real-world version of the G2++ Model for interest rate risk, where parameters are calibrated on current and real market data. Finally, we illustrate a case study on a single-line and a multi-line insurance company in order to see how the risk drivers behave in both a stand-alone and an aggregate framework.

Keywords: Capital requirements; time horizon; risk management; real-world valuation; market risk; non-life premium risk; Geometric Brownian Motion; G2++ Model; Collective Risk Model; Solvency II; ORSA

1. Introduction

The inversion of the production cycle regarding revenues and costs is a peculiar and important feature of insurance companies, and it means that policyholders pay premiums in advance, and contractual benefits or indemnities are paid later only if unfavorable events occur. This characteristic implies that insurance companies have significant financial resources to be invested in order to properly face future liabilities. This is certainly true in the majority of life insurers, for which asset management is often the main business providing profitable margins, but it is also true in non-life insurance (which is the focus of this paper), where financial profits can be quite sizeable when the insurance is issued with a relatively long duration in the technical liabilities. The resources are larger if we consider that insurance companies have also their own equity and cumulative profit margins produced year by year. As a result, while insurance companies must focus on underwriting risk, which is the most representative risk of the insurance business, they also must focus on market risk, which in the Solvency II framework is often one of the most material risks in terms of capital requirements. Indeed, the resources of the non-life insurance business (i.e. the claims reserve and the premiums net of the claim amounts and expenses), that is risky by its nature, are invested in financial markets and create an additional risk, which must be estimated and monitored. Hence, it is important to put in place a risk management system covering both financial and underwriting items.

In recent years, the literature has been mainly focused on stand-alone analyses of these two sources of risk. The aim of this paper is thus the examination of market risk, non-life premium risk (from now on, denoted as premium risk), and their interactions under Solvency II (Directive 2009/138/EC, 2009), in order to quantify a reasonable risk profile with some coherent real-world stochastic models and case studies based on actual data.

Since the introduction of Solvency II, the management of risks has become more important, compared to the pre-existing system of Solvency 0 and Solvency I. Insurance companies are now requested to calculate capital requirements to face all the quantifiable risks for existing business and new business expected to be written over the following 12 months. Moreover, they are allowed to calculate the so-called Solvency Capital Requirement (SCR) under the Standard Formula (SF) or a full or partial Internal Model (IM), and they are required to derive it as the Value-at-Risk of the basic Own Funds, subject to a confidence level of 99.5% over a 1-year period. In this regard, there is an important debate about the fairness of the Value-at-Risk. Indeed, many authors argue that other risk measures, such as the Tail Value-at-Risk, are preferable, notwithstanding the more difficult calibration of the distribution tails of the various risks. Nowadays, another important part of the risk management system is the so-called Own Risk and Solvency Assessment (ORSA) (see Directive 2009/138/EC, 2009). It means that insurance companies must evaluate their overall solvency needs on a continuous basis, with the compliance of both capital requirements and requirements on technical provisions and furthermore the compliance of their risk profile with the assumptions underlying the SCR. In addition, the insurance companies must have a forward-looking perspective with a 3–5 year projection view (see EIOPA, 2013a, 2013b, for further details). As a result, not only are insurance companies encouraged to have a short-term view but also a medium and long-term perspective. Indeed, a short-term view only is not desirable, because in this case the risk is not properly managed, since problems might arise in the future.

In order to manage uncertainty, insurance companies use a wide range of stochastic models. The premium risk is usually described by a Collective Risk Model (well-known in the risk theory literature), i.e. a frequency severity approach in which the number of claims and the single claim amount are separately described by some suitable distributions, under some independence assumptions between the claim count and claim size. A popular choice is the Negative Binomial for the number of claims and the Lognormal for the single claim amount, as proposed in practical analyses for instance by Beard et al. (1984) and Daykin et al. (1994). On the other hand, the market risk is usually described by financial models based on stochastic differential equations, i.e. mathematical equations describing stochastic processes in the continuous time. A popular choice is the Geometric Brownian Motion for stocks and a short-rate model for the term structure of interest rates, as proposed for instance by Ballotta and Savelli (2006). A short-rate model is an interest rate model, based on stochastic differential equations, that describes the behavior of the instantaneous short rate, and it is able to describe the entire term structure. In general, interest rate models are distinguished in two main categories, i.e. the equilibrium models and the arbitrage-free models. Equilibrium models produce a term structure as output, and hence they do not match the current term structure observed in the market. Arbitrage-free models take the observed term structure as an input, and hence, they match the current term structure observed in the market. Some well-known equilibrium models have been introduced by Vasicek (1977), Cox et al. (1985) and Duffie and Kan (1996), and some well-known arbitrage-free models have been introduced by Hull and White (1990) and Heath et al. (1992).

During recent decades, the literature on financial models has been primarily devoted to the pricing of interest rate derivatives; therefore, risk-neutral probabilities have been typically preferred (see Brigo and Mercurio, 2006, among others). As explained by Giordano and Siciliano (2015), risk-neutral probabilities are acceptable for pricing, but not to forecast the future value of an asset. Real-world probabilities should instead be used for risk management purposes.

Unfortunately, over the past few years, only moderate attention has been directed to models under real-world probabilities, hence this paper's objective of supporting the literature in this area.

Solvency II requires a market-consistent valuation for technical provisions to be made using risk-neutral probabilities. In the same way, the new international accounting principles for insurance contract valuation, i.e. IFRS 17 (see IASB, 2017), effective from 2023 in EU regulation, require a market-consistent valuation for technical provisions that will probably be carried out by insurance companies using risk-neutral probabilities as well. On the other hand, Solvency II requires that capital requirements according to an IM are calculated using real-world probabilities to predict the risk drivers' behavior for the first year, even though risk-neutral probabilities must be used again to determine the market-consistent value of the basic Own Funds of the insurer after the first year. We point out that, by specifying the models under real-world probabilities and risk-neutral probabilities, it is possible to carry out either risk management analysis, or market-consistent valuations, as proposed by Gambaro et al. (2018) or Berninger and Pfeiffer (2021). Such model specification can also be used for planning and management purposes.

The paper is organized as follows. In Section 2, we describe the theoretical framework underlying the analysis. More precisely, in Section 2.1 we present the risk reserve equation and the related quantities, such as the annual net cash flows originated by the insurance business and the investments portfolio. In Section 2.2, we present the model used to describe the annual rate of return, while in Section 2.3 we present the model used to describe the total claim amount. In Section 3, we propose a numerical analysis in which we determine the capital requirements for market and premium risk, according to a partial IM and the SF. This case study has been performed on a single-line and a multi-line insurance company, using current and available market data. Finally, in Section 4 we report the main conclusions of our research and further steps to be investigated.

2. Analysis framework

2.1 Risk reserve

As explained by Daykin et al. (1994), the risk reserve[1] $\boldsymbol{U}_t$ represents the funds accumulated by the insurance company over time. For simplicity, in this paper we leave aside the reserve risk, dropping the possibility to have unfavorable claims reserving developments. Moreover, we also ignore reinsurance mitigation, taxes, and dividends. As a result, we only consider the market and premium risk, assuming that the stochastic risk reserve at the end of time t is given by:[2]

$$\boldsymbol{U}_t = (1+\boldsymbol{j}_t)\,\boldsymbol{U}_{t-1} + \left(B_t - \boldsymbol{X}_t - E_t\right) + \boldsymbol{j}_t\, L_{t-1}$$

where $\boldsymbol{j}_t$ is the stochastic annual rate of return (annually compounded) of the investments of the insurance company, B_t is the gross premium amount, $\boldsymbol{X}_t$ is the stochastic total claim amount, E_t is the expense amount, including acquisition and general expenses, and L_{t-1} is the claims reserve (also called loss reserve) at the end of the previous year. The premium reserve is not accounted for, because the premium amount is assumed to refer to a single calendar year only and consequently, in this simplified framework, earned premiums and written premiums are identical[3] (for this reason, from now on, we simply denote the gross premium amount as GPW). The GPW,

[1]The term *Risk Reserve* has historically been used in risk theory literature and continues to this day. Under the Solvency II framework, it represents the Own Funds to face the business uncertainty and avoid the risk of insolvency of an insurance company. By contrast, it is worth reminding that the term *Reserve Risk* refers instead to the risk of unfavorable developments of the claims reserve.

[2]Note that the random variables are indicated by bold letters.

[3]In the more general case, where the coverage does not simply refer to a single calendar year, it is enough to introduce into the model the rule to calculate the premium reserve on a risk-based approach, as in the Solvency II framework, considering the dynamic growth of the earned premiums and taking into account, not only the nominal commercial growth but also the initial premium reserve portion released in the referred year. The main results reported in this paper are certainly not diminished for this simple assumption.

stochastic total claim amount, and expense amount are all assumed to occur at the end of the year, and consequently, it is not necessary to consider the accumulation of interest.

The (deterministic) GPW is given by the following equation:

$$B_t = \pi_t + \lambda\, \pi_t + c\, B_t \tag{1}$$

where π_t is the risk premium amount, λ is the safety loading coefficient, and c is the expense loading coefficient.

The expense amount is assumed to be deterministic and equal to the expense loadings included in the GPW. Notwithstanding that the amount of expenses is linked to the GPW (and hence it is not fixed over time), empirically it has a small volatility in the non-life insurance business, in particular where acquisition costs are high. This is supported by the Solvency II calibration, where expense risk in non-life insurance is roughly incorporated in the volatility of the loss ratio.[4] We have:

$$E_t = c\, B_t \tag{2}$$

In addition, we can observe that the ratio of claims reserve to GPW is empirically highly influenced by the Line of Business (LoB). Hence, we assume that the claims reserve is equal to a constant percentage δ of the GPW, different according to the LoB examined:

$$L_t = \delta\, B_t \tag{3}$$

In this regard, as we will explain later on, it is worth pointing out that the stochastic amount paid for the claims settled in the current year, i.e. the stochastic liability cash outflows, is implicitly provided by this constant percentage, in connection with the dynamic of the GPW and stochastic total claim amount (i.e. sum paid and reserved for the claims occurred in the current year). As a result, the risk reserve is found to be:

$$\boldsymbol{U}_t = (1 + \boldsymbol{j}_t)\, \boldsymbol{U}_{t-1} + \Big[(1+\lambda)\, \pi_t - \boldsymbol{X}_t\Big] + \boldsymbol{j}_t\, \delta\, B_{t-1} \tag{4}$$

Finally, the insurance portfolio is dynamic; therefore, we assume that the risk premium amount increases every year according to the following rule:

$$\pi_t = \pi_{t-1}\,(1+i)\,(1+g) = \pi_0\,(1+i)^t(1+g)^t$$

and the GPW as well:

$$B_t = B_{t-1}\,(1+i)\,(1+g) = B_0\,(1+i)^t(1+g)^t \tag{5}$$

where i is the claims inflation rate and g is the real growth rate. It is useful to observe that empirically these rates usually differ by LoB, according to the practical dependence on economic inflation and commercial strength in that particular segment.

We are aware that the reserve risk is an important source of randomness in non-life insurance (see Daykin *et al.*, 1987), but we postpone its analysis to future researches, since in this paper we are interested in isolating the interactions between the risk arising from premiums and investments. For the same reason, we consider a deterministic evolution of the claims reserve, and we do not include the randomness arising from the discounting. Note that the reserve risk regards only the possibility that the claims reserve at the valuation date will have unfavorable developments in the forthcoming annual time horizon, and the total claim amount volatility regarding the premium risk must not incorporate the just mentioned unfavorable developments of the claims reserve. Clearly, in this way we do not obtain any possible risk due to the claims reserving run-off process, but we also do not have any possible risk compensation in case of an increase of risk-free interest rates (i.e. a decrease of the market value of zero-coupon bonds).

[4]On the contrary, expense risk in life insurance has a specific sub-module, because of the well-known long-term maturity of the policies.

2.1.1 Risk reserve ratio

As previously mentioned, the risk reserve is an absolute amount, that depends more on the capital position of the insurance company than on economic results. Actually, we might have a significant risk reserve, which is low compared with the premium volume of the insurance company and vice versa. Hence, it is preferable, also for comparative analyses, to consider relative amounts (see Savelli, 2003).

The stochastic risk reserve ratio at the end of time t is given by:

$$\boldsymbol{u}_t = \frac{\boldsymbol{U}_t}{B_t}$$

Using equations (4) and (5), the risk reserve ratio is then found to be:

$$\boldsymbol{u}_t = \frac{(1+\boldsymbol{j}_t)}{(1+i)\,(1+g)}\,\boldsymbol{u}_{t-1} + \frac{\pi_t}{B_t}\left[(1+\lambda) - \frac{\boldsymbol{X}_t}{\pi_t}\right] + \frac{\boldsymbol{j}_t\,\delta}{(1+i)\,(1+g)}$$

Using equation (1), we have:

$$\frac{\pi_t}{B_t} = \frac{1-c}{1+\lambda}$$

which then gives:

$$\boldsymbol{u}_t = \frac{(1+\boldsymbol{j}_t)}{(1+i)\,(1+g)}\,\boldsymbol{u}_{t-1} + \frac{1-c}{1+\lambda}\left[(1+\lambda) - \frac{\boldsymbol{X}_t}{\pi_t}\right] + \frac{\boldsymbol{j}_t\,\delta}{(1+i)\,(1+g)} \tag{6}$$

2.1.2 Annual net cash flows

The stochastic annual net cash flows originated by the insurance business at the end of time t are given by:

$$\boldsymbol{F}_t = B_t - E_t - \left(\boldsymbol{C}_t^{\mathrm{CY}} + \boldsymbol{C}_t^{\mathrm{PY}}\right)$$

where $\boldsymbol{C}_t^{\mathrm{CY}}$ is the stochastic amount paid for claims occurring in the current year and settled in the same year, and $\boldsymbol{C}_t^{\mathrm{PY}}$ is the stochastic amount paid for claims occurring in the previous years and settled in the current year. The annual net cash flows are invested in the financial market and increase the asset value, which we will describe in the following subsection. For this reason, they are invested based on the same asset allocation of the asset value.

Absent consideration of the claims reserving run-off, the claims reserve is found to be:

$$L_t = L_t^{\mathrm{CY}} + L_t^{\mathrm{PY}} = \boldsymbol{X}_t - \boldsymbol{C}_t^{\mathrm{CY}} + L_{t-1} - \boldsymbol{C}_t^{\mathrm{PY}}$$

where L_t^{CY} is the claims reserve for claims occurring in the current year, and L_t^{PY} is the claims reserve for claims occurring in the previous years.

Using equations (3) and (5), we have the stochastic amount paid for claims settled in the current year:

$$\boldsymbol{C}_t^{\mathrm{CY}} + \boldsymbol{C}_t^{\mathrm{PY}} = \boldsymbol{X}_t - L_t + L_{t-1} = \boldsymbol{X}_t - \delta\,B_t\left(1 - \frac{1}{(1+i)\,(1+g)}\right)$$

then, using equation (2), the stochastic annual net cash flows originated by the insurance business are found to be:

$$\boldsymbol{F}_t = B_t\left[(1-c) + \delta\left(1 - \frac{1}{(1+i)\,(1+g)}\right)\right] - \boldsymbol{X}_t \tag{7}$$

Finally, the claims reserve evolution is determined by the following deterministic relationship, depending on the real growth rate g, claims inflation rate i, and ratio of claims reserve to GPW δ:

$$L_t = L_{t-1} + \delta\, B_t \left(1 - \frac{1}{(1+i)\,(1+g)}\right)$$

2.1.3 Asset portfolio

In this paper, we consider three investments in stocks and five investments in zero-coupon bonds with time to maturity $w = 1, 2, 3, 5, 10$, even though there are a lot of other investments in the market. Furthermore, we assume that the asset allocation is kept constant year by year.

First of all, the initial asset value of the portfolio is given by:

$$A_0 = U_0 + L_0$$

The stochastic asset value of the portfolio at the end of time t is obtained from the combination of the stochastic values of the stock and bond portfolios:

$$\boldsymbol{A}_t = \boldsymbol{A}_t^S + \boldsymbol{A}_t^P$$

The stochastic value of the stock portfolio is given by:

$$\boldsymbol{A}_t^S = \alpha \left(\boldsymbol{A}_{t-1} \sum_{v=1}^{3} \beta_v \frac{\boldsymbol{S}_v(t)}{\boldsymbol{S}_v(t-1)} + \boldsymbol{F}_t\right)$$

so that the stochastic value of a single stock investment is found to be:

$$\boldsymbol{A}_t^{S_v} = \alpha\, \beta_v \left(\boldsymbol{A}_{t-1} \frac{\boldsymbol{S}_v(t)}{\boldsymbol{S}_v(t-1)} + \boldsymbol{F}_t\right) \quad \text{with} \quad v = 1, 2, 3 \tag{8}$$

and the stochastic value of the bond portfolio is given by:

$$\boldsymbol{A}_t^P = (1-\alpha) \left(\boldsymbol{A}_{t-1} \sum_{w \in \{1,2,3,5,10\}} \gamma_w \frac{\boldsymbol{P}(t, t-1+w)}{\boldsymbol{P}(t-1, t-1+w)} + \boldsymbol{F}_t\right)$$

so that the stochastic value of a single bond investment is found to be:

$$\boldsymbol{A}_t^{P_w} = (1-\alpha)\, \gamma_w \left(\boldsymbol{A}_{t-1} \frac{\boldsymbol{P}(t, t-1+w)}{\boldsymbol{P}(t-1, t-1+w)} + \boldsymbol{F}_t\right) \quad \text{with} \quad w = 1, 2, 3, 5, 10 \tag{9}$$

where α and $1-\alpha$ are the percentages invested in the stock and bond portfolios, respectively, β_v is the percentage invested in the vth stock, and γ_w is the percentage invested in the bond with time to maturity w. Moreover, $\boldsymbol{S}_v(t)$ is the stochastic vth stock price, and $\boldsymbol{P}(t, t+w)$ is the stochastic zero-coupon bond price with time to maturity w. It is noted that the asset value increases each time by the annual net cash flows previously described.

Moreover, the stochastic annual rate of return of the investments of the insurance company, used in equation (4), is given by:

$$\boldsymbol{j}_t = \frac{(\boldsymbol{A}_t - \boldsymbol{F}_t) - \boldsymbol{A}_{t-1}}{\boldsymbol{A}_{t-1}}$$

and it is finally found to be:

$$\boldsymbol{j}_t = \alpha \sum_{v=1}^{3} \beta_v \frac{\boldsymbol{S}_v(t)}{\boldsymbol{S}_v(t-1)} + (1-\alpha) \sum_{w \in \{1,2,3,5,10\}} \gamma_w \frac{\boldsymbol{P}(t, t-1+w)}{\boldsymbol{P}(t-1, t-1+w)} - 1 \tag{10}$$

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2.2 Market model

The inversion of the production cycle implies that insurance companies have a lot of resources to invest in order to make profit.

In this subsection, we discuss models based on differential equations. Once we have described over time the distributions of the average stock and bond prices, we are able to obtain the distribution of the annual rate of return time by time by using equation (10). We assume that the market is frictionless, meaning that all securities are perfectly divisible and that no short-sale restrictions, transaction costs, or taxes are present. The security trading is continuous, and there are no riskless arbitrage opportunities.

2.2.1 Short-rate model

We now describe a short-rate model (see among others Brigo and Mercurio, 2006, for further details) in order to describe over time the distribution of the short-term interest rate at time t (also called instantaneous short rate, since it applies to an infinitesimally short period of time at time t). Once it is specified, we are able to compute the zero-coupon bond price and determine the initial zero curve and its future evolution. As a matter of fact, the price at time t of a zero-coupon bond that provides a terminal payoff equal to 1 at maturity date $T > t$ is given by:

$$P(t, T) = e^{-R(t,T)(T-t)}$$

As a result, the continuously compounded zero-coupon interest rate[5] at time t for a term of $T-t$ is given by:

$$R(t, T) = -\frac{\ln P(t, T)}{T-t} \tag{11}$$

Once we have the zero-coupon bond price, we can use the equation above to get the continuously compounded zero-coupon interest rate. This equation will be useful to calibrate the risk premium parameters of the real-world short-rate model.

In this paper, the short rate follows the Two-Additive-Factor Gaussian Model (i.e. G2++ Model) that is given by:

$$\boldsymbol{r}(t) = \boldsymbol{x}(t) + \boldsymbol{y}(t) + \varphi(t)$$

where $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ are the stochastic state variables, and $\varphi(t)$ is a deterministic function of time that allows the model to fit perfectly the term structure observed in the market.

According to Berninger and Pfeiffer (2021), the real-world processes for the state variables satisfy the following stochastic differential equations:

$$\mathrm{d}\boldsymbol{x}(t) = a\,(d_x(t) - \boldsymbol{x}(t))\,\mathrm{d}t + \sigma\,\mathrm{d}\boldsymbol{W}_x^{\mathbb{P}}(t) \quad \text{with} \quad x(0) = 0$$

$$\mathrm{d}\boldsymbol{y}(t) = b\,(d_y(t) - \boldsymbol{y}(t))\,\mathrm{d}t + \eta\,\mathrm{d}\boldsymbol{W}_y^{\mathbb{P}}(t) \quad \text{with} \quad y(0) = 0$$

where $a > 0$ and $b > 0$ are the constant speeds of mean reversion, $\sigma > 0$ and $\eta > 0$ are the constant diffusion coefficients, while $\boldsymbol{W}_x^{\mathbb{P}}(t)$ and $\boldsymbol{W}_y^{\mathbb{P}}(t)$ are the Standard Brownian Motions under the real measure $\mathbb{P}$ with instantaneous correlation $-1 \leq \rho \leq 1$. Furthermore, $d_x(t)$ and $d_y(t)$ are the deterministic mean reversion levels, and we here assume that they are given by step functions:

$$d_x(t) = \mathbb{1}_{t\leq\tau}\, d_x + \mathbb{1}_{t>\tau}\, l_x$$

$$d_y(t) = \mathbb{1}_{t\leq\tau}\, d_y + \mathbb{1}_{t>\tau}\, l_y$$

[5]Be aware that the continuously compounded zero-coupon interest rate is different to the short-term interest rate. Contrary to the former, the latter applies to an infinitesimally short period of time.

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where d_x, l_x, d_y, and l_y are real-valued constants, τ is a constant time parameter, and $\mathbb{1}$ is the indicator function.

The deterministic function of time that allows the model to fit perfectly the term structure observed in the market is given by:

$$\varphi(t) = f^M(0,t) + \frac{\sigma^2}{2a^2}\left(1-e^{-at}\right)^2 + \frac{\eta^2}{2b^2}\left(1-e^{-bt}\right)^2 + \rho\frac{\sigma\eta}{ab}\left(1-e^{-at}\right)\left(1-e^{-bt}\right)$$

where $f^M(0,t)$ is the instantaneous forward rate at initial time for a maturity t implied by the term structure observed in the market.

The stochastic differential equations above have explicit solutions that are given by:

$$\boldsymbol{x}(t) = \int_0^t e^{-a(t-u)}\, a\, d_x(u)\,\mathrm{d}u + \sigma \int_0^t e^{-a(t-u)}\,\mathrm{d}\boldsymbol{W}_x^{\mathbb{P}}(u)$$

$$\boldsymbol{y}(t) = \int_0^t e^{-b(t-u)}\, b\, d_y(u)\,\mathrm{d}u + \eta \int_0^t e^{-b(t-u)}\,\mathrm{d}\boldsymbol{W}_y^{\mathbb{P}}(u)$$

As a result, the short rate is found to have a Normal distribution with mean and variance at time zero given by:

$$\mathbb{E}_0^{\mathbb{P}}\left\{\boldsymbol{r}(t)\right\} = \int_0^t e^{-a(t-u)}\, a\, d_x(u)\,\mathrm{d}u + \int_0^t e^{-b(t-u)}\, b\, d_y(u)\,\mathrm{d}u + \varphi(t)$$

$$\mathbb{V}\mathrm{ar}_0^{\mathbb{P}}\left\{\boldsymbol{r}(t)\right\} = \frac{\sigma^2}{2a}\left(1-e^{-2at}\right) + \frac{\eta^2}{2b}\left(1-e^{-2bt}\right) + 2\rho\frac{\sigma\eta}{a+b}\left(1-e^{-(a+b)t}\right)$$

The price at time t of a zero-coupon bond with maturity in $T > t$ is thus found to be:

$$\boldsymbol{P}(t,T) = \exp\left\{-\int_t^T \varphi(u)\,\mathrm{d}u - \frac{1-e^{-a(T-t)}}{a}\,\boldsymbol{x}(t) - \frac{1-e^{-b(T-t)}}{b}\,\boldsymbol{y}(t) + \frac{1}{2}\,V(t,T)\right\} \tag{12}$$

The integral has an explicit solution that is given by:

$$\exp\left\{-\int_t^T \varphi(u)\,\mathrm{d}u\right\} = \frac{P^M(0,T)}{P^M(0,t)}\,\exp\left\{-\frac{1}{2}\,V(0,T) + \frac{1}{2}\,V(0,t)\right\}$$

where $P^M(0,t)$ is the price at initial time of a zero-coupon bond with maturity in t implied by the term structure observed in the market. Moreover, we have:

$$\begin{aligned} V(t,T) = {} & \frac{\sigma^2}{a^2}\left[T-t+\frac{2}{a}\,e^{-a(T-t)} - \frac{1}{2a}\,e^{-2a(T-t)} - \frac{3}{2a}\right] \\ & + \frac{\eta^2}{b^2}\left[T-t+\frac{2}{b}\,e^{-b(T-t)} - \frac{1}{2b}\,e^{-2b(T-t)} - \frac{3}{2b}\right] \\ & + 2\rho\frac{\sigma\eta}{ab}\left[T-t+\frac{e^{-a(T-t)}-1}{a} + \frac{e^{-b(T-t)}-1}{b} - \frac{e^{-(a+b)(T-t)}-1}{a+b}\right] \end{aligned}$$

2.2.2 Stock price model

We now describe a stock price model (see among others Hull, 2018, for further details) in order to describe over time the distribution of the stock price.

In this paper, the real-world process for the νth non-dividend-paying stock price follows a Geometric Brownian Motion that satisfies the following stochastic differential equation:

$$\mathrm{d}\boldsymbol{S}_\nu(t) = (\boldsymbol{r}(t) + \mu_\nu)\,\boldsymbol{S}_\nu(t)\,\mathrm{d}t + \sigma_\nu\,\boldsymbol{S}_\nu(t)\,\mathrm{d}\boldsymbol{W}_\nu^{\mathbb{P}}(t)$$

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where μ_ν is the constant risk premium coefficient, i.e. the (annualized) expected excess of return in an infinitesimally short period of time, $\sigma_\nu > 0$ is the diffusion coefficient, and $\boldsymbol{W}_\nu^{\mathbb{P}}(t)$ is the Standard Brownian Motion under the real measure $\mathbb{P}$. We assume that the Standard Brownian Motions of the stock prices have equal instantaneous correlation among themselves, but they are independent of those of the short rate. However, in our numerical analysis we will use copula functions to inject some dependence structure between the stock and bond factors.

The stochastic differential equation above has an explicit solution that is given by:

$$\boldsymbol{S}_\nu(t) = S_\nu(0)\ \exp\left\{\int_0^t \boldsymbol{r}(u)\,\mathrm{d}u + \left(\mu_\nu - \frac{1}{2}\,\sigma_\nu^2\right) t + \sigma_\nu \int_0^t \mathrm{d}\boldsymbol{W}_\nu^{\mathbb{P}}(u)\right\}$$

As a result, and in line with the assumption of independence between the short rate and νth stock price, the latter is found to have a Lognormal distribution with mean and variance at time zero given by:

$$\begin{aligned}\mathbb{E}_0^{\mathbb{P}}\{\boldsymbol{S}_\nu(t)\} = \frac{S_\nu(0)}{P^M(0,t)}\ \exp\Bigg\{&\int_0^t \left(1 - e^{-a(t-u)}\right) d_x(u)\,\mathrm{d}u \\ &+ \int_0^t \left(1 - e^{-b(t-u)}\right) d_y(u)\,\mathrm{d}u + \mu_\nu\, t + V(0,t)\Bigg\}\end{aligned}$$

$$\mathbb{V}\mathrm{ar}_0^{\mathbb{P}}\{\boldsymbol{S}_\nu(t)\} = \mathbb{E}_0^{\mathbb{P}}\{\boldsymbol{S}_\nu(t)\}^2 \exp\left\{V(0,t) + \sigma_\nu^2\, t\right\} - \mathbb{E}_0^{\mathbb{P}}\{\boldsymbol{S}_\nu(t)\}^2$$

2.2.3 Risk-neutral models

The risk-neutral processes for the state variables and νth non-dividend-paying stock price satisfy the following stochastic differential equations:

$$\mathrm{d}\boldsymbol{x}(t) = -a\,\boldsymbol{x}(t)\,\mathrm{d}t + \sigma\,\mathrm{d}\boldsymbol{W}_x^{\mathbb{Q}}(t) \quad \text{with} \quad x(0) = 0$$

$$\mathrm{d}\boldsymbol{y}(t) = -b\,\boldsymbol{y}(t)\,\mathrm{d}t + \eta\,\mathrm{d}\boldsymbol{W}_y^{\mathbb{Q}}(t) \quad \text{with} \quad y(0) = 0$$

$$\mathrm{d}\boldsymbol{S}_\nu(t) = \boldsymbol{r}(t)\,\boldsymbol{S}_\nu(t)\,\mathrm{d}t + \sigma_\nu\,\boldsymbol{S}_\nu(t)\,\mathrm{d}\boldsymbol{W}_\nu^{\mathbb{Q}}(t)$$

where the Standard Brownian Motions, as well as the short rate, are now under the risk-neutral measure $\mathbb{Q}$, and the correlation structure is the same as in the real-world case.

The explicit solutions of the differential equations and the mean of the state variables (or short rate) and of the νth stock price are the same as in the real-world case, but the deterministic mean reversion levels and constant risk premium coefficients are now null. The sum of $d_x(t)$ and $d_y(t)$ can be interpreted as the local risk premium of the short rate, i.e. the amount which is added in the real-world to the risk-neutral short rate and which allows the change of measure according to the Girsanov theorem. The zero-coupon bond price formula is also found to be exactly the same as in the real-world case, even though the state variables correspond now to the values of the processes under the risk-neutral measure. As described by Berninger and Pfeiffer (2021), using the explicit solutions of the differential equations both in the real and risk-neutral world, and using equation (11) and (12), the relation between the expected zero-coupon interest rate (continuously compounded) at time t for a term of $T-t$ under the real measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$ is given by:

$$\mathbb{E}_0^{\mathbb{P}}\{\boldsymbol{R}(t,T)\} = \mathbb{E}_0^{\mathbb{Q}}\{\boldsymbol{R}(t,T)\} + \frac{1}{T-t}\left[\frac{1 - e^{-a(T-t)}}{a}\,\mathrm{RP}_x(t) + \frac{1 - e^{-b(T-t)}}{b}\,\mathrm{RP}_y(t)\right] \tag{13}$$

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where $\mathrm{RP}_x(t)$ and $\mathrm{RP}_y(t)$ are the actual risk premiums of the short rate between time zero and t for the state variables:

$$\mathrm{RP}_x(t) = \int_0^t e^{-a(t-u)}\, a\, d_x(u)\, \mathrm{d}u$$

$$\mathrm{RP}_y(t) = \int_0^t e^{-b(t-u)}\, b\, d_y(u)\, \mathrm{d}u$$

As already described, the parameters of the risk-neutral model (a, b, σ, η, and ρ) are in common with the real-world model. We can calibrate these parameters based on derivative instruments (this is the reason why we need risk-neutral models). Once we estimate the risk-neutral parameters, we can calibrate the additional real-world parameters (d_x, l_x, d_y, and l_y, given the value of τ) using the equations above. To do this, we need some expectations for real-world zero-coupon interest rates (continuously compounded). In our numerical analysis, for example, we will use some interest rate forecasts published by the Organisation for Economic Co-operation and Development.

The relation between the vth stock price under the real measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$ could be derived; however, we do not need it in our numerical analysis.

2.3 Non-life model

The inversion of the production cycle also implies that insurance companies have to measure and manage the future total claim amount, in order to control losses and determine insurance premiums.

In this subsection, we describe the well-known Collective Risk Model based on a compound process, in order to describe over time the distribution of the total claim amount (see among others Daykin et al., 1994, for further details). It is very popular in non-life insurance modeling, because each risk can produce claims of different severity.[6] We consider the entire LoB portfolio, composed of homogeneous risks, and we separately analyze the number of claims and the single claim amount, that is assumed to be independent of the contract that generated it.

Finally, the stochastic total claim amount at the end of time t is given by:

$$\boldsymbol{X}_t = \sum_{k=1}^{\boldsymbol{K}_t} \boldsymbol{Z}_{k,t} \tag{14}$$

where $\boldsymbol{K}_t$ is the stochastic number of claims, and $\boldsymbol{Z}_{k,t}$ is the stochastic amount for the kth claim, for which we assume that:

1. the random variables $\boldsymbol{Z}_{k,t}$ are independent;
2. the random variables $\boldsymbol{Z}_{k,t}$ are identically distributed;
3. the random variables $\boldsymbol{Z}_{k,t}$ and $\boldsymbol{K}_t$ are independent.

The first and second assumption are satisfied, in particular, in limited homogenous portfolios and also only in some certain time period. In order to solve this kind of problem, the portfolio can be split into different more homogenous sub-portfolios, where the mentioned conditions are more properly satisfied.[7] The third assumption is usually satisfied, but it might be refuted in some situations. In case of windstorm or hurricane, for example, the number of claims and single claim amount variables increase both significantly at the same time.

[6]This is opposite to life insurance, where the sum insured is mainly defined at policy issue.

[7]In this case, the sub-portfolios must be aggregated to come back to the full portfolio, assuming linear or non-linear dependence as appropriate.

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2.3.1 Number of claims

There are several distributions associated with the number of claims, such as the Poisson and Negative Binomial. We point out that the total claim amount is found to be zero, when the number of claims is zero.

Since the insurance portfolio is dynamic, we assume that the expected number of claims increases or decreases every year, according to the real growth rate g, which is the reference indicator for the new number of policyholders:

$$n_t = n_{t-1}\,(1+g) = n_0\,(1+g)^t \quad \text{with} \quad n_0 > 0$$

The expected number of claims increases every year in the same way as the insurance portfolio. As a result, the claims frequency of the portfolio is assumed to remain the same over time. This assumption could be refuted in some situations, such as in case of considerable modification of the portfolio, where new policyholders have significantly different claim frequency (e.g. young drivers or policyholders with high-power vehicles). Moreover, we point out that not only does the initial expected number of claims depend on the insurance portfolio size but also on the individual claims frequency of the people insured. Later on, the parameter of the expected number of claims will influence the so-called size factor, which must be taken into account in an IM, rather surprisingly, differently from what is done in the SF of Solvency II.

In this paper, the number of claims is distributed as a Mixed Poisson, i.e. a Poisson with stochastic (and not deterministic) parameter $\boldsymbol{n}_t > 0$, such that:

$$\boldsymbol{n}_t = n_t\,\boldsymbol{q}$$

where $\boldsymbol{q}$ is the stochastic structure variable, which denotes the multiplicative noise term, representing the parameter uncertainty embedded in the distribution. As well-known in the literature, the cumulant generating function of the Mixed Poisson distribution is given by:

$$\Psi_{\boldsymbol{K}_t}(\upsilon) = \Psi_{\boldsymbol{q}}\big[n_t\,(e^{\upsilon}-1)\big] \tag{15}$$

In order to describe the number of claims, considering the short-term fluctuations only, we must assume the structure variable to have an expected value equal to one. In the literature, this distribution is frequently assumed to be a Gamma with equal parameters (h, h), that is defined by the following probability density function:

$$f_{\boldsymbol{q}}(q) = \frac{h^h\,q^{h-1}}{\Gamma(h)}\,e^{-hq} \quad \text{with} \quad q > 0 \text{ and } h > 0$$

with mean, variance, and skewness that are given by:

$$\mathbb{E}\{\boldsymbol{q}\} = 1 \quad \text{and} \quad \mathbb{V}\mathrm{ar}\{\boldsymbol{q}\} = \frac{1}{h} \quad \text{and} \quad \mathbb{S}\mathrm{k}\{\boldsymbol{q}\} = \frac{2}{\sqrt{h}} = 2\,\mathbb{S}\mathrm{td}\big\{\boldsymbol{q}\big\}$$

As a result, the Mixed Poisson distribution is found to be a Negative Binomial with parameters (h, p_t) and to be defined by the following probability mass function:

$$\Pr\big(\boldsymbol{K}_t = k\big) = \binom{k+h-1}{h-1}\,p_t^h\,(1-p_t)^k$$

$$\text{with} \quad k = 0, 1, \ldots \text{ and } h > 0 \text{ and } 0 < p_t = \frac{h}{h+n_t} < 1$$

2.3.2 Single claim amount

Since the insurance portfolio is dynamic, we assume that the single claim amount distribution (dropping the index k because of the assumption of identical distribution) is only rescaled for the claims inflation every year:

$$\boldsymbol{Z}_t \sim \boldsymbol{Z}_{t-1}\,(1+i) \sim \boldsymbol{Z}_0\,(1+i)^t$$

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Hence, the ωth raw moment is simply found to be rescaled as well:

$$\mathbb{E}\left\{Z_t^{\omega}\right\} = \mathbb{E}\left\{Z_0^{\omega}\right\}(1+i)^{\omega t}$$

As a result, we start by describing the initial single claim distribution, and, by changing one parameter, we scale it to obtain the subsequent ones. It follows that all the relative indicators, e.g. the skewness and coefficient of variation, remain the same over time, notwithstanding the absolute indicators, e.g. the expected value and standard deviation, evolve in line with the claims inflation rate assumed. This is because we assume that the new policyholders and new empirical data do not have an effect on the shape of distribution, but only a rescaling effect.

In this paper, we assume that the single claim amount is distributed as a Lognormal with parameters (m_t, s), that is defined by the following probability density function:

$$f_{Z_t}(z) = \frac{1}{z\sqrt{2\pi}\,s}\ \exp\left\{-\frac{1}{2}\left(\frac{\ln(z)-m_t}{s}\right)^2\right\}$$
$$\text{with}\quad z>0 \ \text{ and } \ -\infty < m_t < +\infty \ \text{ and } \ s>0$$

where s is assumed to be constant, because empirically it does not change much over time. It is worth pointing out that the skewness of a Lognormal distribution is provided uniquely by the coefficient of variation, according to the following well-known relation:[8]

$$\mathbb{S}\mathrm{k}\left\{Z_t\right\} = 3\,\mathbb{C}\mathrm{V}\left\{Z_0\right\} + \mathbb{C}\mathrm{V}\left\{Z_0\right\}^3$$

There are also other distributions associated with the single claim amount, such as the Gamma, Weibull, Inverse Normal, and Pareto, and many other potential distributions. The first three distributions, together with the Lognormal, usually fit attritional claims well, i.e. the most frequent and least expensive claims (large frequency and low severity). The last distribution fits large claims well, i.e. the least frequent and most expensive claims. The Lognormal is a distribution that often serves as a reference for the single claim amount. A popular alternative is to use distributions where attritional and large claims are described by different random variables.[9]

2.3.3 Total claim amount

Using the definitions and assumptions above, we are able to determine the total claim amount distribution. Indeed, the cumulant generating function of the total claim amount distribution is found to be:

$$\Psi_{X_t}(\upsilon) = \Psi_{K_t}\left[\Psi_{Z_t}(\upsilon)\right]$$

[8] This relation is quite interesting, because it clearly provides an insight into the obvious relation between the coefficient of variation and skewness of a Lognormal distribution. The higher the relative volatility, the higher the skewness. For this reason, the difference between a quantile and the mean cannot be expressed in terms of the same multiplier of the standard deviation, since the multiplier changes every time the relative volatility changes. This comment will be seen later, when we will present the SF for the calculation of the capital requirement for non-life premium and reserve risk, in which the underlying distribution is assumed to be a Lognormal, and the multiplier is kept constant and equal to 3.

[9] The Lognormal distribution is a basic assumption in the literature of the single claim amount behavior. The latter can be represented by many other distributions, as explained above, and clearly, the most appropriate one should be revealed using fitting procedures on appropriate empirical data. In addition, mixture, composite, or spliced distribution approaches can be used to have a proper distinction between the behavior of small-size amounts and medium-large amounts (see for instance Clemente et al., 2014). It is worth pointing out that we follow a total approach, without any distinction between small or medium-large claims. In some standard procedures, the simple attritional-large approach is followed (e.g. SF of the Swiss Solvency Test), but for an IM the total approach is highly recommended, in order to have more powerful information, in particular when we want to analyze the benefits coming from non-proportional reinsurance strategies (e.g. Excess of Loss).

Using equation (15), the cumulant generating function of the total claim amount distribution according to the mixed compound Poisson process is found to be:

$$\Psi_{\boldsymbol{X}_t}(\upsilon) = \Psi_{\boldsymbol{q}}\left[n_t\left(M_{\boldsymbol{Z}_t}(\upsilon) - 1\right)\right]$$

As a result, the mean, variance, skewness, and coefficient of variation of the total claim amount distribution are found to be:

$$\mathbb{E}\{\boldsymbol{X}_t\} = n_t\,\mathbb{E}\{\boldsymbol{Z}_t\} = \mathbb{E}\{\boldsymbol{X}_0\}\,(1+g)^t(1+i)^t$$

$$\mathbb{V}\text{ar}\{\boldsymbol{X}_t\} = n_t\,\mathbb{E}\{\boldsymbol{Z}_t^2\} + n_t^2\,\mathbb{E}\{\boldsymbol{Z}_t\}^2\,\mathbb{V}\text{ar}\{\boldsymbol{q}\}$$

$$\mathbb{S}\text{k}\{\boldsymbol{X}_t\} = \frac{n_t\,\mathbb{E}\{\boldsymbol{Z}_t^3\} + 3n_t^2\,\mathbb{E}\{\boldsymbol{Z}_t\}\,\mathbb{E}\{\boldsymbol{Z}_t^2\}\,\mathbb{V}\text{ar}\{\boldsymbol{q}\} + n_t^3\,\mathbb{E}\{\boldsymbol{Z}_t\}^3\,\mathbb{S}\text{k}\{\boldsymbol{q}\}\,\mathbb{S}\text{td}\{\boldsymbol{q}\}^3}{\mathbb{S}\text{td}\{\boldsymbol{X}_t\}^3}$$

$$\mathbb{CV}\{\boldsymbol{X}_t\} = \sqrt{\frac{1+\mathbb{CV}\{\boldsymbol{Z}_0\}^2}{n_t} + \mathbb{V}\text{ar}\{\boldsymbol{q}\}}$$

Consequently, the skewness of the total claim amount approaches the skewness of the structure variable as n_0 or g increase, i.e. when the size of the insurance company increases, and the coefficient of variation approaches the standard deviation of the structure variable. Nevertheless, a variation of claim inflation rate i does not affect the skewness nor the coefficient of variation, because of the single claim amount distribution rescaling property previously mentioned.

2.4 Copula functions

We finally describe copula functions, in order to describe the dependence structure between random variables, such as the total claim amounts of different LoBs. Copula functions are very popular in insurance, because they are able to describe a wide range of dependence structures, including but not limited to linear dependence (see among others Embrechts et al., 2003; Nelsen, 2006, for further details).

A n-dimensional copula $C{:}[0,1]^n \to [0,1]$ is a multivariate cumulative distribution function of uniformly distributed marginals, and it satisfies the following properties:

1. $C(u_1, \ldots, u_n)$ is non-decreasing in each component u_i;
2. $C(u_1, \ldots, u_n)$ is null if at least one component u_i is null;
3. $C(u_1, \ldots, u_n)$ is equal to u_i if all the components are equal to one, except u_i.

The most popular copulas in the insurance business are the Gaussian, Student's t, Clayton, and Gumbel copulas. The first and second one are Elliptical copulas (i.e. copula functions based on multivariate elliptical distributions), while the third and fourth one are Archimedean copulas. The Gaussian copula does not include any tail dependence, while the other mentioned copulas include some dependence structure on the tails of the marginal distributions. In this paper, the dependence structure is described by using the Gaussian copula, Student's t copula (which includes both upper and lower tail dependence), or Gumbel copula (with upper tail dependence only), which are the benchmark in the practical financial-actuarial modeling. European insurance supervisory authorities are aware that some tail dependence is very often present, and it cannot be disregarded, but they prefer that the IM is based on Elliptical copulas, such as the Gaussian one, because it is quite manageable. However, they require more conservative linear correlation coefficients. This point has been introduced also in the SF, where the correlation matrix includes some prudence to implicitly incorporate the tail dependence. Consequently, in this paper we consider both the Gaussian and Gumbel copulas for the aggregation in the premium risk framework, in

Table 1. Average of all Euro Area government rates on September 30, 2020 (continuously compounded and expressed in %)

Maturity in years	Average government rate
1	−0.540
2	−0.514
3	−0.481
5	−0.389
10	−0.079

order to assess the effect of having an upper tail dependence in the total claim amount distributions and the consequent change in capital requirements. We also consider a Student's t copula for the aggregation in the market risk framework.

3. Numerical results

As proposed by Ballotta and Savelli (2006), in this section we calculate the capital requirements for market and premium risk of an insurance company, according to a partial IM and the SF (in this second case the capital requirement is denoted by SCR). We utilize the following assumptions:

- the capital requirements according to our IM are calculated over a period of 1, 2 and 3 years, using a Monte Carlo simulation approach based on 100,000 simulation paths;
- the insurance contracts are not multi-annual, their exposure is referred to the full calendar year, and there is no geographical diversification for SF purposes;
- the interest rate risk and equity risk are the only sources of market risk, and the premium risk is the only source of non-life underwriting risk;
- the market risk only affects the investments, in particular the interest rate risk only affects the bond investments, and the equity risk only affects the stock investments;
- the security trading is continuous, all securities are perfectly divisible, and there are no transaction costs, taxes, short-sale restrictions, or riskless arbitrage opportunities;
- the bond investments are zero-coupon bonds that remunerate the average of all Euro Area government rates on September 30, 2020, presented in Table 1;
- the bond investments are risk-free (i.e. no spread risk, liquidity risk, or default risk), and they thus evolve according to the dynamic of the European interest rate swaps;
- the stock investments are listed in regulated markets of the Euro Area, hence they can be considered as type 1 equities under the SF;
- the stock investments are non-dividend-paying stocks without strategic nature, because of the absence of a clear decisive strategy to continue holding them for long period;
- the symmetric adjustment required by the SF is not present;
- the asset allocation is recalibrated at year end only.

As already mentioned, the loss absorbing capacity and reserve risk arising from the claims reserve are not considered here, because at this stage of our research we want to isolate the interactions between the market and premium risk only. Moreover, spread risk, liquidity risk, and default risk are also not considered, and this is consistent with the SF, according to which, government bonds are assumed not to be affected by these sources of risk. Finally, we point out that taxes and dividends usually have a rescaling impact on the stochastic result of the insurance company, but we do not consider them for comparability with the SF.

According to our IM, the capital requirements are here calculated according to the minimum Risk-Based Capital (RBC), which is a risk measure that takes into account the expected return

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Table 2. General parameters of our numerical analysis

B_0	u_0	F_0	i	g	λ	c	δ
100,000,000	25%	0	1.5%	2%	0.87%	21.24%	156.10%

produced by the investment of the resources. The RBC over the time horizon $(0, t)$ within the confidence level $1 - \varepsilon$ is given by:

$$\mathrm{RBC}(0, t) = U_0 - \frac{U_\varepsilon(t)}{\prod_{k=1}^{t}\left(1 + \mathbb{E}\{\boldsymbol{j}_k\}\right)} \tag{16}$$

where $U_\varepsilon(t)$ is the εth order quantile of the risk reserve. As a result, the ratio of RBC and initial GPW is found to be:

$$u_{\mathrm{RBC}}(0, t) = \frac{\mathrm{RBC}(0, t)}{B_0} = u_0 - u_\varepsilon(t)\, \frac{(1+i)^t(1+g)^t}{\prod_{k=1}^{t}\left(1 + \mathbb{E}\{\boldsymbol{j}_k\}\right)}$$

where $u_\varepsilon(t)$ is the εth order quantile of the risk reserve ratio, that is given by:

$$u_\varepsilon(t) = \frac{U_\varepsilon(t)}{B_t}$$

Clearly, the RBC above decreases (or increases) for expected profits (or losses) above the initial risk reserve. In this paper, we use this approach, even though the European insurance supervisory authorities are reluctant to include expected profits or losses in the capital requirement calculation according to an IM, because they want to target consistency with the SF framework, which does not consider this possibility.

3.1 Single-line insurance company

At the beginning of our analysis, we consider a single-line insurance company, with underwriting business in Motor Third-Party Liability (MTPL) only. We point out that we use the same notation as in the previous sections.

The general parameters of our numerical analysis are presented in Table 2. The initial risk reserve ratio (25%) is roughly one and a half times the Required Solvency Margin for non-life insurance, required by Solvency 0 and Solvency I. The initial GPW of our company is 100 million, and other general parameters are estimated based on the Italian market data until the end of 2018, provided by the National Association of Insurance Companies (ANIA, 2020) and Italian Insurance Supervisory Authority (IVASS, 2019). In particular, the safety loading coefficient is estimated by the complement of 100% of the average of the last five annual observations of the combined ratio (claims reserving run-off excluded), and then, it is transformed into a coefficient of the risk premium amount, as shown in equation (1). The complement of 100% of the combined ratio indeed represents the expected profit of the insurance company included in the pricing process. On the one hand, the expense loading coefficient is estimated by the average of the last five annual observations of the expense ratio, on the other hand, the ratio of claims reserve to GPW is estimated by the average of the last five annual observations. The role of the expense loading is to cover future expenses, including both acquisition and general expenses. As a result, the initial claims reserve is found to be equal to 156.1 million, and together with 25 million of initial risk reserve we obtain an initial asset value of the portfolio equal to 181.1 million. We remind that there is no premium reserve, since we assume exposure referred to the full calendar year only. Moreover, we assume some a priori estimate for the claims inflation rate and real growth rate, and we point out that there is no interaction between these variables and the economic scenario. We point out that for the annual non-life insurance business, the claims inflation uncertainty

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Table 3. Asset allocation of our numerical analysis

α	β_1	β_2	β_3	γ_1	γ_2	γ_3	γ_5	γ_{10}
15%	60%	30%	10%	40%	25%	15%	10%	10%

Table 4. Real-world parameters of the Geometric Brownian Motions

μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	Corr vth and wth stocks
0.0269	0.0359	0.0448	0.1547	0.2063	0.2579	+0.7500

Table 5. Real-world parameters of the G2++ Model

a	b	σ	η	ρ	d_x	d_y	l_x	l_y
0.2810	0.0554	0.0154	0.0121	−0.9877	−0.0008	0.0178	−0.0296	0.0182

has a minor impact if we do not consider reserve risk. This is instead relevant for multi-annual insurance business.

The asset allocation of our numerical analysis, i.e. the proportion and composition of the bond and stock portfolio, is presented in Table 3. We remind that our numerical analysis assumes five zero-coupon bonds and three stocks. Both the stock portfolio (15% of the total) and the bond portfolio (85% of the total) are more invested in low-risk assets rather than in high-risk ones (e.g. 60% of the stock portfolio is invested in a low-risk stock, and 40% of the bond portfolio is invested in a 1-year zero-coupon bond). It is thus possible to obtain the initial values of the bond and stock portfolios and investments.

The real-world parameters of the Geometric Brownian Motions of our numerical analysis are presented in Table 4. As proposed by Hull (2018), the risk premium and diffusion coefficients of the stock 2 are respectively equal to 3.6% and 20.6%, and they are estimated by using the method of moments on the continuously compounded daily interest rates of the Euro Stoxx 50 between September 30, 2010, and September 30, 2020, assuming that the short rate is absent (since it is empirically very small). The risk premium and diffusion coefficients of the stock 1 (or the stock 3) are assumed to be 25% lower (or higher) than the corresponding parameters of the stock 2. The instantaneous correlation between different stocks is assumed to be equal to the linear correlation coefficient between type 1 and type 2 equities under the SF, which is defined in the Commission Delegated Regulation (EU) 2015/35 (2015).

The real-world parameters of the G2++ Model of our numerical analysis are presented in Table 5. As proposed by Brigo and Mercurio (2006), the risk-neutral parameters are estimated by minimizing the sum of the squares of the percentage differences between model and market swaption volatilities. In doing so, a genetic algorithm is used to solve the optimization problem. The model swaption volatilities are Normal volatilities implied by the approximation proposed by Schrager and Pelsser (2006) for swaption prices under the G2++ Model. The market swaption volatilities are the most liquid at-the-money European swaption volatility quotes on September 30, 2020 (i.e. those with maturity and tenor combination from 1 to 10 years). As proposed by Berninger and Pfeiffer (2021), the local risk premium functions (i.e. the components of the amount which is added in the real-world to the risk-neutral short rate) are estimated by using equation (13) and the interest rate forecasts published by the Organisation for Economic Co-operation and Development (OECD, 2020), which can be seen as real-world expectations. In particular, the local risk premiums in the short horizon are estimated by the latest available forecasts of the 3-month and 10-year Euro Area interest rates on September 30, 2020, for the longest available projection horizon (i.e. the fourth quarter of 2021). The local risk premiums in the long horizon are estimated by the monthly average of 3-month and 10-year Euro Area interest rates

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Table 6. Main indicators of the distributions of the single claim amount and number of claims

$\mathbb{E}\{\boldsymbol{Z}_0\}$	$\mathbb{CV}\{\boldsymbol{Z}_0\}$	n_0	$\mathbb{S}\mathrm{td}\{\boldsymbol{q}\}$
4,000	7	19,520	0.0821

Table 7. Parameters of the Lognormal and Negative Binomial distributions

m_0	s	h	p_0
6.3380	1.9779	148.47	0.0075

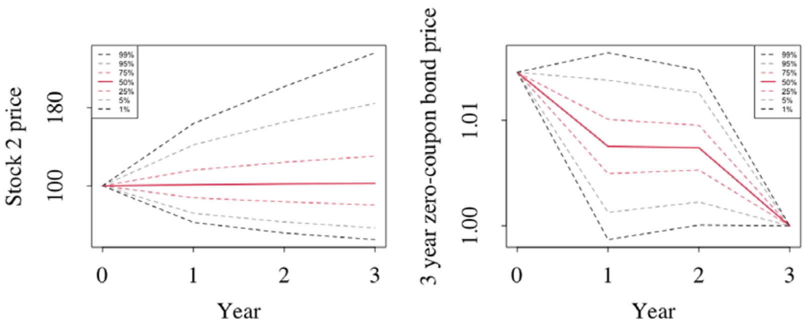

Figure 1. Quantiles of the simulated stock 2 price and 3-year zero-coupon bond price over the years.

over the last 15 years from September 30, 2020. Moreover, the parameter τ is assumed to be 15 months, which is the time horizon from September 30, 2020, to the fourth quarter of 2021.

The main indicators of the distributions of the single claim amount and number of claims are presented in Table 6. The initial mean and coefficient of variation of the single claim amount distribution are determined through empirical evidence, and the initial expected number of claims is obtained in function of the size of the insurance company. The standard deviation of the structure variable (i.e. the parameter uncertainty of the number of claims distribution) is estimated based on the Italian market data until the end of 2018, provided by ANIA (2020) and IVASS (2019) In particular, it is given by the product of the standard deviation of the loss ratio on accrual basis and the ratio of gross and risk premium amount. This result holds because we can see the Italian market data as the portfolio history of a huge-sized insurance company, and thus the standard deviation of the pure loss ratio (i.e. the coefficient of variation of the total claim amount) approaches the standard deviation of the structure variable. Currently, there is no method in the literature to estimate this relevant parameter excluding the variability coming from the underwriting cycle.

The parameters of the Lognormal and Negative Binomial distributions are presented in Table 7. The parameters of the Lognormal distribution are estimated by using the method of moments on the main indicators of the single claim amount distribution above, i.e. the initial mean and coefficient of variation. The parameters of the Negative Binomial distribution are estimated by using the relations in Section 2.3.1 and the main indicators of the distribution of the number of claims above, i.e. the initial expected number of claims and standard deviation of the structure variable.

Figure 1 shows the quantiles of the simulated stock 2 price and 3-year zero-coupon bond price over time, as representative of their respective asset classes. We can clearly see that the volatility of the stock increases as the time horizon raises. On the other hand, the increasing risk of the zero-coupon bond is offset by the convergence to its nominal value.

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Table 8. Single-line descriptive statistics of the simulated annual rates of return (with amounts in %) and of the simulated total claim amounts (with amounts in millions)

Year	Mean	St. dev.	Skew.	Min.	1st qu.	Median	3rd qu.	Max.
Annual rate of return								
1	−0.17	2.63	0.4961	−9.40	−2.01	−0.38	1.45	18.25
2	0.41	2.64	0.4973	−8.68	−1.42	0.19	2.02	15.46
3	0.20	2.64	0.5084	−9.06	−1.65	−0.01	1.82	20.44
Total claim amount								
1	80.80	7.78	0.4962	53.62	75.48	80.47	85.76	241.99
2	83.70	8.09	0.9834	55.17	78.20	83.34	88.80	376.52
3	86.67	8.24	0.3362	53.89	81.01	86.35	91.93	173.76

Using equations (10) and (14), we are able to simulate the annual rate of return and the total claim amount over time. Table 8 shows some descriptive statistics of the annual rate of return and total claim amount after 1, 2, and 3 years. We point out that the mean of the annual rate of return should increase over time, because the instantaneous forward rate is an increasing function of time. However, for the second year the mean (0.41%) is higher than for the third year (0.20%). We indeed remind that the local risk premium functions are step functions with jumps after 15 months. As explained above, in our real-world model, the expected value of the short rate increases because of the instantaneous forward rate and actual risk premiums. On the other hand, the standard deviation of the annual rate of return remains quite stable over time and is not affected by the real-world assumptions. Moreover, because of the dynamic portfolio assumption, the mean and standard deviation of the total claim amount increase over time, because of increasing size for real growth and claims inflation rates, and the skewness decreases over time, approaching the skewness of the structure variable. In particular, in the first year we obtain an expected value of 80.80 million, corresponding to a combined ratio of roughly 99.3% and in line with the parameters.

Building on the above and in order to inject some dependence structure between the stock and bond factors, we use Student's t copulas with 2 degrees of freedom. The correlation coefficient is assumed to be null, i.e. the linear correlation coefficient between interest rate risk and equity risk under the SF, which is defined in the Commission Delegated Regulation (EU) 2015/35 (2015). Moreover, in order to inject some dependence structure between the annual rate of return and total claim amount, we use rotated Gumbel copulas with parameter 1.1904. It corresponds to a probability of 50% that the annual rate of return experiences an event with a probability lower than 15%, while the total claim amount experiences an event with 99% probability. This approach allows the modeling of a reasonable tail dependence between market and premium risk and is in line with what is actually sometimes used in the insurance industry.

3.1.1 Market risk

We now isolate the effect of the market risk, and so we leave aside the premium risk. In this regard, we drop the underwriting result by the risk reserve, but still consider the interest on its investment.

Moreover, we assume that the total claim amount is deterministic and equal to its mean, hence the underwriting result is equal to the safety loadings. We believe that this is not a simplification and is a reasonable solution to make a stand-alone analysis of the market risk with respect to premium risk. As a result, starting from equation (4), the risk reserve is found to be:

$$\boldsymbol{U}_t = (1 + \boldsymbol{j}_t)\, \boldsymbol{U}_{t-1} + \boldsymbol{j}_t\, \delta\, B_{t-1} + \boldsymbol{j}_t \sum_{k=1}^{t-1} \lambda\, \pi_k$$

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Table 9. Single-line capital requirements over the initial GPW (expressed in %) and diversification benefits under the SF and our IM over a period of 1, 2, and 3 years

	Year 1 (SF)	Year 1 (IM)	Year 2 (IM)	Year 3 (IM)
Stock 1	6.36	5.19	6.79	7.92
Stock 2	3.18	3.24	4.12	4.67
Stock 3	1.06	1.27	1.58	1.76
Total stocks	**10.59**	**9.70**	**12.49**	**14.35**
Bond with maturity in 1	0.61	0.29	0.82	1.64
Bond with maturity in 2	0.76	0.39	0.40	0.61
Bond with maturity in 3	0.68	0.36	0.36	0.32
Bond with maturity in 5	0.75	0.57	0.66	0.66
Bond with maturity in 10	1.46	1.70	2.23	2.52
Total bonds	**4.27**	**3.30**	**4.46**	**5.76**
Market risk	**11.42**	**10.63**	**14.98**	**18.73**
Market diversification benefit	*23.15%*	*18.20%*	*11.63%*	*6.88%*

and, starting from equation (6), the risk reserve ratio is found to be:

$$\boldsymbol{u}_t = \frac{(1+\boldsymbol{j}_t)}{(1+i)\,(1+g)}\,\boldsymbol{u}_{t-1} + \frac{\boldsymbol{j}_t\,\delta}{(1+i)\,(1+g)} + \sum_{k=1}^{t-1} \frac{1-c}{1+\lambda}\,\frac{\boldsymbol{j}_t\,\lambda}{(1+i)^k\,(1+g)^k}$$

Furthermore, the annual net cash flows are now deterministic, because their behavior only depends on a deterministic total claim amount. Starting from equation (7), they are found to be:

$$F_t = B_t\left[(1-c) + \delta\left(1 - \frac{1}{(1+i)\,(1+g)}\right)\right] - \pi_t$$

Using the annual rate of return given by the financial market model, we are able to calculate the capital requirements over a period of 1, 2, and 3 years. In doing so, we compute the result for each sub-module (i.e. investment category), and we determine the overall diversification benefit with the usual function used in the insurance industry (see Bürgi et al., 2008):

$$\mathrm{DB} = 100\% - \frac{\mathrm{RBC}_{\sum_i Y_i}}{\sum_i \mathrm{RBC}_{Y_i}}$$

The results are presented in Table 9, together with the comparison to the SF (obviously over a 1-year period only).

In order to be consistent with equation (16), the RBC of a stock investment is found to be:

$$\mathrm{RBC}_{S_\nu}(0,t) = \alpha\,\beta_\nu\,U_0 - \frac{\alpha\,\beta_\nu\,U_\varepsilon^{S_\nu}(t)}{\prod_{k=1}^{t}\left(1+\mathbb{E}\left\{\boldsymbol{j}_k^{S_\nu}\right\}\right)} \quad \text{with} \quad \nu = 1,2,3$$

where $U_\varepsilon^{S_\nu}(t)$ is the εth order quantile of a new variable, used to isolate the risk of the νth stock investment, and $\boldsymbol{j}_k^{S_\nu}$ is the corresponding stochastic annual rate of return. We have:

$$\boldsymbol{U}_t^{S_\nu} = \left(1+\boldsymbol{j}_t^{S_\nu}\right)\boldsymbol{U}_{t-1}^{S_\nu} + \boldsymbol{j}_t^{S_\nu}\,\delta\,B_{t-1} + \boldsymbol{j}_t^{S_\nu}\sum_{k=1}^{t-1}\lambda\,\pi_k$$

On the other hand, the RBC of a bond investment is found to be:

$$\mathrm{RBC}_{P_w}(0,t) = (1-\alpha)\,\gamma_w\,U_0 - \frac{(1-\alpha)\,\gamma_w\,U_\varepsilon^{P_w}(t)}{\prod_{k=1}^{t}\left(1+\mathbb{E}\left\{\boldsymbol{j}_k^{P_w}\right\}\right)} \quad \text{with} \quad w = 1, 2, 3, 5, 10$$

where the variables involved are analogous to the stock case, and they can be found by replacing the character S_v with P_w. We point out that the stochastic annual rates of return of the stock and bond investments can be obtained by using equations (8) and (9).

We rely on some key assumptions to interpret the pattern of the capital requirements in our IM. On one side, we have the riskiness of the investment. On the other side, we have several forms of expected profit, i.e. the real-world return on the investments, the profit produced by the investment of the claims reserve and safety loadings and finally the real-world discounting effect. The higher the risk, the bigger the RBC. The higher the expected profit, the smaller the RBC. Furthermore, the dynamic portfolio assumption (i.e. the presence of a claims inflation rate and a real growth rate) has a scaling effect on the RBC, because it raises the size of the insurance company. We point out that not only do the capital requirements depend on the riskiness of the investment, but also on the amount invested, hence the RBC of the stock investments is bigger in the case of the low-risk stocks. The RBC of the bond investments increases as the time to maturity increases. Furthermore, for the first year, the 1-year zero-coupon bond factor is not stochastic, because the initial zero-coupon bond price is given, and after 1 year it must equal one (we remind that there is no default risk). Despite this, there is an expected loss, due to the investment of the claims reserve and safety loadings at a negative interest rate (−0.540%). Consequently, we can observe a strong increase in the RBC between the first and second year. Moreover, the RBC of the stock investments have a slow growth over time. This is because the expected profit of the stock investments is quite high in a real-world framework (e.g. the risk premium coefficient ranges from 2.7% to 4.5%). In conclusion, the RBC of each investment category, as well as the overall one, increases over time, because the risk increases more than the expected profits.

We remind that stock investments are considered as type 1 equities without a strategic nature and a long-term holding strategy and that the symmetric adjustment is null. Hence, in the SF framework, the SCR for equity risk, calculated on a single stock investment, is found to be:

$$\mathrm{SCR}_{S_v} = 39\%\,A_0^{S_v} \quad \text{with} \quad v = 1, 2, 3$$

and the SCR for interest rate risk, calculated on a single bond investment, is found to be:

$$\mathrm{SCR}_{P_w} = A_0^{P_w} - A_0^{P_w}\left[1+R_a^{\mathrm{euro}}(0,w)\right]^{w}\left[1+R_a^{\mathrm{eiopa\ up}}(0,w)+R_a^{\mathrm{euro}}(0,w)-R_a^{\mathrm{eiopa}}(0,w)\right]^{-w}$$

$$\text{with} \quad w = 1, 2, 3, 5, 10$$

where $R_a^{\mathrm{euro}}(0,w)$, $R_a^{\mathrm{eiopa}}(0,w)$, and $R_a^{\mathrm{eiopa\ up}}(0,w)$ are respectively the average of all spot Euro Area government rates (annually compounded) with maturity w and the spot Eiopa risk-free rates (annually compounded) without Volatility Adjustment (VA) and with maturity w, either without any shock or with positive interest rate shock.

The average of all Euro Area government rates and the Eiopa risk-free rates without VA on September 30, 2020, are presented in Table 10. We point out that in the SF calculation, we assume that the spread remains equal to its value before the interest rate shock.

Furthermore, we remind that the scenario of the interest rate risk sub-module only affects the bond investments, and the scenario of the equity risk sub-module only affects the stock investments. Hence, the SCR for equity risk is found to be:

$$\mathrm{SCR}_{\mathrm{equity}} = \sum_{v=1}^{3}\mathrm{SCR}_{S_v} = 39\%\,A_0^{S}$$

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Table 10. Average of all Euro Area government rates and Eiopa risk-free rates without VA on September 30, 2020 (annually compounded and expressed in %)

Maturity in years	Average government rate	Eiopa rate	Eiopa rate – Shock up
1	−0.539	−0.578	0.422
2	−0.513	−0.587	0.413
3	−0.480	−0.575	0.425
5	−0.389	−0.528	0.472
10	−0.079	−0.331	0.669

and the SCR for interest rate risk is found to be:

$$\mathrm{SCR}_{\text{interest rate}} = \sum_{w \in \{1,2,3,5,10\}} \mathrm{SCR}_{P_w}$$

The equity risk and interest rate risk are the only sources of market risk we have. Hence, under an assumption of null correlation, the SCR for market risk is found to be:

$$\mathrm{SCR}_{\text{market}} = \sqrt{\mathrm{SCR}^2_{\text{equity}} + \mathrm{SCR}^2_{\text{interest rate}}}$$

The capital requirements of the SF and our IM are quite similar, because of their similar assumptions. In general, the capital requirements of the bond investments are bigger in the SF than in our IM, as are the overall capital requirement. We indeed remind that in our IM the asset allocation is recalibrated at year end only, hence we do not suffer from changes in the market value during the year. In addition, the SF does not consider any form of expected profit in its calculation. The diversification benefits are slightly bigger in the SF than in our IM. We indeed remind that in our IM we model the tail dependence between stock and bond factors using Student's t copulas, while in the SF, the SCR for market risk is obtained through a linear correlation aggregation process.

3.1.2 Non-life premium risk

We now isolate the effect of the premium risk, and so we leave aside the market risk. In this regard, we drop the investment result obtained by the risk reserve, assuming that the annual rate of return is null. Once again, we believe that this is not a simplification and is a reasonable solution to make a stand-alone analysis of the premium risk with respect to the market risk. As a result, the risk reserve is found to be:

$$\boldsymbol{U}_t = \boldsymbol{U}_{t-1} + \left[(1+\lambda)\, \pi_t - \boldsymbol{X}_t \right]$$

and the risk reserve ratio is found to be:

$$\boldsymbol{u}_t = \frac{1}{(1+i)\,(1+g)}\, \boldsymbol{u}_{t-1} + \frac{1-c}{1+\lambda} \left[(1+\lambda) - \frac{\boldsymbol{X}_t}{\pi_t} \right]$$

Using the total claim amount given by the Collective Risk Model, we are also able to calculate the capital requirements over a time span of 1, 2, and 3 years. The results are presented in Table 11, together with the comparison to the SF.

In order to understand the pattern of the capital requirements in our IM, we should now focus on the riskiness of the total claim amount and on the expected profit produced by the safety loadings. The first one contributes to a bigger RBC, and the second one contributes to a smaller RBC. Again, the dynamic portfolio assumption has a scaling effect on the RBC. The RBC is bigger than

Table 11. Single-line capital requirements over the initial GPW (expressed in %) under the SF and our IM over a period of 1, 2, and 3 years

	Year 1 (SF)	Year 1 (IM)	Year 2 (IM)	Year 3 (IM)
Market risk	11.42	10.63	14.98	18.73
NL Premium risk	**31.06**	**21.63**	**29.66**	**36.47**

in the previous subsection. It is quite common that in non-life insurance the capital requirements for underwriting risk are quite big, because the business is mainly devoted to the underwriting side. However, in our numerical analysis the capital requirements for premium risk are much higher than for market risk. This is because we have a quite risky insurance portfolio resulting from the upper tail of the total claim amount distribution being quite heavy. Furthermore, the investment portfolio is composed of simple assets expressed in euro currency and issued by Euro Area governments, so that many important sources of market risk (i.e. the spread risk, liquidity risk, and default risk) are not considered.

Under the SF, the SCR for non-life premium and reserve risk (in our case referring only to the premium risk) is calculated using the following simplified formula:[10]

$$\mathrm{SCR}_{\mathrm{nl\ prem\ res}} = 3\,\sigma_{\mathrm{nl}}\,V_{\mathrm{nl}}$$

where σ_{nl} and V_{nl} are respectively the volatility factor (i.e. the standard deviation, in relative terms) and the volume measure for premium and reserve risk. The latter is distinguished in earned premiums and best estimate of the provisions for claims outstanding. We remind that the reserve risk and reinsurance are not considered, and the single-line insurance company only underwrites MTPL policies. The volatility factor for premium and reserve risk is thus found to be 10%, and the volume measure for premium and reserve risk is found to be 103.53 million, i.e. the GPW at the end of the first year. Furthermore, since there are no other sources of risk, the SCR for non-life underwriting risk refers to the premium risk only.

Once again, the capital requirement is bigger in the SF than in our IM. This is because the expected profit produced by the safety loadings is not considered in the SF. Moreover, the size factor is not considered, so that in the SF the volatility factors for premium and reserve risk are assumed to be the same for each European insurance company. However, in our IM we also consider the insurance company size, coming from the expected number of claims. In addition, we remind that in the SF, the underlying distribution for premium and reserve risk is assumed to be a Lognormal, and the multiplier 3 is consistent only when the volatility factor is roughly equal to 14.5%. As previously mentioned, the assumption of a constant multiplier is flawed, because it should change as the relative volatility changes.

3.1.3 Market and non-life premium risk

In Section 3.1.1, we assumed that the underwriting result was always equal to the safety loadings and that it produced interest each year. Actually, the interest is obtained from the investment of the occurring value of the underwriting result. Finally, we take into account both the market risk and premium risk, considering equations (4) and (6).

Using the annual rate of return given by the financial market model and the total claim amount given by the Collective Risk Model, we are able to calculate the capital requirements over a period

[10]Note that until QIS 5 (i.e. the fifth Quantitative Impact Study, carried out in 2010) a very elegant and consistent solution had been adopted. It assumed the exact formula for the quantile calculation in case of a Lognormal distribution underlying the premium and reserve risk. Under QIS 5, the higher the volatility factor, the higher the multiplier. The latter was equal to 3 only when the volatility factor was roughly 14.5%. We point out that the multiplier should approach 2.58 (i.e. the multiplier of a Standard Normal distribution) when the skewness is extremely close to zero.

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Table 12. Single-line capital requirements over the initial GPW (expressed in %) and diversification benefits under the SF and our IM over a period of 1, 2, and 3 years

	Year 1 (SF)	Year 1 (IM)	Year 2 (IM)	Year 3 (IM)
Market risk	11.42	10.63	14.98	18.73
NL Premium risk	31.06	21.63	29.66	36.47
Market and NL Premium risk	**35.67**	**28.84**	**38.29**	**45.51**
Market diversification benefit	*23.15%*	*18.20%*	*11.63%*	*6.88%*
NL Premium diversification benefit	–	–	–	–
Market and NL Premium diversification benefit	*16.03%*	*10.62%*	*14.22%*	*17.55%*
Total diversification benefit	*22.32%*	*16.73%*	*17.85%*	*19.57%*

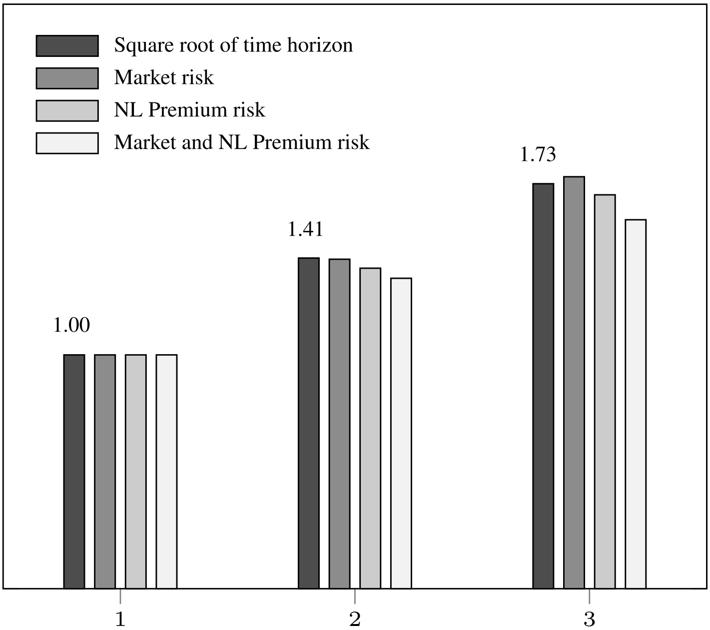

Figure 2. Percentage increase in capital requirements according to our IM over a period of 1, 2, and 3 years, against the square root of time horizon.

of 1, 2, and 3 years. The diversification benefit is calculated at module level (i.e. market risk and premium risk) and sub-module level (i.e. equity risk, interest rate risk, and premium risk), so that we can observe both the partial diversification benefit and the total one.[11] The results are presented in Table 12, together with the comparison to the SF.

It can be observed that the capital requirements (for market and premium risk both separately and combined) increase over time by approximately the square root of time horizon, as presented in Figure 2. This pattern is commonly relied upon when analyzing capital requirements with respect to time. Note that it is just a *rule of thumb*, and there are many elements (e.g. expected profits) that can invalidate it.

[11] A neat solution to calculate the diversification benefit, without using stand-alone models, is to determine the threshold of the Tail Value-at-Risk of the portfolio that gives the same result of our Value-at-Risk and to use this threshold to allocate the capital using Tasche's approach (see Boonen et al., 2020, for further details).

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Table 13. General parameters of our numerical analysis

LoB	B_0	λ	c	δ
MTPL	50,000,000	0.87%	21.24%	156.10%
MOD	25,000,000	13.81%	30.30%	22.88%
GTPL	25,000,000	6.61%	32.30%	416.52%
Total	**100,000,000**	**4.99%**	**26.27%**	**187.90%**

Under an assumption of moderate correlation (+0.25), the SCR for market and non-life underwriting risk (in our case referring only to the premium risk) is found to be:

$$\mathrm{SCR} = \sqrt{\mathrm{SCR}^2_{\text{market}} + \frac{1}{2}\,\mathrm{SCR}_{\text{market}}\,\mathrm{SCR}_{\text{non-life}} + \mathrm{SCR}^2_{\text{non-life}}}$$

where $\mathrm{SCR}_{\text{non-life}}$ is the SCR for non-life underwriting risk.

Finally, we can observe that the total capital requirement is bigger in the SF than in our IM, mostly because of the premium risk. We remind that in our IM the high upper tail dependence between the annual rate of return and total claim amount is modeled using a rotated Gumbel copula. For this reason, the diversification benefits according to our IM are smaller than for the SF, which is based on a linear correlation aggregation process.

3.2 Multi-line insurance company

We now investigate the case in which a similar insurance company has underwriting business, not only in MTPL but also in Motor Other Damages (MOD) and General Third-Party Liability (GTPL). The GPW is the same as in the single-line case (100 million), and the business mix is 50% for MTPL, 25% for MOD, and 25% for GTPL. For this reason, we use Gaussian or Gumbel copulas to inject some dependence structure in the total claim amount of the three different LoBs. This step is not necessary in case of a single-line insurer. We point out that for many results within this subsection, the same comments apply as applied to the single-line insurance company; hence, we focus only on the main developments and differences.

Almost all the parameters are the same as in the case of the single-line insurance company, and the new ones are estimated by the same procedure (see IVASS 2020, in this case). As a result, the parameters of MTPL are identical to those used for the single-line insurer. Furthermore, we maintain the same claims inflation rate i and real growth rate g for each single LoB, even if empirically they might differ.

The new general parameters of our numerical analysis are presented in Table 13. The GPW is now found to come from three different sources, and the safety loading coefficient of MOD (13.8%) is much bigger than in the other two LoBs (0.9% and 6.6%). Furthermore, the ratio of claims reserve to GPW of MOD is very low (22.9%), since the claims settlement speed is fast. On the other hand, the same ratio for GTPL is very high (416.5%), since the settlement speed is considerably lower. This is because MOD deals only with material damages to property, whereas GTPL (as well as MTPL) deals with a significant portion of bodily injuries, from which a long settlement process derives. As a result, the initial claims reserve of the insurance company is found to be equal to 187.9 million (mainly affected by the ratio of claims reserve to GPW of GTPL), and together with 25 million of initial risk reserve we obtain an initial asset value of the portfolio equal to 212.9 million.

The new main indicators of the distributions of both the single claim amount and number of claims are presented in Table 14 for each LoB, while the new parameters of the Lognormal and Negative Binomial distributions are presented in Table 15. The coefficient of variation of the single claim amount $\mathbb{CV}\{\boldsymbol{Z}_0\}$ and the standard deviation of the structure variable $\mathbb{S}\mathrm{td}\{\boldsymbol{q}\}$ are the main

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Table 14. Main indicators of the distributions of the single claim amount and number of claims

LoB	$\mathbb{E}\{\boldsymbol{Z}_0\}$	$\mathbb{CV}\{\boldsymbol{Z}_0\}$	n_0	$\mathbb{S}\text{td}\{\boldsymbol{q}\}$
MTPL	4,000	7	9,760	0.0821
MOD	2,500	2	6,124	0.0500
GTPL	10,000	12	1,587	0.1478

Table 15. Parameters of the Lognormal and Negative Binomial distributions

LoB	m_0	s	h	p_0
MTPL	6.3380	1.9779	148.47	0.0150
MOD	7.0193	1.2686	399.40	0.0612
GTPL	6.7220	2.2309	45.78	0.0280

Table 16. Parameters of the Gaussian copula for the aggregate total claim amount

MTPL and MOD	MTPL and GTPL	MOD and GTPL
+0.5000	+0.5000	+0.2500

Table 17. Parameters of the Gumbel copula for the aggregate total claim amount

Year	MTPL and MOD	MTPL aggregated to MOD and GTPL
1	1.5000	1.4893
2	1.5000	1.4893
3	1.5000	1.4892

risk drivers, together with the expected number of claims n_0, which determines the LoB size. We have the highest values for GTPL, because the large portion of claims relating to bodily injuries makes the segment rather volatile compared to the others. The coefficient of variation of MOD is quite small (2), with respect to GTPL (12) and MTPL (7). Furthermore, the standard deviation of the structure variable plays a significant role, even if the LoB sizes are limited, since it is around 15% for GTPL, 8% for MTPL, and 5% for MOD, and it affects the asymptotic behavior of the total claim amount volatilities.

The parameters of the Gaussian copula, used to inject some dependence structure in the total claim amounts, are presented in Table 16. They are assumed to be equal to the linear correlation coefficients between the three LoBs, as reported in the SF for premium and reserve risk and defined in the Commission Delegated Regulation (EU) 2015/35 (2015).

The parameters of the Gumbel copula are presented in Table 17. As proposed by Savelli and Clemente (2011), we here consider a hierarchical structure in which we join MTPL and MOD at the first step, and we add GTPL at the second step. The first column of parameters is estimated by using the Kendall's rank correlation coefficient corresponding to the linear correlation coefficient of the Gaussian copula mentioned above. The second column of parameters is estimated by using the Kendall's rank correlation coefficient corresponding to the implicit linear correlation coefficient between GTPL and the sum of MTPL and MOD. This is done in order to have the same linear correlation structure with or without a tail dependence, so that we can make a proper comparison between the Gaussian copula, Gumbel copula, and SF.

The distribution of the annual rate of return is the same as in the case of the single-line insurer. Figure 3 shows the simulated distribution of the total claim amount of each LoB after 1, 2, and

Table 18. Multi-line descriptive statistics of the simulated total claim amounts after 1, 2, and 3 years (amounts in millions)

LoB	Year	Mean	St. dev.	Skew.	Min.	1st qu.	Median	3rd qu.	Max.
MTPL	1	40.39	4.40	1.0170	24.42	37.42	40.14	43.05	166.19
MTPL	2	41.82	4.55	1.3330	25.29	38.76	41.55	44.58	214.07
MTPL	3	43.32	4.76	3.6336	27.17	40.14	43.04	46.15	340.53
MOD	1	15.85	0.91	0.1156	11.85	15.22	15.83	16.45	19.95
MOD	2	16.41	0.94	0.1224	12.48	15.77	16.39	17.03	21.32
MOD	3	16.99	0.97	0.1207	13.11	16.33	16.97	17.63	21.70
GTPL	1	16.44	6.20	42.6100	5.53	13.32	15.63	18.46	976.36
GTPL	2	16.98	5.49	6.6964	5.55	13.80	16.18	19.08	250.63
GTPL	3	17.60	5.45	4.7099	5.28	14.30	16.78	19.80	253.37

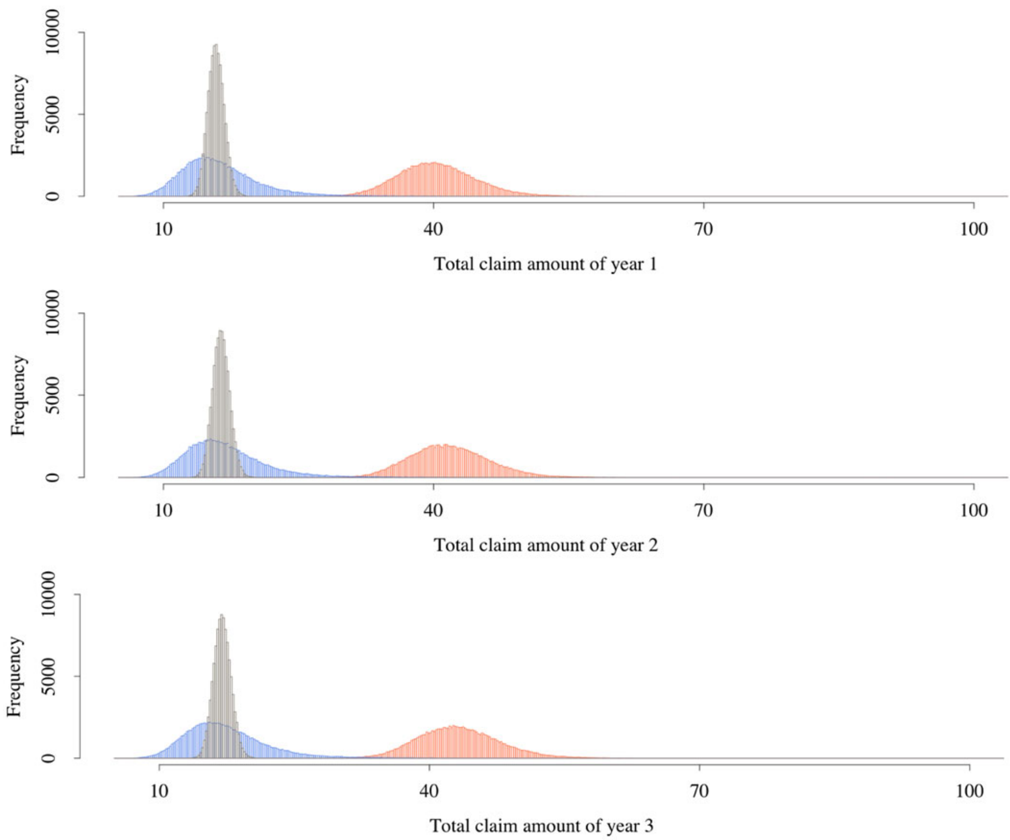

Figure 3. Multi-line simulated total claim amounts after 1, 2, and 3 years (MTPL in red, MOD in black, GTPL in blue, and *x*-axis values in millions).

3 years, and Table 18 shows some descriptive statistics. Table 19 shows the same indicators for the simulated distribution of the aggregate total claim amount given by Gaussian and Gumbel copulas.

As shown in the figures, the mean and standard deviation of the total claim amount of each LoB rise slightly over time, because of a limited increase in the GPW (a nominal growth rate around 3.5% for all the LoBs). It is also noted that the distribution of MOD is heavily concentrated around

Table 19. Multi-line descriptive statistics of the simulated aggregate total claim amount given by the Gaussian copula and by the Gumbel copula after 1, 2, and 3 years (amounts in millions)

Year	Mean	St. dev.	Skew.	Min.	1st qu.	Median	3rd qu.	Max.
Gaussian copula								
1	72.65	9.13	11.8311	46.95	67.04	71.81	77.12	964.42
2	75.24	8.93	2.4206	49.33	69.47	74.39	79.90	320.53
3	77.91	9.10	2.2536	49.60	71.96	77.05	82.69	382.94
Gumbel copula								
1	72.63	8.94	3.6824	47.92	66.98	71.51	76.85	437.72
2	75.30	9.51	5.1145	49.96	69.50	74.14	79.57	456.19
3	77.94	9.28	2.6501	53.51	71.97	76.78	82.42	405.32

the mean compared to GTPL, notwithstanding that they have a similar volume. This is mainly due to the coefficient of variation of the single claim amount (2 and 12 respectively). Besides, it can be seen from the values in the table that the skewness trend is affected by some outliers. It is evident at the third year for MTPL and, more significantly, at the first year for GTPL (in order to better understand, it is useful to see the maximum value in the last column of the table). In any case, the skewness realized for the three LoBs is quite different. It is very significant for GTPL (around $+5$), it has a medium value for MTPL (around $+1$), and it is less significant for MOD (around $+0.1$). The skewness is mainly influenced by the single claim amount distribution, due to the quite small portfolio size, in particular for GTPL. In case of bigger insurance companies, all these skewness values would be closer to the skewness of the structure variable. In any case, these skewness values will be helpful to properly understand the effective multipliers of our IM, in comparison to the SF.

Using the annual rate of return given by the financial market model, the aggregate total claim amount given by the Collective Risk Model, and copula functions, we are now able to calculate the capital requirements over a period of 1, 2, and 3 years. The results are presented in Table 20, together with the comparison to the SF.

Considering market risk, the main difference compared to the single-line insurance company is that the invested resources are now larger, mainly because of the higher initial claims reserve (187.9 million against 156.1 million of the single-line insurer, which corresponds roughly to $+20\%$). This is the reason we now have higher capital requirements, both in our IM and in the SF. As already explained, the main difference between our IM and the SF is that only in the former is expected profit considered, represented by the real-world return on the investments and by the interest on the investment of the claims reserve and safety loadings. The European insurance supervisory authorities are reluctant to allow insurance companies to include an expected profit or loss in their IM for capital requirement calculation, because of the desire for consistency with the SF framework. Even if we believe that a prudent approach is suitable and desirable, we also think that a correct quantification of the solvency position of an insurance company should consider not only the sources of risk but also the risk mitigation elements (at least partially). Another difference between our IM and the SF is that the latter does not take account of the peculiarity of the investments. For instance, under the SF, all equity instruments of a certain type and with a particular characteristic (e.g. strategic nature) have equal capital requirements. Accordingly, we remind that obviously the SF is a simplified and standardized approach, and an IM can be used to better fit the various characteristics of a particular portfolio.

Focusing on the premium risk, in our IM the RBC of each LoB increases over time, except for MOD, in which the limited risk is more than compensated by the significant expected profit created by the safety loadings. The aggregate total claim amount is now riskier with respect to the single-line insurance company, because of the presence of GTPL, and it is even riskier adopting the Gumbel copula in the aggregation process, where there is a high upper tail dependence

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Table 20. Multi-line capital requirements over the initial GPW (expressed in %) and diversification benefits under the SF and our IM over a period of 1, 2, and 3 years

	Year 1 (SF)	Year 1 (IM)	Year 2 (IM)	Year 3 (IM)
Stock 1	7.47	6.09	8.03	9.43
Stock 2	3.74	3.80	4.86	5.56
Stock 3	1.25	1.49	1.87	2.09
Total stocks	**12.45**	**11.38**	**14.76**	**17.09**
Bond with maturity in 1	0.72	0.35	0.98	1.99
Bond with maturity in 2	0.90	0.46	0.48	0.74
Bond with maturity in 3	0.80	0.43	0.42	0.39
Bond with maturity in 5	0.88	0.67	0.78	0.78
Bond with maturity in 10	1.72	2.00	2.64	3.00
Total bonds	**5.02**	**3.91**	**5.30**	**6.91**
MTPL	15.53	13.01	18.27	21.89
MOD	6.21	0.27	−0.96	−2.43
GTPL	10.87	21.08	28.74	33.94
Total LoBs	**32.61**	**34.36**	**46.05**	**53.41**
Gaussian copula for NL Premium risk				
Market risk	**13.43**	**12.51**	**17.71**	**22.28**
NL Premium risk	**26.40**	**27.69**	**35.97**	**40.73**
Market and NL Premium risk	**32.47**	**35.67**	**44.70**	**52.14**
Market diversification benefit	*23.15%*	*18.20%*	*11.71%*	*7.12%*
NL Premium diversification benefit	*19.05%*	*19.40%*	*21.89%*	*23.74%*
Market and NL Premium diversification benefit	*18.47%*	*11.27%*	*16.73%*	*17.25%*
Total diversification benefit	*35.16%*	*28.16%*	*32.39%*	*36.63%*
Gumbel copula for NL Premium risk				
Market risk	**13.43**	**12.51**	**17.71**	**22.28**
NL Premium risk	**26.40**	**31.11**	**39.82**	**45.91**
Market and NL Premium risk	**32.47**	**38.90**	**48.89**	**56.37**
Market diversification benefit	*23.15%*	*18.20%*	*11.71%*	*7.12%*
NL Premium diversification benefit	*19.05%*	*9.44%*	*13.54%*	*14.03%*
Market and NL Premium diversification benefit	*18.47%*	*10.83%*	*15.01%*	*17.34%*
Total diversification benefit	*35.16%*	*21.66%*	*26.05%*	*27.16%*

(consequently a smaller diversification benefit and higher capital requirements). However, it is noted that under the SF, the total SCR is smaller than in case of the single-line insurer (we now have the same volume measure as before, but a smaller volatility factor, equal to 8.5%). It is also important to see that, in contrast to the other LoBs, the SCR of GTPL is lower in the SF than in our IM. Overall, the numerous differences seen between our IM and the SF are due to the simplified logic behind the latter approach. In contrast with the SF, in our IM we indeed calibrated the parameters on specific data, considering the size factor and using an aggregation procedure different to the linear correlation one. Moreover, as previously mentioned, the multiplier 3 used in the SF for the premium and reserve risk is not fully consistent, because it is fixed regardless of the level of volatility. For this formula, EIOPA declared an underlying Lognormal distribution. In this case, when the volatility is around 14.5% we have a multiplier equal to 3, otherwise the appropriate multiplier would be different. The drawback of this formula is that it does not respect the relationship satisfied by the Lognormal distribution. In particular, when we have a quite low

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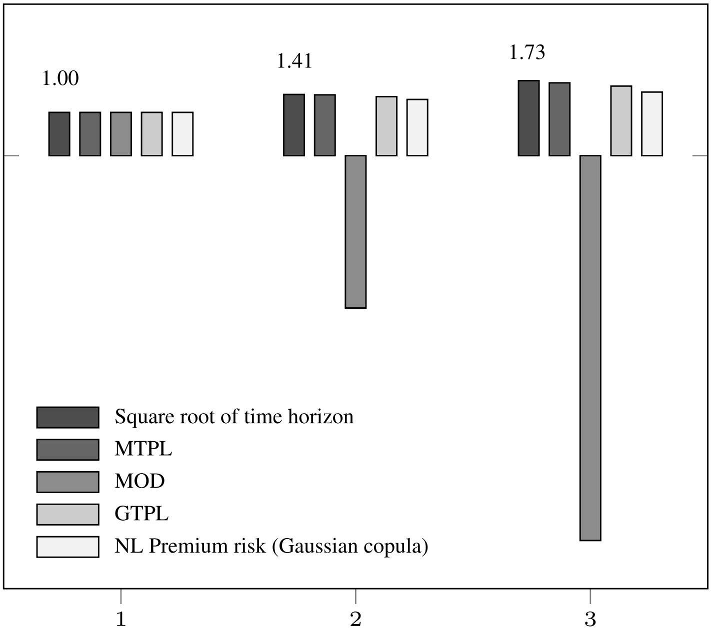

Figure 4. Percentage increase in capital requirements according to our IM over a period of 1, 2, and 3 years, against the square root of time horizon.

volatility, the skewness is small, and then the multiplier will be closer to 2.58, which is the quantile of order 99.5% of a Standard Normal distribution (e.g. with a volatility of 5% the multiplier is equal to 2.72). On the other side, with a higher volatility, the skewness is bigger, and then the multiplier is larger (e.g. with a volatility of 25% the multiplier is equal to 3.32). Clearly, in the first case the SF overestimates the capital requirement, whereas in the second case, it underestimates it. In our IM the multipliers for the first year are equal to 3.05 for MTPL, 2.71 for MOD, and 3.57 for GTPL. Hence, the SF creates an inconsistent benefit in the first and third case. In addition, the SF does not allow expected profit to be considered, for example that produced by the safety loadings. All the limitations described above can be clearly solved by implementing an IM as we have outlined in this paper.

Finally, in Figure 4 we show the yearly percentage increase of the capital requirements for premium risk, against the square root of time horizon. We can observe that for MTLP and GTPL the expected trend is again confirmed. However, this is not the case for MOD, because its capital requirement decreases, in the opposite direction from the square root of time horizon. This is because in MOD we have a highly significant expected profit, that over time is higher than the risk of this LoB. Consequently, the total capital requirement for premium risk (for simplicity, in the figure there is the case of Gaussian copula only) is not fully consistent with the above *rule of thumb*. This is one situation in which the proxy described above is not supported, hence care must be taken to merely use this as an indicator.

4. Conclusion

In this paper, we showed that as the cash flows produced by the insurance business are invested, they consequently create a risk, above the underwriting risk (here represented by the premium risk only). This highlights that non-life insurers not only face underwriting risk but also market risk. More specifically, we pointed out that premium risk, equity, and interest rate are all relevant risks for the non-life insurer. Consequently, we described an approach to modeling the distributions of the annual rate of return and aggregate total claim amount, in order to calculate the capital requirements for market and premium risk and to illustrate their combined effect. We produced a numerical analysis for a single-line and a multi-line insurance company in a multi-annual dynamic perspective, using current and available market data, in order to show a realistic and heterogeneous time-dependent non-life insurance context.

In this paper, we also showed that capital requirements are clearly more demanding from a methodological point of view when an IM is applied than when using the SF. Moreover, notwithstanding that an IM must be approved by an insurance supervisory authority, the calibration is critical, because it influences the final result of the capital requirements. For this reason, supervisory authorities pay close attention to cases of so-called *model change*.

We explained the main differences between our IM and the SF according to our numerical analysis, where market and underwriting risks are examined in connection to each other. In particular for our single-line MTPL insurance company, the SF results in higher capital requirements than our IM (35.7% against 28.8% as percentages of the initial GPW, see Table 12). This is caused by three main reasons. Firstly, they are given by the higher volatility factor for premium risk (10% for MTPL in the SF), secondly by a more conservative approach regarding the expected profits (not counted as a mitigation of risk in the SF), and thirdly by the fixed multiplier in the SF, irrespective of the relation between the skewness and volatility underlying the Lognormal distribution assumption.

Compared to the single-line insurer, the multi-line insurance company (having the same GPW) has higher market risk for both the SF and our IM, because of larger investment resources (in particular, the initial claims reserve) given by the GTPL characteristic of extremely high ratio of claims reserve to GPW (more than 400%).

Considering the premium risk, the multi-line insurance company has smaller capital requirements than the single-line insurer if the SF approach is adopted (26.4% against 31.1% as percentages of the initial GPW, see Tables 11 and 20). The significant reduction is mainly given by the diversification benefit among the different LoBs (due to the linear correlation matrix), which is absent for the single-line insurer. By contrast, for our IM the multi-line capital requirement is higher than in the single-line, because of the high volatility and skewness of GTPL, which is not counterbalanced by either the limited values registered for MOD or the diversification benefit. Consequently, in our case study, the total capital requirement of the SF is lower than our IM when using the Gaussian copula for premium risk (32.5% against 35.7% as percentages of the initial GPW, see again Table 20), while for the single-line insurance company it is the opposite (35.7% against 28.8% as percentages of the initial GPW, see again Table 12). In addition, in case of Gumbel copula for the premium risk, which is distinguished by a higher upper tail dependence than the Gaussian copula, the multi-line capital requirement in our IM increases from 35.7% to 38.9%. We remind that our IM can only be accounted as partial, because a full IM would obviously consider all the sources of risk (e.g. reserve risk, cat risk, counterparty default risk, operational risk, . . .). Moreover, the analysis was performed with simple assets expressed in euro currency and issued by Euro Area governments, which means many important sources of market risk (i.e. the spread risk, liquidity risk, and default risk) were not considered. Hence, we obtained a market risk much smaller than premium risk, unlike in practice, where market risk is often larger than underwriting risk.

Further studies can regard, for instance, the introduction of a more complex and dynamic asset allocation to better describe a real investment portfolio and to further exploit the diversification benefit of the overall market risk. Other future studies building on our research could be addressed to the Risk Appetite Framework (RAF) limits, using the IM results. We remind that in our IM no dividends are taken into consideration. Hence, it could be interesting to analyze the impact of different dynamic dividend policies on the solvency ratio and capital requirement, fixing a target zone and an extreme downside zone, where a strong limitation occurs in case the solvency ratio approaches or oversteps either 100% of the target or RAF lower bound. In addition, we believe the analysis of multi-year modeling could be very promising, in order to derive a natural proxy as a benchmark for the capital requirement calculation on a time span longer than 1 year. For instance, the square root of the time could be studied as a *rule of thumb*. In this paper, we made some studies using a period of 2 and 3 years and the same risk measure. In our analysis, we observed similar results to this *rule of thumb* for both the market risk and premium risk. The differences with respect to the square root of time horizon are mainly given by the expected profits (on both market and underwriting sides) and volume increase time by time. This might be relevant in future, for instance in case EIOPA decides to modify the capital requirement metrics, lengthening the 1-year time horizon (e.g. to a 2-year or 3-year time horizon) and introducing a multi-approach (e.g. a double approach, in which both the 1-year and multi-year risk measures are taken into account). This would necessitate an update in the confidence levels and an enhancement of risk strategies to take a medium-term view.

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